Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2020

Propositional Logic

2.1. Introduction and Basics

2.2 Conditional Statements

2.3 Inferencing

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 5 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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Propositional Logic

2.3 Logical Inferencing

In this lecture:

- ❑ Part 1: **Numeration Method**
- ❑ Part 2: **Rules of Inference**
- ❑ Part 3: **Example**

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Numeration Method
Example 1

If today is Friday then today is holiday

Today is Friday

∴today is holiday?

$p \rightarrow q$

p

$\therefore q?$

		premises		conclusion	
p	q	$p \rightarrow q$	p	q	
T	T	T	T	T	← critical row
T	F	F	T		
F	T	T	F		
F	F	T	F		

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Numeration Method
Example 2

$p \to q \vee \sim r$

$q \to p \wedge r$

$\therefore p \to r$?

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

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Numeration Method

What is wrong with this Numeration Method?

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Rules of Inference

1. Modus Ponens:

$p \to q$

p

$\therefore q$

If today is Friday then today is holiday

Today is Friday

∴ **Today is holiday**

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Rules of Inference

2. Modus Tollens:

$p \rightarrow q$
$\sim q$
$\therefore \sim p$

If today is Friday then today is holiday
Today is not holiday

∴ **Today is not Friday**

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Rules of Inference

3. Generalization:

p

$\therefore p \lor q$

q

$\therefore p \lor q$

Today is Saturday

∴ **Today is Saturday or today is Sunday**

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Rules of Inference

4. Specialization:

$p \wedge q$	Today is Friday and today is holiday
$\therefore \boldsymbol{p}$	**∴ Today is Friday**

$p \wedge q$	
$\therefore \boldsymbol{q}$	**∴ Today is Holiday**

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Rules of Inference

5. Conjunction:

p	Today is Friday
q	Today is Holiday
$\therefore p \wedge q$	**∴ Today is Friday and today is holiday**

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Rules of Inference

6. Elimination:

$p \vee q$
$\sim p$
$\therefore \mathbf{q}$

$p \vee q$
$\sim q$
$\therefore \mathbf{p}$

Today is Saturday or today is Sunday
Today is not Saturday

∴ **Today is Sunday**

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Rules of Inference

7. Transitivity:

$p \rightarrow q$
$q \rightarrow r$
$\therefore \mathbf{p \rightarrow r}$

If today is Friday then Today is holiday
If today is holiday then I am happy

∴ **If today is Friday then I am happy**

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Rules of Inference

8. Division into Cases:

$p \vee q$
$p \to r$
$q \to r$

$\therefore r$

Today is Friday or today is Sunday
If today is Friday then I am happy
If today is Sunday then I am happy

∴ **I am happy**

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Rules of Inference

9. Contradiction Rule:

$\sim p \to c$

$\therefore p$

If "Today is not Friday" is false

∴ **Today is Friday**

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Rules of Inference
Summary

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	a. $p \lor q$ $\sim q$ $\therefore p$ b. $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	a. p $\therefore p \lor q$ b. q $\therefore p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	a. $p \land q$ $\therefore p$ b. $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

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Inferencing Example

Formalize the following text in propositional logic and use the inference rules find the glasses.

If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.	$RK \rightarrow GK$
If my glasses are on the kitchen table, then I saw them at breakfast.	$GK \rightarrow SB$
I did not see my glasses at breakfast.	$\sim SB$
I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.	$RL \vee RK$
If I was reading the newspaper in the living room then my glasses are on the coffee table.	$RL \rightarrow GC$

Where are the glasses?

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Inferencing Example

Let
- RK = I was reading the newspaper in the kitchen.
- GK = My glasses are on the kitchen table.
- SB = I saw my glasses at breakfast.
- RL = I was reading the newspaper in the living room.
- GC = My glasses are on the coffee table.

$RK \rightarrow GK$
$GK \rightarrow SB$
$\sim SB$
$RL \vee RK$
$RL \rightarrow GC$

$RK \rightarrow GK$
$GK \rightarrow SB$
$\therefore RK \rightarrow SB$ by transitivity

$RL \vee RK$
$\sim RK$
$\therefore RL$ by elimination

$RK \rightarrow SB$
$\sim SB$
$\therefore \sim RK$ by modus tollens

$RL \rightarrow GC$
RL
$\therefore GC$ by modus ponenes

Thus the glasses are on the coffee table.

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