ch.8

ch.7 Prove that $f(x) = \frac{(-1)^x(4x+1)}{2}$ $f: \mathbb{Z}^{nonneg} \to \mathbb{Z}$

is injective (One-to-One).

~ if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$$2 \Big| \; \frac{(-1)^{x_1}(4x_1+1)}{\cancel{2}} = \frac{(-1)^{x_2}(4x_2+1)}{\cancel{2}}$$

$$(-1)^{x_1}(4x_1+1) = (-1)^{x_2}(4x_2+1)$$

$x_1, x_2 \in \mathbb{Z}^{noneg}$

$\{-1, 1\}$

$\Rightarrow (X)$ Suppose that x_1 is even and x_2 is odd

$$(4x_1+1) = -(4x_2+1)$$

$$4x_1+1 = -4x_2-1$$

$4x_1+4x_2 = -2$

$$\boxed{x_1+x_2 = -\frac{1}{2}}$$

Contradiction $\notin \mathbb{Z}^{noneg}$

x_1, x_2 is even $\quad 4x_1 \cancel{+1} = 4x_2 \cancel{+1}, \Rightarrow x_1 = x_2$

x_1, x_2 is odd $\quad -4x_1 - 1 = -4x_2 - 1 \Rightarrow x_1 = x_2$

Ch.8

Inverse Relation:

Def: Let R be a Relation from A to B

define inverse relation R^{-1} from B to A

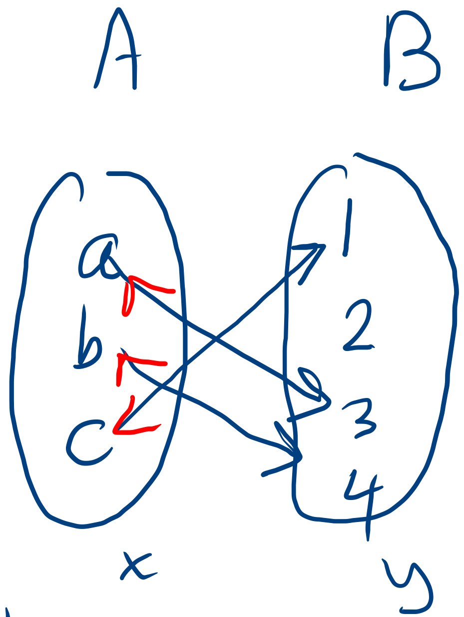
as follows:

$$R^{-1} = \{ (y,x) \in B \times A \mid (x,y) \in R \}$$

$$\forall x,y \in B \times A \,,\; (y,x) \in R^{-1} \iff (x,y) \in R$$

Ex:

R:



$R = \{(c,1), (a,3), (b,4)\}$

find R^{-1} ?

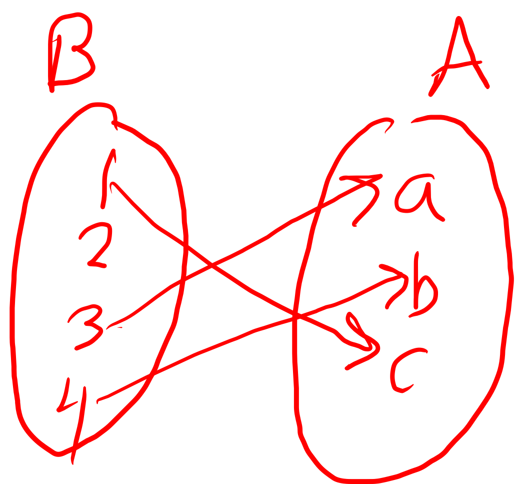
$(x,y) \in R$, xRy $\rightarrow$ $A \times B$

R^{-1} $\quad (y,x) \in B \times A \qquad (y,x) \in R^{-1}$

$\hookrightarrow (x,y) \in R$

$\hookrightarrow R^{-1} = \{(1,c), (3,a), (4,b)\}$

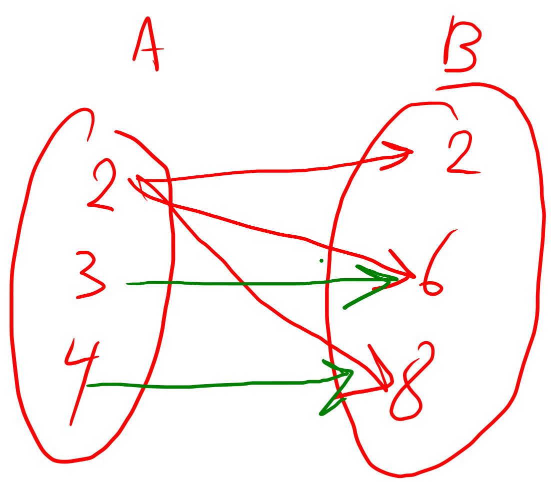
R^{-1}

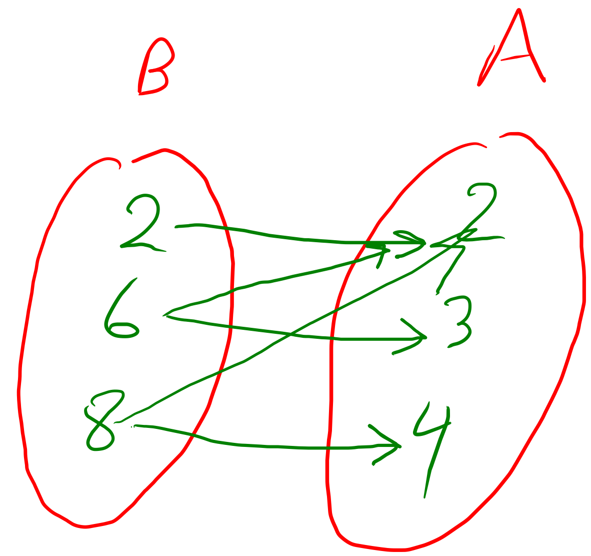


$A = \{\underset{x}{2}, 3, 4\}$ $\qquad$ $B = \{\underbrace{2, 6,}_{y} 8\}$

$A R B \iff \underline{\underline{x \mid y}}$ $\qquad R^{-1}$

Find R, R^{-1} ?





$B R A$

y is multiple of x

$R = \{(2,2), (2,6), (2,8), (3,6), (4,8)\}$

$2 = 2 \times 1$

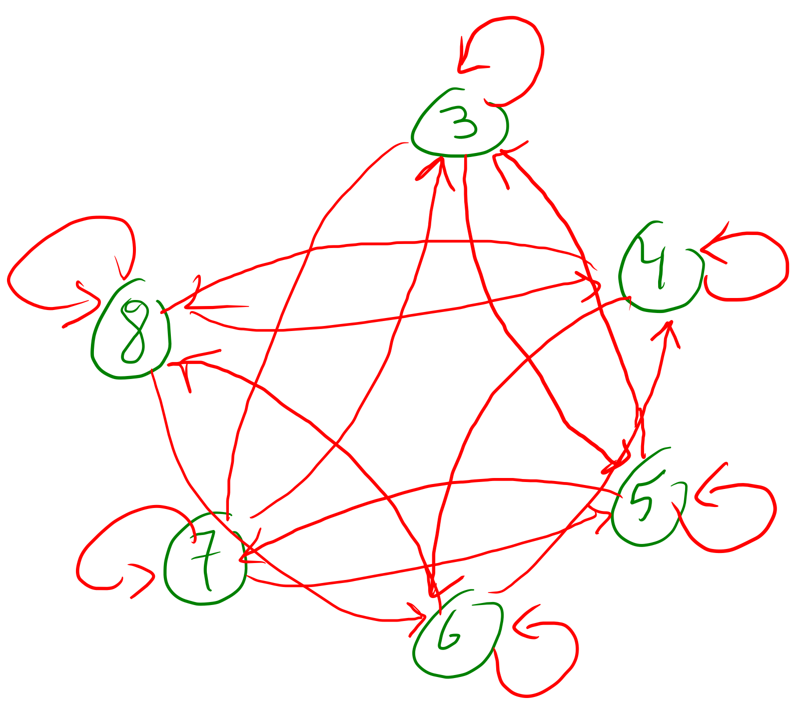
$6 = 3 \times 2,\ 2 \times 3$

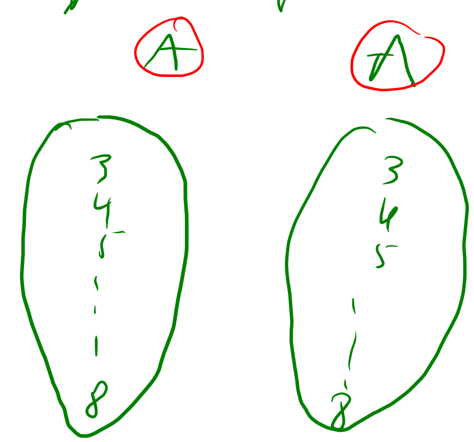
$8 = 2 \times 4,\ 4 \times 2$

Def: A relation on a set A is a relation from A to A.
[direct graph].

Ex: $A = \{3, 4, 5, 6, 7, 8\}$, define Relation R as follows: $\forall (x, y) \in A$, $x \, R \, y \iff 2 \mid (x - y)$

Draw direct graph





$\frac{0}{2} = 0$

x y

$(3, 5)$ $\quad 3 - 5 = \frac{-2}{2} = -1$

$(5, 3)$ $\quad 5 - 3 = \frac{2}{2} = 1$

N-ary Relational DB: ?

No Question

Binary Relational

3-ary, 4-ary - - - - Fields

Table

A_1	A_2	A_3	A_4
ID	Name	Date	Diagnosis
100	Ali	010220	asmtha
200	Alma	07/1/20	-
300	Sad		
100	Ali	01120	- - -

S

010320

$A_1 \times A_2 \times A_3 \times A_4$

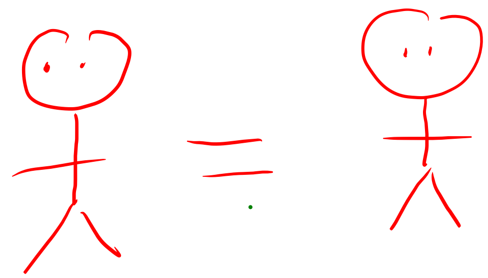
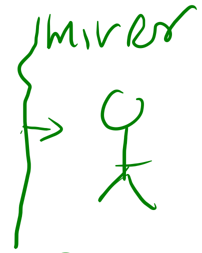
$A_1 \times A_2 \times \{010320\} \times A_4$

Intersection

Select ID, Name
From table_S
where date = '010320'

Project → ID, Nam

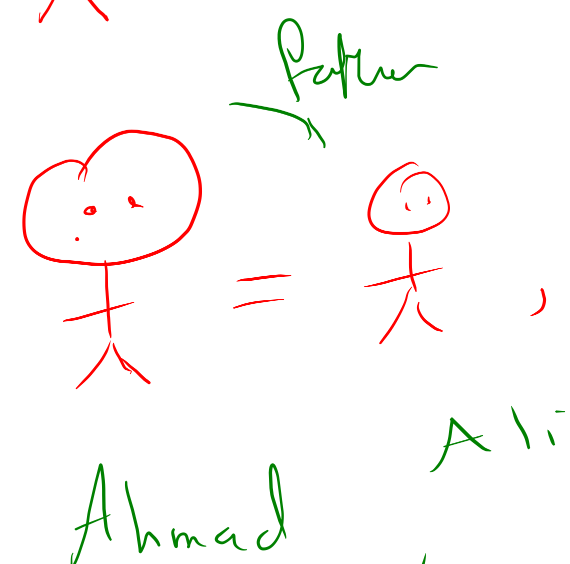
Reflexive, Symmetric, Transitive

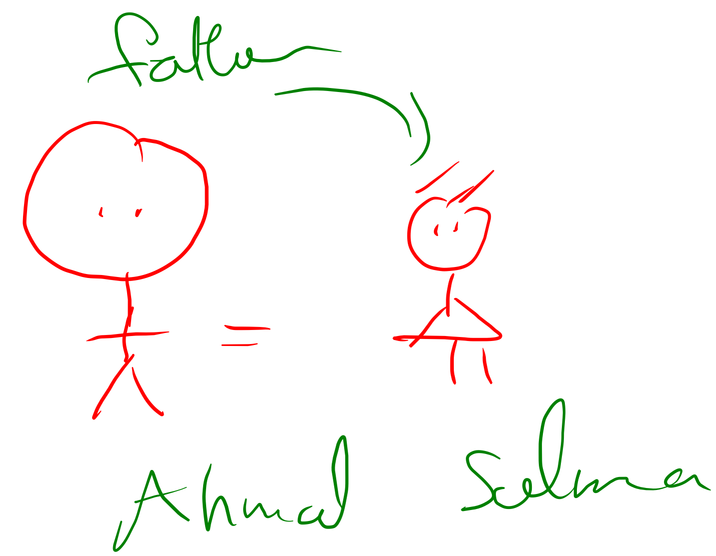
1)  $\Rightarrow$ 

$\Rightarrow$ $\boxed{x = x}$

2) 

$\Rightarrow$ $x = y$, $y = x$

3) 

$\therefore$


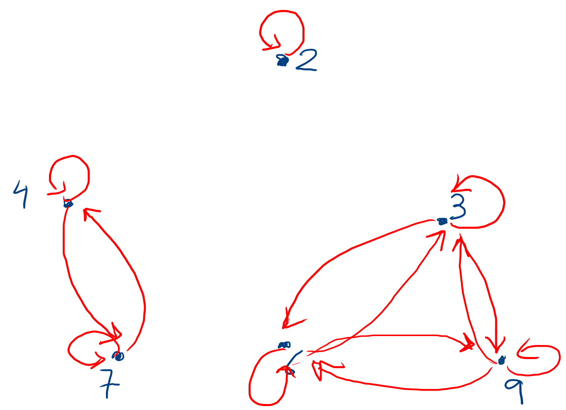
$\hookrightarrow$ $x = y$, $y = z$ $\therefore$ $x = z$

Ex: Let $A = \{2, 3, 4, 6, 7, 9\}$

$xRy \iff 3 \mid (x-y)$. Is this Relation
- Reflexive
- Symmetric
- Transitive ?

Ex: $(3,9), (9,6)$ (x y, y z) $\to (3,6)$ ✓

Solution: [Draw direct graph]



pair

Reflexive: R is reflexive, $x \in A$, xRx. $\forall x \in A$, $(x,x) \in R$

Yes $\{(2,2), (3,3), (4,4), (7,7), (6,6), (9,9)\}$ ✓

Symmetric: R is Symmetric, $(x,y) \in R$, xRy; $(y,x) \in R$

yRx.

$\forall (x,y) \in A$, if xRy then yRx.

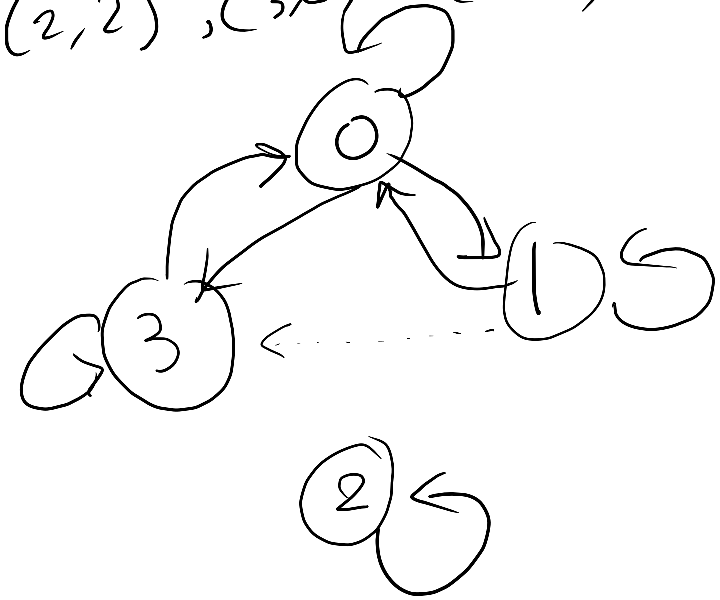
$\{(3,6), (6,3), (3,9), (9,3), (6,9), (9,6), (4,7), (7,4)\}$ ✓

Transitive: R is transitive $(x,y) \in R$, $(y,z) \in R$ then $(x,z) \in R$

$\forall x, y, z \in R$ if xRy, and yRz then xRz.

Let $A = \{0, 1, 2, 3\}$ define

A) $R = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$ is R {Reflexive, Symmetric, Transitive}?



Reflexive : Yes $\{(0,0), (1,1), (2,2), (3,3)\}$ ✓

Symmetric : Yes $\{(0,1), (1,0), (3,0), (0,3)\}$

Transitive : ~~Yes~~ $\{(\overset{x}{1}, \overset{y}{0}), (\overset{y}{0}, \overset{z}{3})$, but $(1,3)$ ↳ $1 \not R 3$

NOT transitive

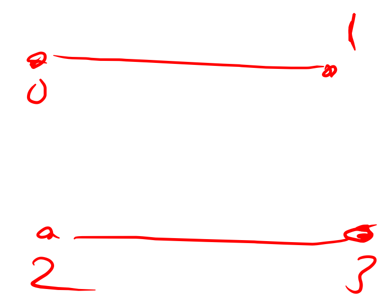
$\{(3,0), (0,1)$, $3 \not R 1$

$S = \{(0,0), (\overset{x}{0},\overset{y}{2}), (0,3), (\overset{y}{2},\overset{z}{3})\}$ $\qquad (x,y) \in R \,;\, (y,x) \in R$

Reflexive: Not $\{\, 2 \not R\, 2 ,\ 3 \not R\, 3 ,\ 1 \not R\, 1 \,\}$

Symmetric: NOT $\{\, (0,2) \in R ,\ (2,0) \notin R \,\}$

Transitive: <u>Yes</u>



$T = \{ (0,1), (2,3) \}$

Reflexive: NO $0 \not R\, 0,\ 1 \not R\, 1,\ 2 \not R\, 2,\ 3 \not R\, 3$

Symmetric: NO $2 R 3$, but $3 \not R\, 2$

transitive: <u>Transitive</u> ✓