Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2017

Propositional Logic

2.1. Introduction and Basics

2.2 Conditional Statements

2.3 Inferencing

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Watch this lecture and download the slides

Course Page: http://www.jarrar.info/courses/DMath/
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Acknowledgement:
This lecture is based on (but not limited to) to chapter 2 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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If-Then Statements

If 4,686 is divisible by 6, then 4,686 is divisible by 3

hypothesis — conclusion

If you study, then you pass

hypothesis — conclusion

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If-Then Statements

Remark that this if-then is a logical (not a causal) condition

p	q	$p \to q$	
T	T	T	you studied and you passed
T	F	F	you studied and you didn't passed
F	T	T	you didn't study and you passed
F	F	T	you didn't study and you didn't pass

Your mam said {**you study → you pass**}, is it True?

If p and q are statement variables, the **conditional** of q by p is "If p then q" or "p implies q" and is denoted $p \to q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** (or **antecedent**) of the conditional and q the **conclusion** (or **consequent**).

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Conditional Statement with a False Hypothesis

If 0 = 1 then 1 = 2.

The statement as whole is true.

Notice that we don't test the correctness of the conclusion

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Truth Tables involving →

Construct a truth table for the statement form $p \vee \sim q \rightarrow \sim p$.

p	q	$\sim p$ (conclusion)	$\sim q$	$p \vee \sim q$ (hypothesis)	$p \vee \sim q \rightarrow \sim p$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

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Logical Equivalences involving →

Division into Cases: Showing that $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$p \vee q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$p \vee q \to r$ and $(p \to r) \wedge (q \to r)$ always have the same truth values, so they are logically equivalent

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Representation of If-Then As Or

$$p \to q \equiv {\sim}p \vee q$$

Examples?

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The Negation of a Conditional Statement

The negation of "if p then q" is logically equivalent to "p and not q."

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

$$\begin{aligned} \sim(p \rightarrow q) &\equiv \sim(\sim p \vee q) \\ &\equiv \sim(\sim p) \wedge (\sim q) && \text{by De Morgan's laws} \\ &\equiv p \wedge \sim q && \text{by the double negative law} \end{aligned}$$

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The Negation of a Conditional Statement

Examples

If my lecture is at TEC109, then I cannot buy coffee

my lecture is at TEC109 and I can buy coffee

If Amjad loves Zatar, then Amjad is smart

Amjad loves Zatar and Amjad is not smart

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Contrapositive Statements

Conditional statement = its contrapositive.

If you don't pass then you didn't study

• Definition

The **contrapositive** of a conditional statement of the form "If p then q" is

If $\sim q$ then $\sim p$.

Symbolically,

The contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$.

Try the truth table

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Contrapositive Statements

Examples

If my lecture is at TEC109, then I cannot buy coffee

If I can buy coffee then my lecture is not at TEC109

If Amjad loves Zatar, then Amjad is smart

If Amjad is not smart then Amjad does not love Zatar

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Converse and Inverse

• Definition

Suppose a conditional statement of the form “If p then q” is given.

1. The **converse** is “If q then p.”
2. The **inverse** is “If $\sim p$ then $\sim q$.”

Symbolically,

$$\text{The converse of } p \to q \text{ is } q \to p,$$

and

$$\text{The inverse of } p \to q \text{ is } \sim p \to \sim q.$$

مقلوب Converse
معكوس Inverse

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Converse and Inverse

Examples

If my lecture is at TEC109, then I cannot buy coffee

Converse: If I cannot buy coffee then my lecture is at TEC109
Inverse: If my lecture is not at TEC109 then I can buy coffee

If Amjad loves Zatar, then Amjad is smart

Converse: If Amjad is smart then Amjad loves Zatar
Inverse: If Amjad does not love Zatar, then Amjad is not smar

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Converse and Inverse

Caution! Many people believe that if a conditional statement is true, then its converse and inverse must also be true. This is not correct!

1. A conditional statement and its converse are *not* logically equivalent.
2. A conditional statement and its inverse are *not* logically equivalent.
3. The converse and the inverse of a conditional statement are logically equivalent to each other.

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Converse and Inverse

Caution! Many people believe that if a conditional statement is true, then its converse and inverse must also be true. This is not correct!

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Only If

"*p* only if *q*" means that "*p* can take place *only* if *q* takes place also"

Definition

It *p* and *q* are statements,

p **only if** *q* means "if not *q* then not *p*,"

or, equivalently,

"*if p* then *q*."

John will break the world's record for the mile run ***only if*** he runs the mile in under four minutes.

If John does ***not*** run the mile in under four minutes, ***then*** he will ***not*** break the world's record.

or, equivalently

If John breaks the world's record, ***then*** he will have run the mile in under four minutes.

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Biconditional

If and only if

iff

• Definition

Given statement variables *p* and *q*, the **biconditional of *p* and *q*** is "*p* if, and only if, *q*" and is denoted $p \leftrightarrow q$. It is true if both *p* and *q* have the same truth values and is false if *p* and *q* have opposite truth values. The words *if and only if* are sometimes abbreviated **iff.**

p	*q*	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Only If and the Biconditional

Truth Table Showing that $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \land (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ and $(p \rightarrow q) \land (q \rightarrow p)$ always have the same truth values, so they are logically equivalent

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محذوف

Only If and the Biconditional

Examples

You get an ID card only if you are above 16

If you are not above 16 Then you don't get an ID card

If you got an ID card then you must be above 16

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Biconditional

Examples

This computer program is correct if, and only if,
it produces correct answers for all possible sets of input data.

If this program produces the correct answers for all possible sets of input data, then it is correct.

If this program is correct, then it produces the correct answers for all possible sets of input data;

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Order of Operations for Logical Operators

1.	$\sim$	Evaluate negations first.
2.	$\wedge, \vee$	Evaluate $\wedge$ and $\vee$ second. When both are present, parentheses may be needed.
3.	$\rightarrow, \leftrightarrow$	Evaluate $\rightarrow$ and $\leftrightarrow$ third. When both are present, parentheses may be needed.

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Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."
r is a **necessary condition** for s means "if not r then not s."

r is a necessary condition for s also means "if s then r."

r is a necessary and sufficient condition for s means "r if, and only if, s."

Example:

Studying is a **sufficient** condition for passing. Study → Pass

In order to pass, it is **sufficient** to study. Study → Pass

It is **sufficient** to study in order to pass. Study → Pass

Studying is a **necessary** condition for passing. ~Study → ~Pass

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Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."
r is a **necessary condition** for s means "if not r then not s."

r is a necessary condition for s also means "if s then r."

r is a necessary and sufficient condition for s means "r if, and only if, s."

Example 2

Being above 16 is a sufficient condition for getting ID Card.

Above (16) → Get ID Card

Being above 16 is a necessary condition for getting ID Card

~Above (16) → ~Get ID Card

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