Lecture Notes on **Discrete Mathematics**.
Birzeit University, Palestine, 2016

Number Theory
and Proof Methods

Mustafa Jarrar

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 4 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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Number Theory

4.2 Rational Numbers

In this lecture:

- ❑Part 1: **Rational and irrational Numbers**
- ❑ Part 2: Proving Properties of Rational Numbers
- ❑ Part 3: Using rational numbers in Programing

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Relational and Irrational Numbers

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• Definition

A real number r is **rational** if, and only if, it can be expressed as a quotient of two integers with a nonzero denominator. A real number that is not rational is **irrational.** More formally, if r is a real number, then

$$r \text{ is rational } \Leftrightarrow \exists \text{ integers } a \text{ and } b \text{ such that } r = \frac{a}{b} \text{ and } b \neq 0.$$

Example

- ✓ Is 10/3 a rational number?
- ✓ Is -(5/39) a rational number?
- ✓ Is 0.281 a rational number?
- ✓ Is 7 a rational number?
- ✓ Is 0 a rational number?
- ✗ Is 2/0 a rational number?
- ✗ Is 2/0 an irrational number? Not number
- ✓ Is 0. 1212... a rational number (where 12 are assumed to repeat forever)? 12/99
- ✓ If m, n are integers and neither m nor n is zero, is $(m + n)/mn$ a rational number?
- ✗ Is (Sqr root of 2) an rational number?

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Examples of Irrational Numbers

Prepared by Student Thana’

1. π =3.1415926535897932384… (Is approximately 22/7).
2. √2 =1.4142135623730951… (Is approximately 14/10).
3. √3 =1.7320508075688772… (Is approximately 17/10).
4. √5 =2.23606797749979… (Is approximately 22/10).
5. √7 =2.6457513110645907… (Is approximately 26/10).
6. phi=ϕ =(1+ √5)/2=1.61803398875… “النسبة الذهبية”
7. e =2.718281828… “العدد النيبيري”

References:-
([BOOK] Numbers: rational and irrational)
(*study.com/academy/lesson/what-is-the-golden-ratio-in-math-definition-examples.html*) *{6}*
(*mathforum.org/dr.math/faq/faq.e.html*) *{7}*

Integers are rational numbers

Theorem 4.2.1
Every integer is a rational number.

$$n = \frac{n}{1}$$ which is a quotient of integers and hence rational.

$$7 = \frac{7}{1}$$ which is a quotient of integers and hence rational.

$$-12 = \frac{-12}{1}$$ which is a quotient of integers and hence rational.

$$0 = \frac{0}{1}$$ which is a quotient of integers and hence rational.

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Proving Properties of Rational Numbers

Theorem 4.2.2

The sum of any two rational numbers is rational.

Proof:

$$r + s = \frac{a}{b} + \frac{c}{d} \quad \text{by substitution}$$

$$= \frac{ad + bc}{bd} \quad \text{by basic algebra.}$$

Let $p = ad + bc$ and $q = bd$.

$$r + s = \frac{p}{q} \text{ where } p \text{ and } q \text{ are integers and } q \neq 0.$$

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Example

Derive the following as a corollary of Theorem 4.2.2.

Corollary 4.2.3

The double of a rational number is rational.

Solution:

Suppose r is any rational number. Then $2r = r + r$ is a sum of two rational numbers. So, by Theorem 4.2.2, $2r$ is rational.

Deriving Additional Results about Even and Odd Integers

Suppose you already proved the following properties of even and odd integers:

1. The sum, product, and difference of any two even integers are even.
2. The sum and difference of any two odd integers are even.
3. The product of any two odd integers is odd.
4. The product of any even integer and any odd integer is even.
5. The sum of any odd integer and any even integer is odd.
6. The difference of any odd integer minus any even integer is odd.
7. The difference of any even integer minus any odd integer is odd.

Use the properties listed above to prove that if *a* is any even integer and *b* is any odd integer, then $\frac{a^2+b^2+1}{2}$ **is an integer.**

➔ Try it at home

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Real Numbers In Programing: Example 1

What would be the datatype of the following

```
#Define P = 3.14
void main()
{   int a; float b; x ?
    x = a+ b+ P
    printf("The sum is: %?\n", x)
}
```

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Real Numbers In Programing: Example 2

When this program produces an error?

Revise example

```
int main (void)
{
   int j, a, b;
   Double x;
   scanf ("%d", a);
   scanf ("%d", b);
   x = (a*a + b*b + 1)/2;
   Y = (int) x
   X = floor(x)+1;
   If (x % 2== 0) | (x % 1 == 0)
     For (j = 1; j<y; ++j)
        Prinf("*");
   retrun (0);
```

add this statement to prevent errors!

➔ Number theory helps you write more robust programs

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Real Numbers in Real Life

Read at home

Two mechanics were working on a car. One can complete a given job in 6 hours. But, the new guy takes 8 hours. They work together for first two hours. But then, the first guy left to help another mechanic on a different job. How long will it take for the new guy to finish the car work?

The first guy can do 1/6 part of job per hour and the second guy can do 1/8 part of job per hour and together they can do 1/6 + 1/8part of job per hour. Now, let 't' hours is the time to complete the car job. So, 1/t job will be completed per hour, Equating the two expressions, we get:

1/6 + 1/8 = 1/t
7/24 = 1/t
As they work for 2 hours, 2 . 7/24 = 14/24 part of job will be done.
The work remaining is 1 - 1/t = (1 - 14/24)
= 10/24
∴ 10/24 job is left which has to be completed by the second guy, who will take 10/24 ÷ 1/8
= 40/12
= 10/3
= 3.33 hours to complete the car job.

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