Chapter 9

Counting and Probability

Ex: 3 officers { President, Treasurer, Secretary

{Abeer, Basil, Carmen, Dalia}

→ Abeer can't be president for some reason.

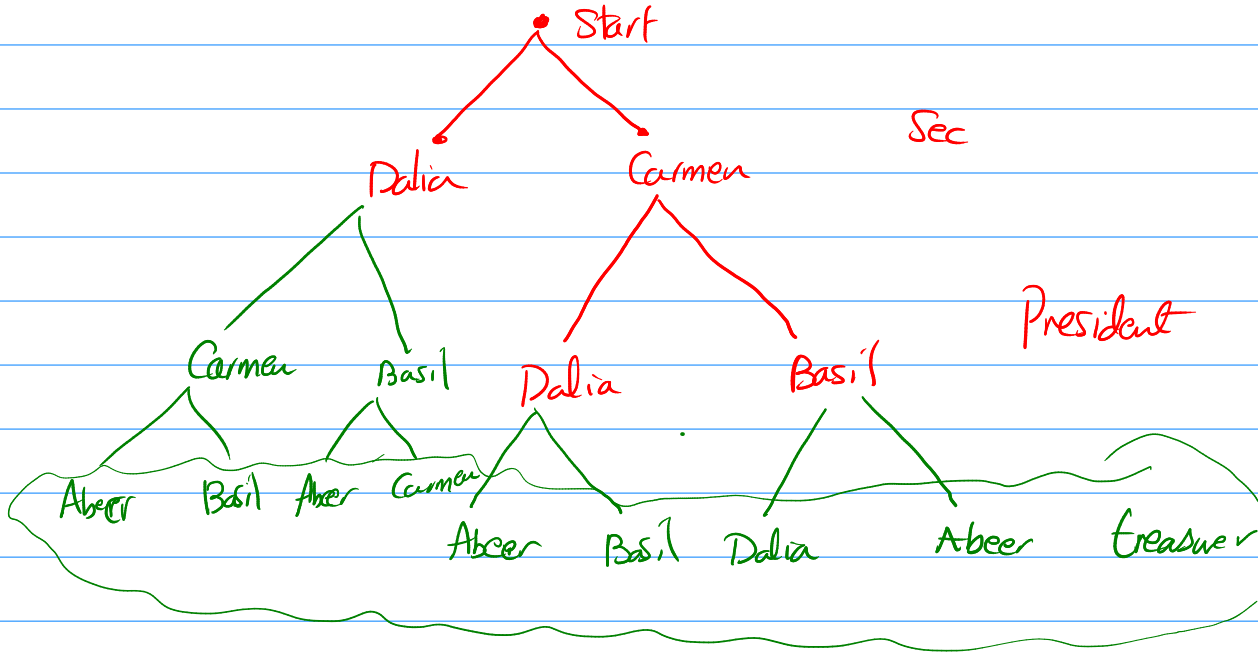
→ Carmen or Dalia must be secretary.

How many ways can be the officers be chosen?

As: ~~4 X 3~$\cdot$

{ President ⇒ 3 {B, C, D}
treasurer ⇒ 4 {A, B, C, D}
sec ⇒ 2 {C, D}

$3 \times 3 \times 2 = 18$ X



Permutations: (without Reapt)

Ex: $\{a,b,c\} = \{abc, acb, bac, bca, cab, cba\}$

6

$n!$

r-Permutations:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Ex Computer

a) How many words can get from this word?

Computer ⇒ $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

8 7 6 5 4 3 2 1

$P(8,8) = \frac{8!}{0!} = 8!$

b) If we fixed two letters (Co) how many words?

fixed ↳ Computer ? $P(7,7) = 7!$

a) $\times$ = #of letters 7-Character $= 7!$

c) If letters of word Computer are randomly arranged in a row, what probability that the letter Co remain next to each other (inorder) as a unit?

$$P(E) = \frac{N(E)}{N(S)} = \frac{7!}{8!} = \frac{7!}{8 \times 7!} = \frac{1}{8}$$

(1) $P(5,2) = \frac{5!}{3!} = \frac{5\times4\times3!}{3!} = 20$

5
$S_1\ S_2\ S_3\ S_4\ S_5$
↑ 2 Student

S_3 $S_1\ S_2\ S_4\ S_5$

$S_1 S_2$

(2) r-Permutation
2-Permutation
$= P(5,2) = \frac{5!}{3!} = 20$

(3) $P(5,5) = \frac{5!}{0!} = 5! = 120$

(4) Prove that $\forall n \in \mathbb{Z}\ n \geq 2$

$P(n,2) + P(n,1) = n^2$

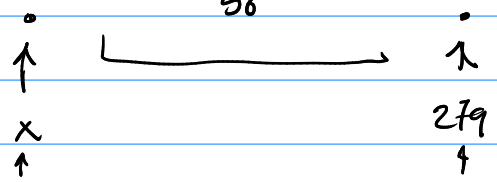
$$\frac{n!}{(n-2)!} + \frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)!}{(n-2)!} + \frac{n(n-1)}{(n-1)!}$$

$$= n^2 - n + n = n^2 \quad \text{XX}$$

Ex: 56 consecutive numbers [integers], largest 279

what is the min #?

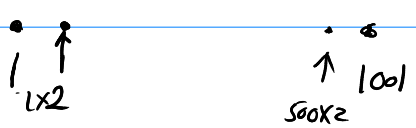
(a)



$(279 - X) + 1 = 56$

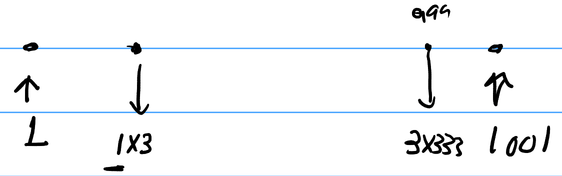
$279 + 1 - 56 = X \Rightarrow X = 224$

(b) 1 - 1001 // # of even numbers



$(500 - 1) + 1 = 500$ ✓

c) 1 – 1001 ⟹ are divisable by 3



$(333-1)+1 = 333$ ※

Ex: 4 letters 3 digits → 26, {0-9} 10

Serial: |26, 26, 26, 26 | 10 | 10 | 10| — char, digits

a) ~ different Licenses plates are possible

$26^4 \times 10^3$

(b) A _ _ _ | _ _ 0 → 1, 26 26 26, 10 10 × 1

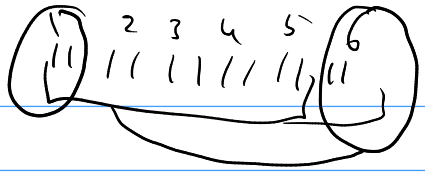
$1 \times 26^3 \times 10^2 \times 1$

(c) TGIF 10 10 10 → 1, 10^3 $= 1 \times 1000 = 1000$ { TGIF000, TGIF001, ---- TGIF999 }

(d) How may plates are possibles [In which the letter & digit are distinct] ? (without repeat)

$[26 \times 25 \times 24 \times 23][10 \times 9 \times 8]$

Cinma 6 person



(a) How may ways can they be seat together in row?

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$

or

(b) Doctor ⇒ 1 _____ 1

$= 5! + 5! = 120 + 120 = 240$

(c) 

$\Rightarrow 3 \times 2 \times 1 = 3! = 6$

$\Rightarrow 12 \Rightarrow 6 + 6$

(4) Prove that $P(n+1, 3) = n^3 - n$

Home work

(b) $P(n+1, 2) - P(n, 2) = 2P(n, 1)$

(c) $P(n+1, 3) - P(n, 3) = 3P(n, 2)$

d) $P(n, n) = P(n, n-1)$