

Chaper 9

$${}^n C_r = \frac{n!}{(n-r)! r!} \quad \text{Combination}$$

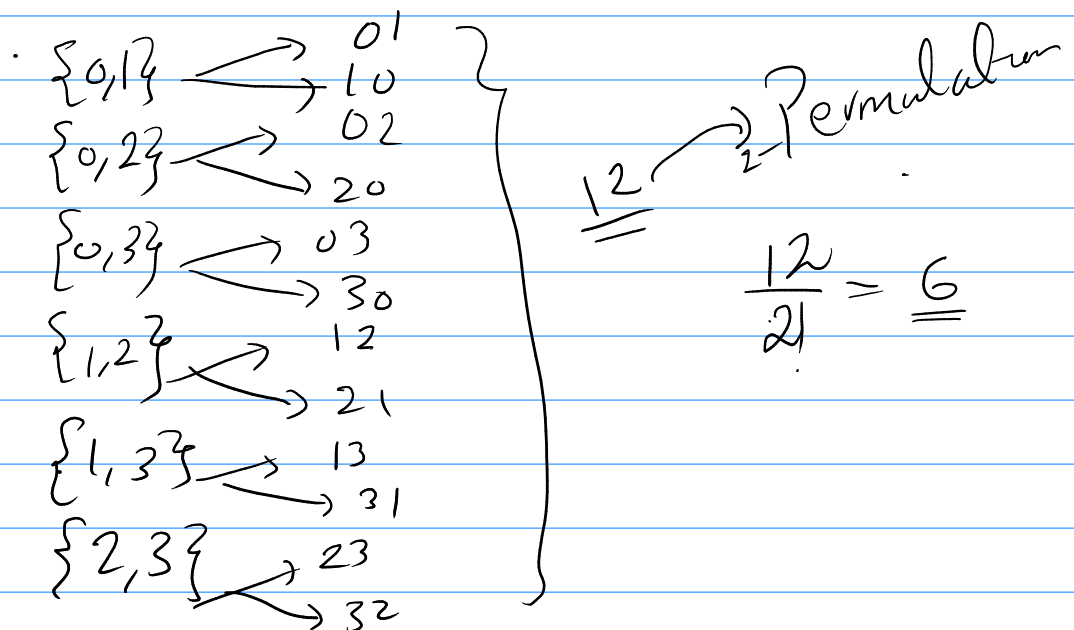
Ex. $\{0, 1, 2, 3\} = \{0, 1\}, \{0, 2\}, \{0, 3\}$
 $\{1, 2\}, \{1, 3\}$
 $\{2, 3\}$ 6

$$\binom{4}{2} = \frac{4!}{2! 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 2} = 6$$

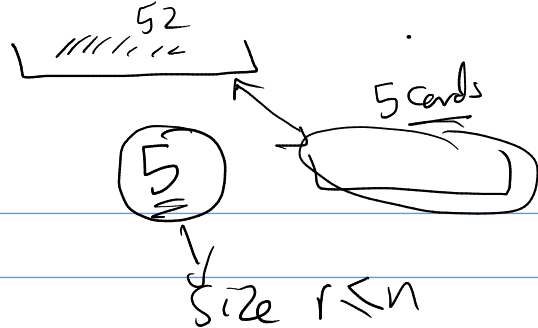
$$\binom{n}{r} = \frac{P(n, r)}{r!}$$

↑ size of subset.

write all 2-permutation of set $\{0, 1, 2, 3\}$



Ex: 52 deck



$$\binom{52}{5} = \frac{52!}{47!5!} = \underline{\underline{2,598,960}} \quad 5 \leq 52$$

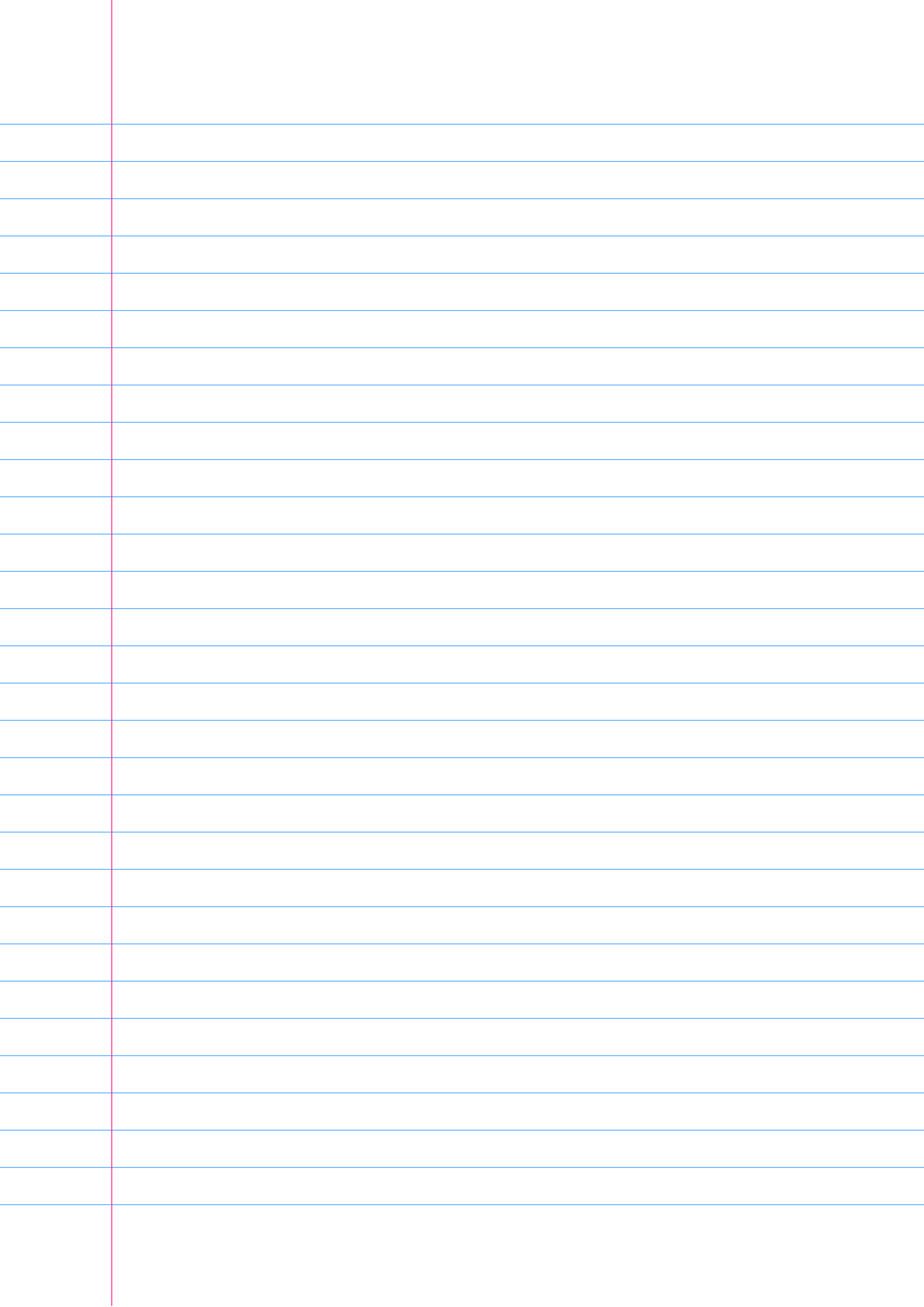
Ex: 5 essay \Rightarrow 3 Question Testbank

$$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \times 4 \times 3!}{3! \cdot 2!} = \underline{\underline{10}} \quad \begin{matrix} \text{5} \\ \text{3 Questions} \end{matrix}$$

Ex: Team 5 from 12

$$\binom{12}{5} = \frac{12!}{5!7!} = \underline{\underline{792}}$$

Note phrase at least n means n or more
at most n means n or fewer

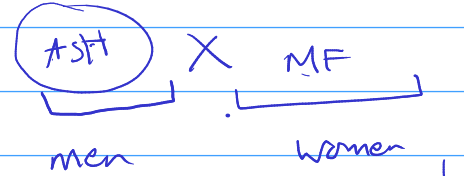


Ex: Group 12 person [5 men, 7 women]

(a) team 5 person { 3 men, 2 women }

Ans $\binom{5}{3} * \binom{7}{2}$

↑ men ↑ women



(a)

$$= \frac{5!}{3!2!} \times \frac{7!}{5!2!} = \underline{\underline{210}}$$

(b) 5 person [at least one man]

Ans 1, 2, 3, 4, 5

$$\binom{5}{1}\binom{7}{4} + \binom{5}{2}\binom{7}{3} + \binom{5}{3}\binom{7}{2} + \binom{5}{4}\binom{7}{1} + \binom{5}{5}\binom{7}{0} = \dots = \underline{\underline{771}}$$

Yes

$\binom{5}{0}\binom{7}{5}$ عدم وجود أي رجل في الفريق

$$\binom{12}{5} - \binom{7}{5} = \underline{\underline{792}} - 71 = \underline{\underline{771}}$$

Ans: $\underbrace{0 \text{ or } 1}_{\text{men}} \Rightarrow \binom{5}{0} \binom{7}{5} + \binom{5}{1} \binom{7}{4}$
 $= \underline{\underline{196}}$

Ex: MISSISSIPPI

Ans: $\underline{\underline{11!}}$

11 characters

permutation

Groups $\left\{ \begin{array}{l} \text{'s} - 4 \\ \text{I's} - 4 \\ \text{P's} - 2 \\ \text{M's} - 1 \end{array} \right.$ $\frac{11!}{4! \cdot 4! \cdot 2! \cdot 1!}$

$= \binom{11}{4} \binom{7}{4} \binom{3}{2} \binom{1}{1} = 34650$

* Note Computer ! \Rightarrow 8 characters $\underline{\underline{8!}}$

Comp $4! = 4 \times 3 \times 2 \times 1 = 24$

$\left. \begin{array}{l} C \ 12 \\ O \ 1 \\ m \ 1 \\ P \ 1 \end{array} \right\} \binom{4}{1} \binom{3}{1} \binom{2}{1} \binom{1}{1} = \frac{4!}{3!1!} \times \frac{3!}{2!1!} \times \frac{2!}{1!1!} \times 1$
 $= 24$ $4 \times 3 \times 2 = \underline{\underline{24}}$

$$\begin{aligned}
 & \left. \begin{array}{l} \{ 2, \text{center} \} \\ \{ 5 \text{ forwards} \} \\ \{ 4 \text{ guards} \} \end{array} \right\} 11 \\
 & \{ \text{One center, 2 forwards, 2 guards} \} \quad \underline{\underline{5}} \\
 & = \binom{2}{1} \binom{5}{2} \binom{4}{2} = \underline{\underline{120}}
 \end{aligned}$$

Ex. 15 Students, Committee (Six person)

(a) $\binom{15}{6} = \frac{15!}{6!9!} = \frac{5 \times 7 \times 13 \times 11 \times \dots}{\dots}$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 9!}$$

(b) $\binom{15}{2} * \binom{13}{5} + \binom{13}{6} = \underline{\underline{4290}}$

we can see $\frac{15!}{2!13!} + \frac{13!}{6!7!}$

$\binom{2}{1} * \binom{13}{5}$
 or $\binom{2}{0} \binom{13}{6}$

6 person

Ans $\cdot \binom{2}{2} \binom{13}{4} + \binom{2}{0} \binom{13}{6} = ??$

$=$

(*) 8 men 7 women \rightarrow { 3 men, 3 women } ⁶

(I) $= \binom{8}{3} \binom{7}{3}$

(II) at least one women

6 person

1 or, 2, 3, 4, 5, 6

$$\binom{7}{1} \binom{8}{5} + \binom{7}{2} \binom{8}{4} + \binom{7}{3} \binom{8}{3} + \binom{7}{4} \binom{8}{2} + \binom{7}{5} \binom{8}{1} + \binom{7}{6} \binom{8}{0} = X$$

Another way $\Rightarrow \binom{15}{6} - \binom{8}{6} = X$