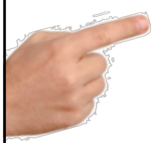
Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2015

Functions

7.1. Introduction to Functions

7.2 One-to-One, Onto, Inverse functions



7.3 Application: The Pigeonhole Principle

1

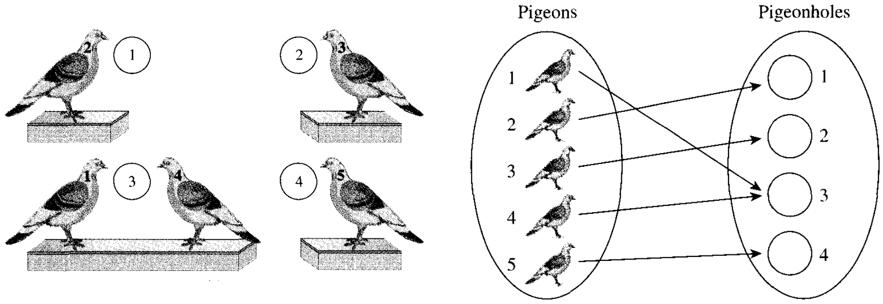
Watch this lecture and download the slides

Course Page: http://www.jarrar.info/courses/DMath/
More Online Courses at: http://www.jarrar.info

Acknowledgement:
This lecture is based on (but not limited to) to chapter 7 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

2

The Pigeonhole Principle



Pigeonhole Principle

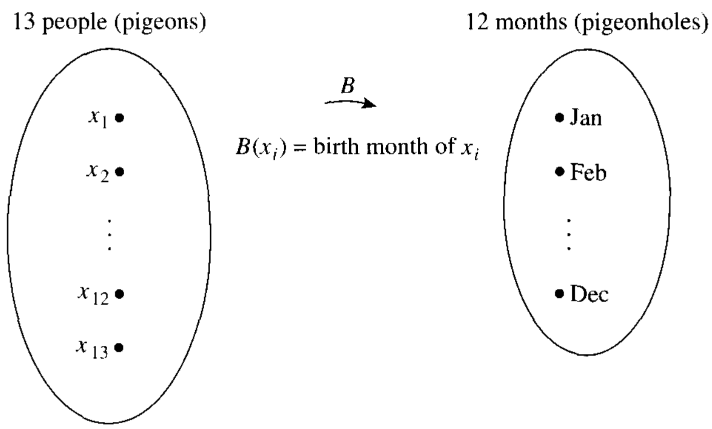
A function from one finite set to a smaller finite set cannot be one-to-one: There must be a least two elements in the domain that have the same image in the co-domain.

We will study this idea and use it to solve several types of problems

3

Applying the Pigeonhole Principle

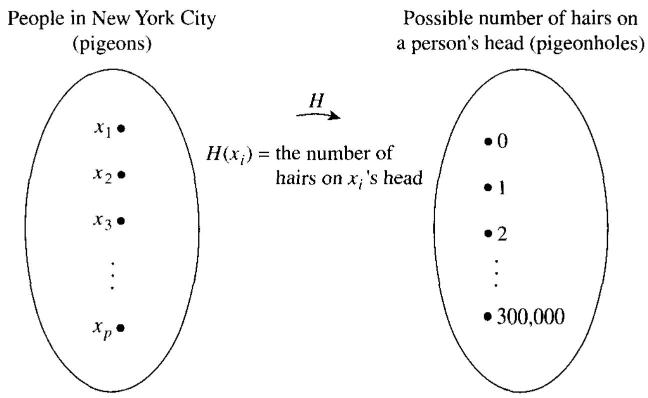
In a group of six people, must there be at least two who were born in the same month? If the the group is thirteen?



4

Applying the Pigeonhole Principle

Among the residents of New York City, must there be at least two people with the same number of hairs on their heads?



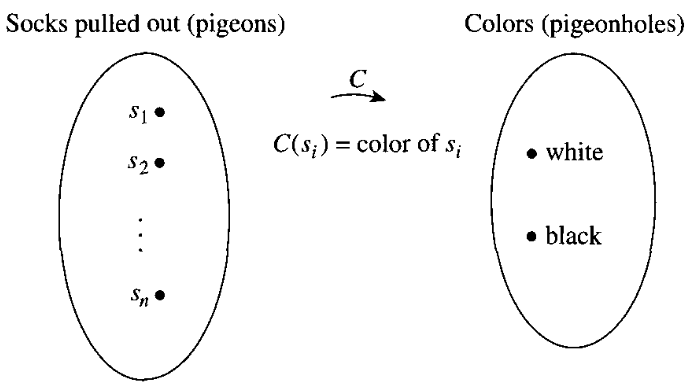
Given that (5M in New York, 3M hairs in a head)

,

5

Finding the Number to Pick to Ensure a Result

A drawer contains ten black and ten white socks. You reach in and pull some out without looking at them. What is the *least* number of socks you must pull out to be sure to get a matched pair? Explain how the answer follows from the pigeonhole principle



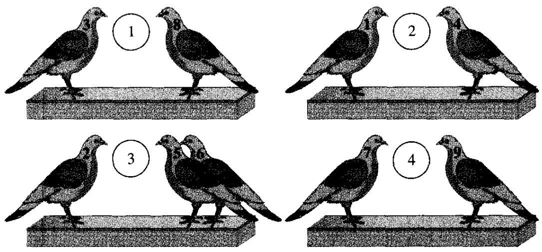
,

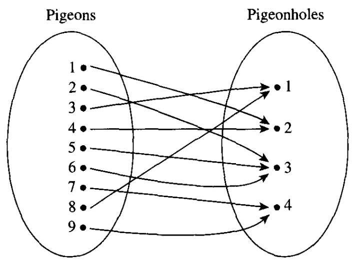
6

Generalized Pigeonhole Principle

if n pigeons fly into m pigeonholes and, for some positive integer *k*, $n > km$, then at least one pigeonhole contains $k+1$ or more pigeons.

For example: Let m = 4, n = 9, and k = 2.





,

7

Generalized Pigeonhole Principle

if n pigeons fly into m pigeonholes and, for some positive integer *k*, $n > k.m$, then at least one pigeonhole contains $k+1$ or more pigeons.

Generalized Pigeonhole Principle

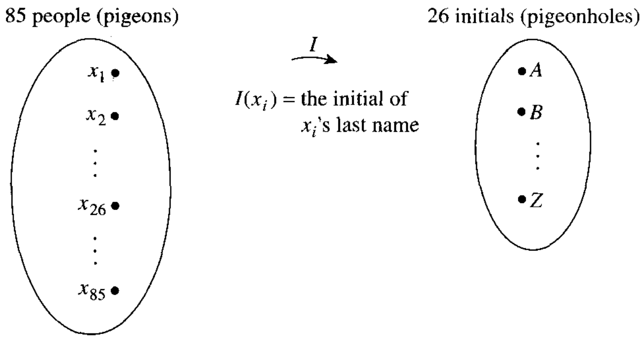
For any function f from a finite set X to a finite set Y and for any positive integer k, if $N(X) > k \cdot N(Y)$, then there is some $y \in Y$ such that y is the image of at least $k + 1$ distinct elements of X.

,

8

Applying the Generalized Pigeonhole Principle

Show how the generalized pigeonhole principle implies that in a group of 85 people, at least 4 must have the same last initial.



85 > 3 . 26 = 78. thus at least (3+1) must have the same last initial

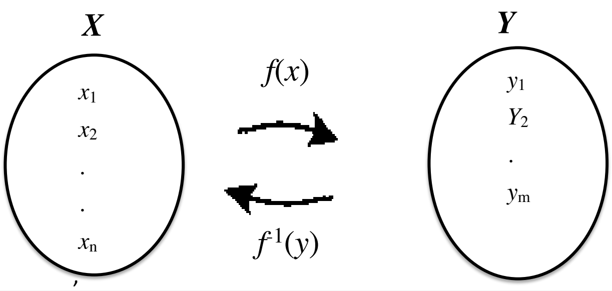
9

Generalized Pigeonhole Principle

An other way (Contrapositive) of forming the Pigeonhole Principle:
if we have m pigeonholes and n pigeons; for some positive integer, if each pigeonhole has at most k pigeons, then then are at most k. n pigeons.

Generalized Pigenohole Principle (Contrapositive Form)

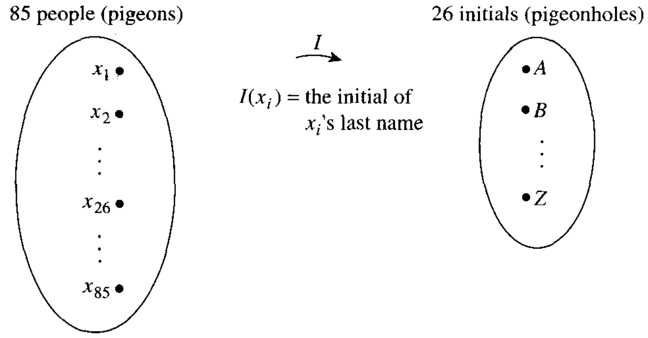
For any function f from a finite set X to a finite set Y and for any positive integer k, if for each $y \in Y$, $f^{-1}(y)$ has at most k elements, then X has at most $k \cdot N(Y)$ elements.



10

Applying the Generalized Pigeonhole Principle

Suppose no 4 people out of the 85 had the same last initial. Then at most 3 would share any particular one?



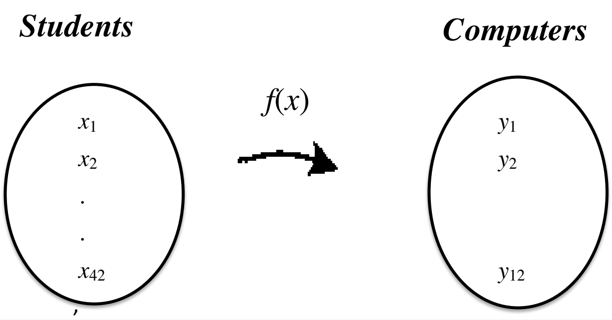
Total number of people is at most 3 . 26 = 78. But this contradicts the fact that there are 85 people in all. Hence at least 4 people share a last initial.

11

Using the Contrapositive Form of the Generalized Pigeonhole Principle

There are 42 students who are to share 12 computers. Each students uses exactly 1 computer, and no computer is used by more than 6 students. Show that at least 5 computers are used by 3 or more students.

k: number of computers used by 3 or more students.
➔We must show that k ≥ 5



12

Using the Contrapositive Form of the Generalized Pigeonhole Principle

There are 42 students who are to share 12 computers. Each students uses exactly 1 computer, and no computer is used by more than 6 students. Show that at least 5 computers are used by 3 or more students.

k: number of computers used by 3 or more students.
➔We must show that $k \geq 5$

6k: # of students using computers with 3 or more
12-k: # of computers used by at most 2 students
2(12-k) = 24-2k: # of students on computers used by 2 st. at most.
(6k) + (24-2k) = 4k + 24: max # of students
42: all students
$4k + 24 \geq 42$
So **$k \geq 5$**

13