Ex:

$m D n \Longleftrightarrow 3 \mid m^2 - n^2$

$\Rightarrow$ we proved // equivalence relation.

$\Rightarrow$ find classes.

$[a] = \{x \in A \mid xRa\}$

$[a] = \{x \in \mathbb{Z} \mid xDa\}$

$= \{x \mathbb{Z} \mid 3 \mid x^2 - a^2\}$

$= \{x \in \mathbb{Z} \mid \underbrace{x^2 - a^2 = 3q}\}$

$=$

from Division theorem

$m \in \mathbb{Z} \Rightarrow m = 3q + r$

$0 \leq r < 3$

$\{3q+0, 3q+1, 3q+2\}$

theorem $m = 3q + 0$ $\quad m \in \mathbb{Z},\ m = 3q+1$

$m = 3q + 1$ $\qquad m = 3q + 2$

$m^2 - 0^2 = 3(t)$

$(3q)^2 = 9q^2$

$= 3(3q^2) + 0$

t

$m^2 - 0^2 = 3(3q^2)$

$\therefore 3 \mid m^2 - 0^2$

✓ [0]

$m^2 = (3q + 1)^2$

$= 9q^2 + 6q + 1$

$= 9q^2 + 6q + 1$

$m^2 = 3(3q^2) + 3(2q) + 1$

$m^2 - 1^2 = 3[(3q^2) + 2q]$

$\therefore 3 \mid m^2 - 1^2$

[1]

$m = 3q + 2$ $\boxed{a = 2}$

distinct classes

$[0], [1]$

$m^2 = (3q + 2)^2$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3.3q^2 + 3.4q + 3 + 1$

$m^2 = 3[3q^2 + 4q + 1] + 1$

$m^2 - 1^2 = 3[w]$

$\therefore 3 \mid m^2 - 1^2$ $\therefore [2] = [1]$

Ch. 9

Counting & Probability

Sample space: is a set of all possible outcomes of a random process or experiment.

An Event: is a subset of Sample space

Coins Sample $\Omega = \{(H,H), (T,T), (H,T), (T,H)\}$

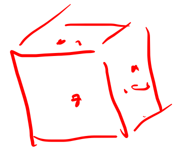
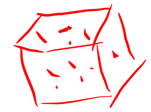
$(4) = 2 \times 2$

Event $((H,H)) = 1$, $E(T,T) = 1$ 2

$P = \frac{N(E)}{N(S)} = \frac{1}{4}$, $P_2 = \frac{1}{4}$ $P_3 = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{2} = 1$

dice

(6) (6)

Sample Space = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2) - - - - - (2,6)
(3,1), (3,2) - - - - - (3,6)
⋮
(6,1) - - - - - - (6,6) }

$N(S) = 6 \times 6 = 36$ Equally likely

(1) Find probability that showing up face of dice have the sum 6 :

Event = { (1,5), (2,4), (3,3), (4,2), (5,1) }

$N(E) = ?$

$$P(E) = \frac{N(E)}{N(S)} = \frac{5}{36}$$

$m \quad , n$

$m, n \in \mathbb{Z}$

5 6 7 8 9 10 11 12

m - - - - - - - - n

$m \leq n$

5 12

m n

Theorm: $n - m + 1$

$(n - m + 1) = 12 - 5 + 1 = 8$

Question: 100 - - - - 999

$(999 - 100) + 1 = 900$

divisable by 5.

Numbers of all number are divisable by 5.

$$P(E) = \frac{N(E)}{N(S)} = \frac{N(E)}{900}$$

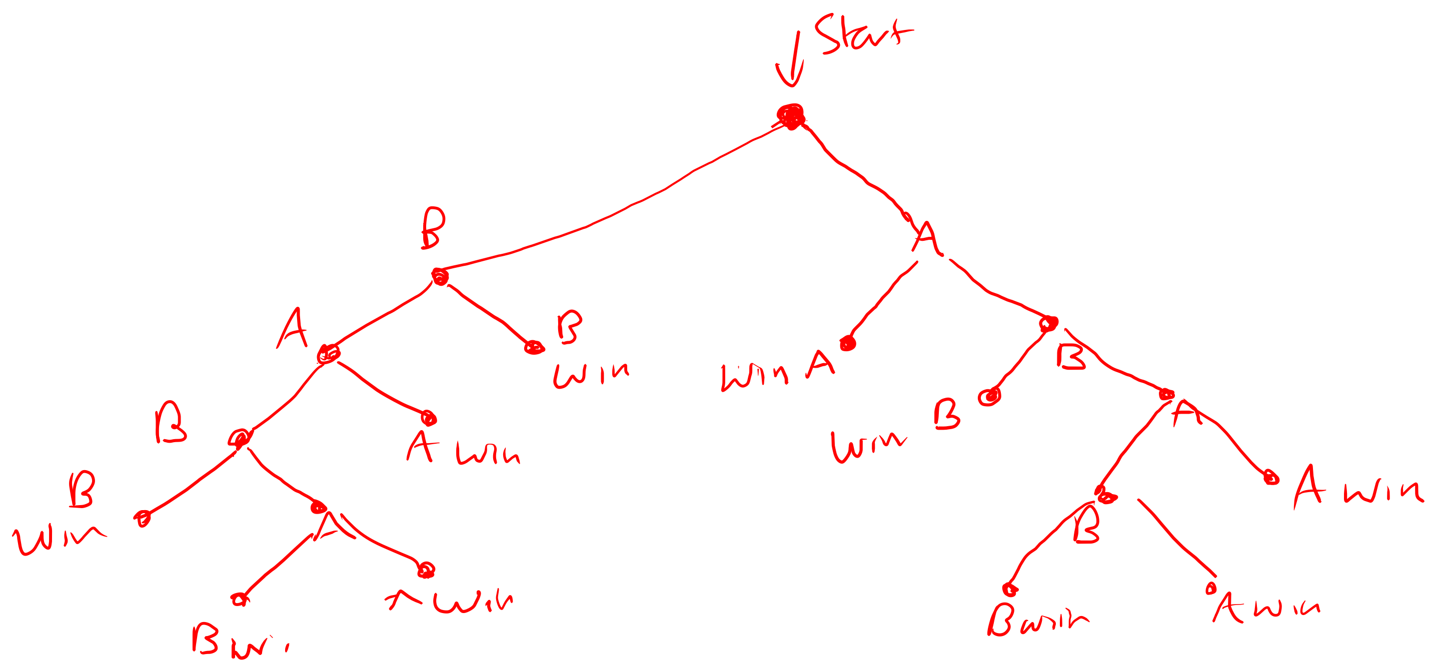
$100 \quad 105 \quad 110 \quad \cdots \quad 999 \quad 1000$

$100 \rightarrow 5 \times \underbrace{20}$, $\quad 105 \rightarrow 5 \times 21$, $\quad 110 \rightarrow 5 \times 22$, $\quad \cdots \rightarrow 5 \times \underbrace{199}$, $\quad 1000 \rightarrow 5 \times 200$

$$(199 - 20) + 1 = \underline{\underline{180}}$$

$$P(E) = \frac{18\cancel{0}}{90\cancel{0}} = \frac{18}{90} = \frac{9}{45} = \frac{1}{5}$$

Play A, B until one wins two games in a row or total of three games.



Space Sample

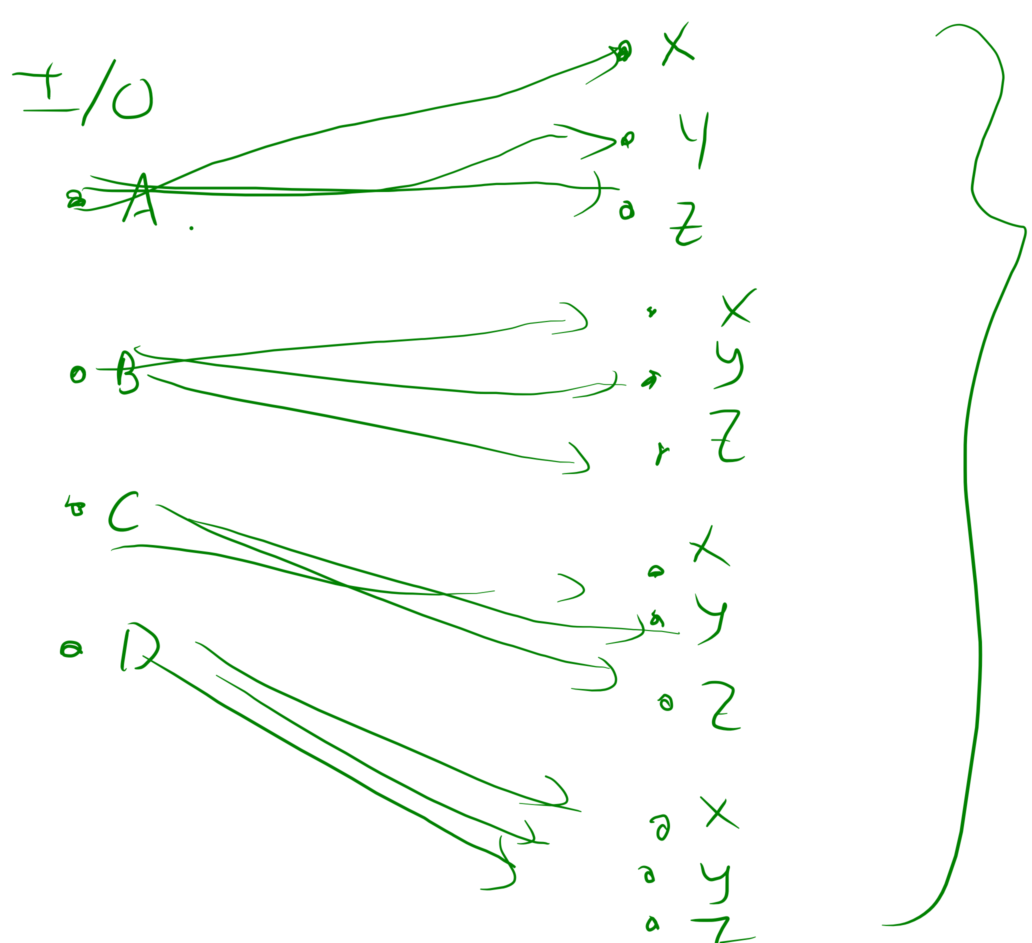
1) A – A
2) A – B – A – A
3) A – B – A – B – A
4) A – B – A – B – B
5) A – B – B
6) B – A – A
7) B – A – B – A – A
8) B – A – B – A – B
9) B – A – B – B
10) B – B

$$P(E) = \frac{N(E)}{N(S)} = \frac{4}{10} = \frac{2}{5}$$

Suppose that a computer installation has 4 I/O units (A, B, C, D) and 3 CPU (x, y, z)

Any I/O unit can be paired with any CPU

How many ways are there to pair an I/O unit with CPU?



$$3 \times 4 = 12$$

PIN (sequence of alpha/with numbers)

4 digits

26 + 10 digits

A184

9B25

Repetition is allowed

(1) Sample Space

36 36 36 36

$= 36^4$

(2) If PIN are equally likely what is the prob. choose a number (random) with not repeated

without Repeat

$N(E) = 36 \times 35 \times 34 \times 33$

$P(E) = \frac{N(E)}{N(S)} = \frac{36 \times 35 \times 34 \times 33}{36^4} \simeq 0.84$

84%