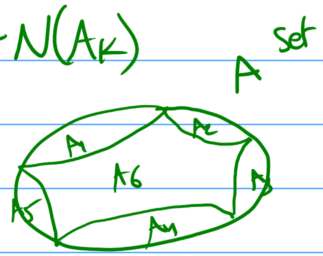
Chapter 9

Addition Rule:

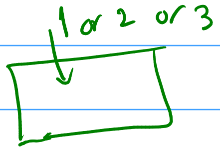
Theorm: Suppose a finite set A equals the union of K distinct mutually disjoint subsets $A_1, A_2, \ldots A_K$. Then

$$N(A) = N(A_1) + N(A_2) + \cdots + N(A_K)$$



$$A_1 \cap A_2 = \phi$$
$$A_1 \cap A_2 \cap \cdots \cap A_K = \phi$$

Ex: Password : $\{1, 2, 3\}$ pass :

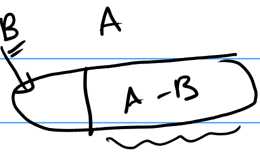


26 - Character $\{A - Z\}$
How may passwords can be generated ?

Group1 : $\{A, B, C, \ldots, Z\}$ 26
Group2 : $\{AA, AB, AC \ldots ZZ\}$ 26×26
Group3 : $\{AAA, AAB, \ldots, ZZZ\}$ 26×26×26

Ans $= 26 + 26^2 + 26^3 = 18,278$ passwords

⇒ difference rule:



Theorm: If A is a finite set and B is a subset of A, then
$N(A-B) = N(A) - N(B)$ ⇒ $\boxed{N(B) + N(A-B) = N(A)}$

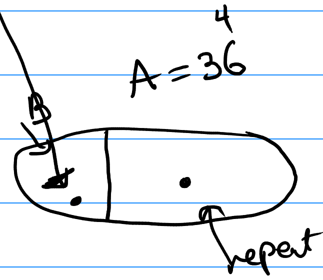
PIN (4-digits) → 26 Characters, 10 digits, Repeat is allowed

(a) How may PIN's are there? 36^4

$An = 36 \cdot 36 \cdot 36 \cdot 36 = 36^4$

(b) without repeat? How many PIN?

$Ans = 36 \cdot 35 \cdot 34 \cdot 33$



(c) How many PIN's has repeated digits?

$$N(A-B) = 36^4 - 36 \cdot 35 \cdot 34 \cdot 33 = \underline{265{,}896}$$

(d) $P(E) = \dfrac{N(E)}{N(S)} = \dfrac{265{,}896}{36^4} = 15.8\%$

choose any PIN ⇒ repeated

⇒ Previous lecture $P(E_2) = \dfrac{N(E_2)}{N(S)} = \dfrac{36 \cdot 35 \cdot 34 \cdot 33}{36^4} = 84.2\%$

⇒ Prove that $P(S-A) = 1 - P(A)$

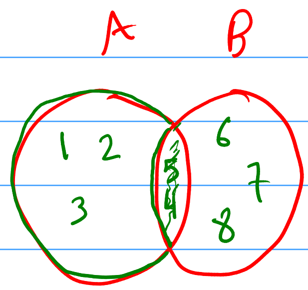
$$\underbrace{P(S-A)}_{E} = \frac{N(S-A)}{N(S)} = \frac{N(S)-N(A)}{N(S)} = 1 - \frac{N(A)}{N(S)} = 1 - P(A) \quad \#$$

↳ $1 - 0.842 = 0.158$

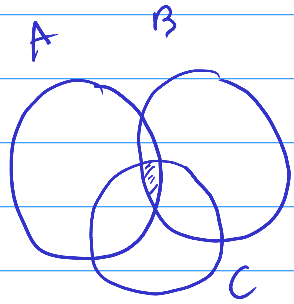
Note: $\boxed{P(A^c) = 1 - P(A)}$

Inclusion / Exclusion Rule:

How many elements are in $A \cup B$



$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$



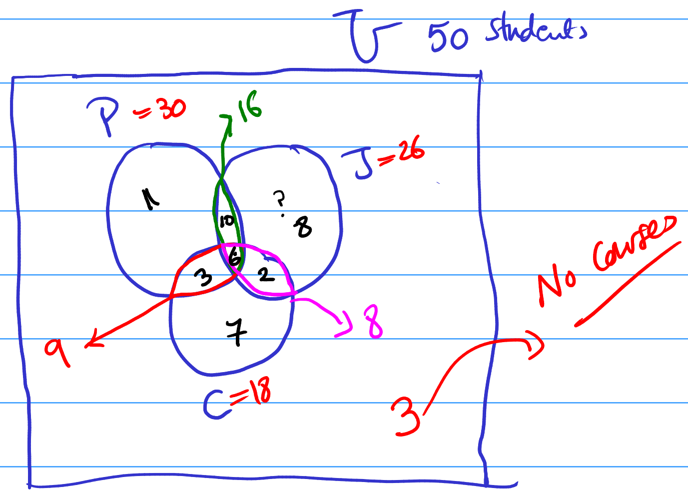
$$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$$

Ex: 50 Students

30	Pre-Cal	16 -	Pre + Java
18	Calc	8 -	Calc + Java
26	Java	47 -	took at least 1 of 3
9	Pre-Calc		

(a)

$N(P \cup C \cup J) = 47$



$$N(P \cup C \cup J) = N(P) + N(C) + N(J) - N(P \cap C) - N(P \cap J) - P(C \cap J) + N(P \cap C \cap J)$$

$$47 = 30 + 18 + 26 - 9 - 16 - 8 + N(P \cap C \cap J)$$

$$\Rightarrow N(P \cap C \cap J) = 6$$

Ex: (a) How many integers from 1 – 1000 is multiple of 3

3 999

1 1000

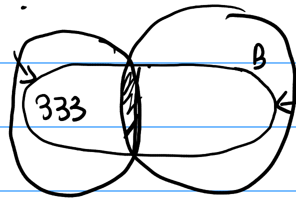
1×3 3×333

$\Rightarrow (333-1)+1 = \underline{\underline{333}}$

(b) How many integers from 1 – 1000 is neither multiple of 3 nor multiple of 5 ?

multiple 3 multiple 5

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$



$= 333 + 200 - 66 = 467$

is multiple of 3 and multiple of 5

990

1 1000

3×5 = 15 15×66

1×15

$\Rightarrow (66-1)+1 = \underline{\underline{66}}$

$\Rightarrow 1000 - 467 = \underline{\underline{533}}$ difference rule

$N(A \cup B) = N(A) + N(B) - N(A \cap B)$

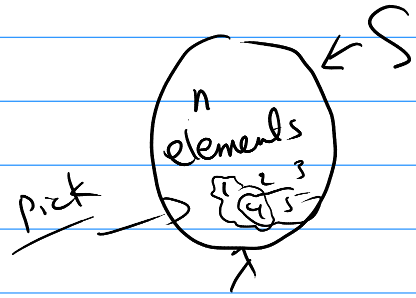
$\Leftrightarrow$

$N(A \cup B)^c = N(U) - A(A \cup B)$ Demorgan's law

Combinations:

Given a set S with n elements, how many Subsets of Size r can be chosen from S?

Size of group is r



r-Combination [unordered]

Ex: . {A, B, C, D} 3 person ⇒ from 4

G1: {A, B, C} − D
G2: {A, B, D} − C
G3: {A, C, D} − B
G4: {C, B, D} − A

} 4 groups

r-Combination ⇒ 3-Combination $C(n,r)$, nCr, $C_{n,r}$, ${}^{n}C_{r}$

$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$C(4,3) = \binom{4}{3} = \frac{4!}{3!\,1!} = \frac{4 \times \cancel{3!}}{\cancel{3!}} = 4$$

⇒ Order ⇒ Permutations
unOrder ⇒ Combination