Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2015

Graphs and Trees

10.1 Introduction to Graphs

10.2 Introduction to Trees

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 10 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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Graphs and Trees

10.2 Trees

In this lecture:

- Part 1: **Concept and Terminology**
- Part 2: Examples of Trees
- Part 3 Important Theorems

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What is a Tree

• Definition

A graph is said to be **circuit-free** if, and only if, it has no circuits. A graph is called a **tree** if, and only if, it is circuit-free and connected. A **trivial tree** is a graph that consists of a single vertex. A graph is called a **forest** if, and only if, it is circuit-free and not connected (i.e., a disjoint union of trees).

Tree is a special case of a graph

Trees

Non-Trees

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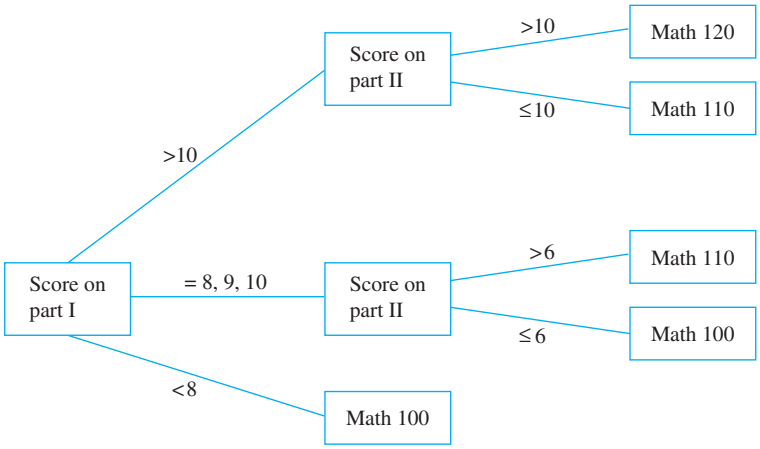
Decision Tree



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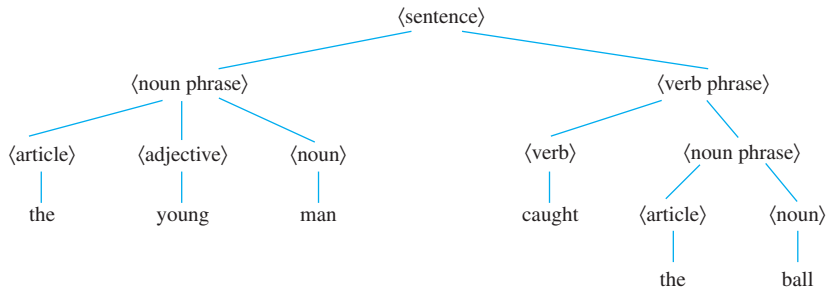
Parse Tree

The Yang man caught the ball



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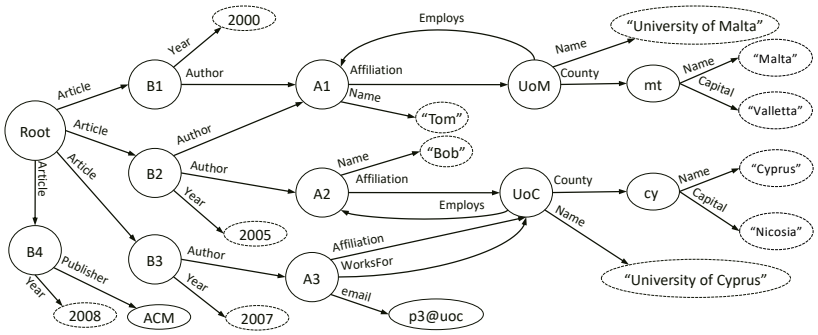
Data Tree

XML



Directed labeled and rooted tree

Number of Vertices Vs Number of Edges

- Any tree with n vertices has $n - 1$ edges.
- Any connected graph with n vertices and n – 1 edges is a tree?
- Every connected graph has a subgraph that is a tree.
- Any graph with n vertices and *fewer* than $n - 1$ edges is not connected.

Lemma 10.5.1

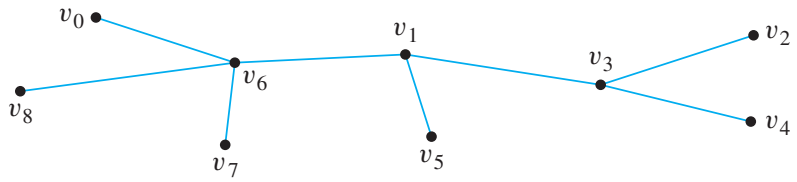
Any tree that has more than one vertex has at least one vertex of degree 1.

i.e., Any nontrivial tree must have at least one vertex of degree 1.

Terminal and Internal Vertices

• Definition

Let T be a tree. If T has only one or two vertices, then each is called a **terminal vertex.** If T has at least three vertices, then a vertex of degree 1 in T is called a **terminal vertex** (or a **leaf**), and a vertex of degree greater than 1 in T is called an **internal vertex** (or a **branch vertex**).



Terminal vertices: v_0 , v_2 , v_4 , v_5 , v_7 , v_8
Internal vertices: v_6 , v_1 , v_7 , v_3

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Determining whether a Graph Is a Tree

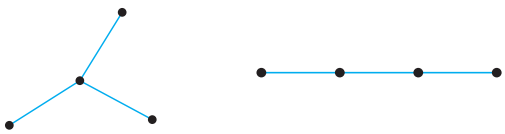
Theorem 10.5.2

For any positive integer n, any tree with n vertices has $n - 1$ edges.

A graph G has 10 vertices and 12 edges. Is it a tree?

NO

Nonisomorphic trees with 4 vertices?



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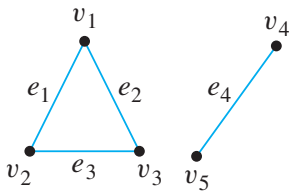
Graph with *n* Vertices and *n*−1 Edges that is Not a Tree

Theorem 10.5.4

For any positive integer n, if G is a connected graph with n vertices and $n - 1$ edges, then G is a tree.

Give a graph with 5 vertices and 4 edges that is not a tree.

➔ such a graph cannot be connected.



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Exercise

Draw a connected graph with 6 vertices and 5 edges, has a nontrivial circuit?

➔Not possible

Draw a Graph with two vertices and one edge that is not a tree?

*

A connected graph has 12 vertices and 11 edges. Does it have a vertex of degree 1?

➔ Yes

A graph has 8 vertices and 6 edges. Is it connected?

➔ NO

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