Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2015

Relations

8.1. Introduction to Relations

8.2 Properties of Relations

8.3 Equivalence Relations

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 8 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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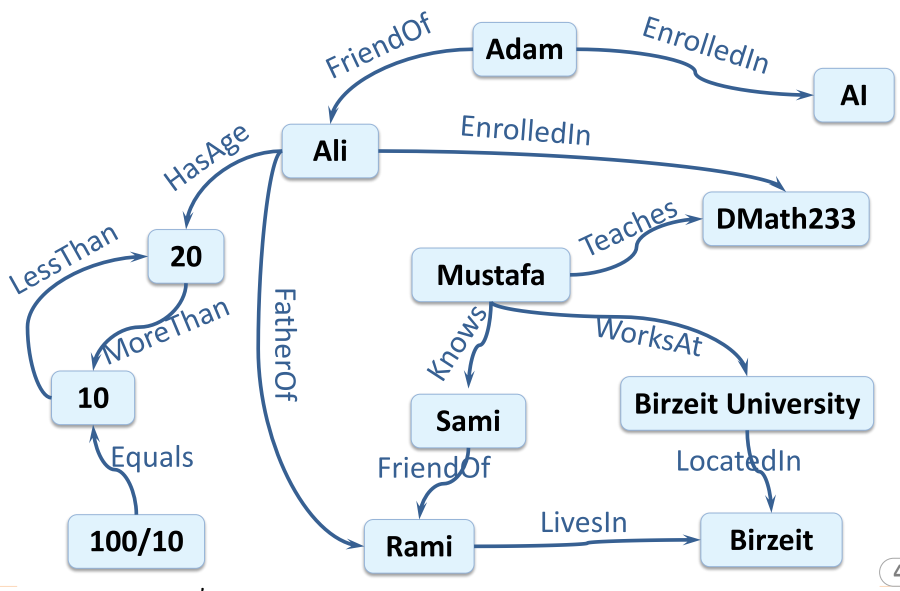
8.1 Introduction to Relations

In this lecture:

- ❑ Part 1: **What is a Relation**
- ❑ Part 2: Inverse of a Relation;
- ❑ Part 3: Directed Graphs;
- ❑ Part 4: n-ary Relations,
- ❑ Part 5: Relational Databases

What is a Relation?



What is a Relation?

Definition

Let A and B be sets. A **(binary) relation R from A to B** is a subset of $A \times B$. Given an ordered pair (x,y) in $A \times B$, x is related to y by R, written $x\ R\ y$, if and only if, (x, y) is in R.

$$x\ R\ y \Leftrightarrow (x,y) \in R$$

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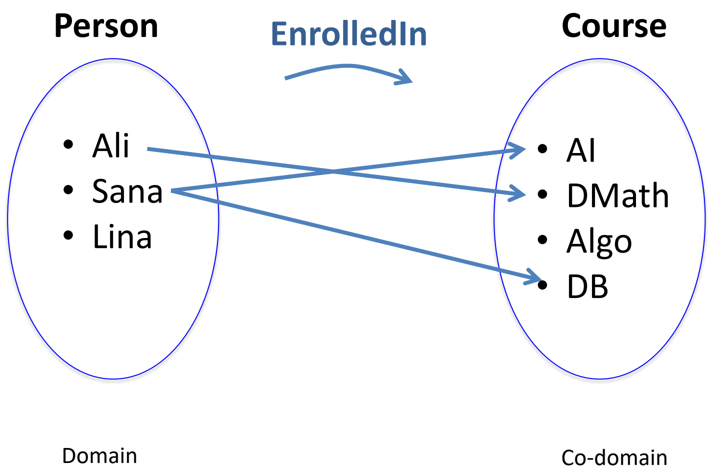
Example



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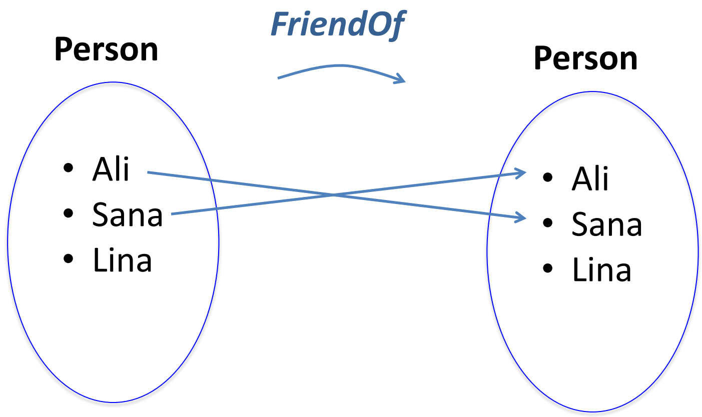
Example



$$FriendOf^I = \{(\text{Ali}, \text{Sana}), (\text{Sana}, \text{Ali})\}$$

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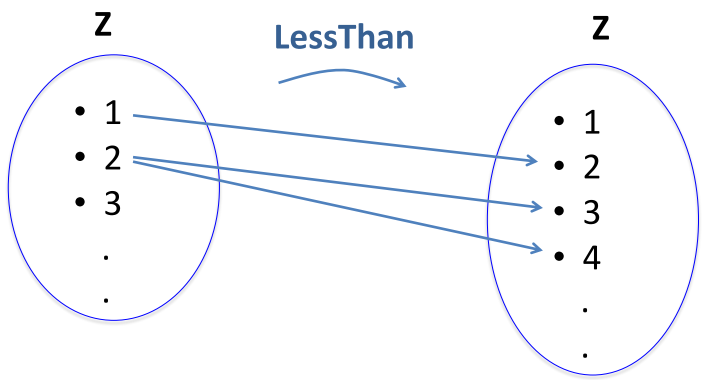
Example



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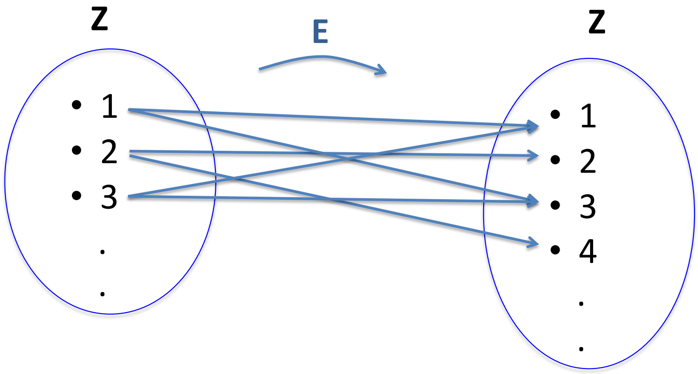
Example



Define a relation E from $\mathbf{Z}$ to $\mathbf{Z}$ as follows:

For all $(m, n) \in \mathbf{Z} \times \mathbf{Z}$, $m \; E \; n \Leftrightarrow m - n$ is even.

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Example: a relation on a Power Set

Let $X = \{a, b, c\}$.
Then $P(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.
Define a relation S from $P(X)$ to $\mathbf{Z}$ as follows: For all sets A and B in $P(X)$ (i.e., for all subsets A and B of X),

A S $B \Leftrightarrow A$ has at least as many elements as B.

Is $\{a,b\}$ S $\{b,c\}$? ✓ both sets have two elements.

Is $\{a\}$ S $\emptyset$? ✓ $\{a\}$ has one element and $\emptyset$ has zero elements, and $1 \geq 0$.

Is $\{b,c\}$ S $\{a,b,c\}$? ✗ $\{b, c\}$ has two elements and $\{a, b, c\}$ has three elements and $2 < 3$

Is $\{c\}$ S $\{a\}$? ✓ both sets have one element.

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Relations and Functions

Definition

A **function *F* from a set *A* to a set *B*** is a relation from *A* to *B* that satisfies the following two properties:

1. For every element x in A, there is an element y in B such that $(x,y) \in F$.
2. For all elements x in A and y and z in B,

$$\text{If } (x,y) \in F \text{ and } (x,z) \in F, \text{ then } y=z.$$

If F is a function from A to B, we write

$$Y = F(x) \Leftrightarrow (x,y) \in F.$$

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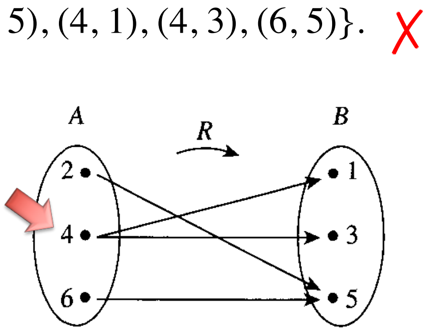
Example

Let A = {2 , 4, 6} and B = {1, 3 , 5}.
Is relation R a function from A to *B?*

$$R = \{(2, 5), (4, 1), (4, 3), (6, 5)\}.$$



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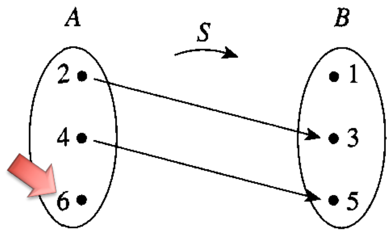
Example

Let A = {2 , 4, 6} and B = {1, 3 , 5}.
Is relation R a function from A to *B?*

For all $(x,y) \in A \times B$, $(x,y) \in S$ $y=x+1$. ✗



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- ❑ Part 1: What is a Relation
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- ❑ Part 4: *n*-ary Relations
- ❑ Part 5: Relational Databases

Inverse Relation

Definition

Let R be a relation from A to B. Define the inverse relation R^{-1} from B to A as follows:

$$R^{-1} = \{(y,x) \in B \times A \mid (x,y) \in R\}.$$

For all $x \in A$ and $y \in B$, $(y,x) \in R^{-1} \Leftrightarrow (x,y) \in R$.

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Example

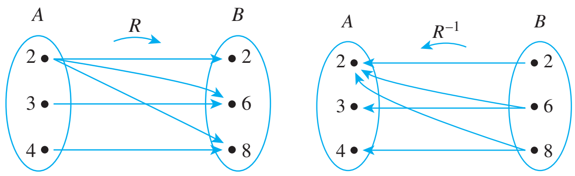
Let $A = \{2,3,4\}$ and $B = \{2,6,8\}$ and let R be the "divides" relation from A to B: For all $(x, y) \in A \times B$,

$$x \; R \; y \Leftrightarrow x \mid y \qquad x \text{ divides } y.$$

State explicitly which ordered pairs are in R and R^{-1}, and draw arrow diagrams for R and R^{-1}

$R = \{(2,2),(2,6),(2,8),(3,6),(4,8)\}$

$R^{-1} = \{(2,2),(6,2),(8,2),(6,3),(8,4)\}$



Describe R^{-1} in words:

For all $(y, x) \in B \times A$, $y \; R^{-1} \; x \Leftrightarrow y$ is a multiple of x.

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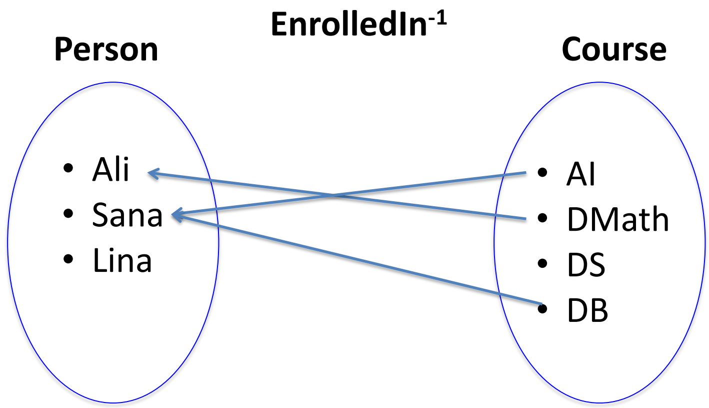
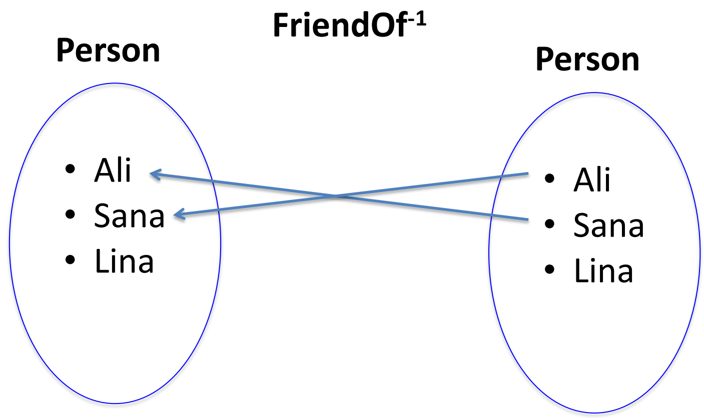
Example



Example



Inverse of Relations in Language

What would be the inverse of the following relations in English

SonOf $^{-1}$ = ?

WifeOf $^{-1}$ = ?

WorksAt $^{-1}$ = ?

EnrolledOf $^{-1}$ = ?

PresidentOf $^{-1}$ = ?

BrotherOf $^{-1}$ = ?

SisterOf $^{-1}$ = ?

....

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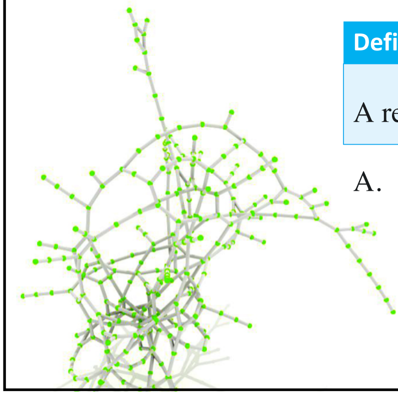
Directed Graph of a Relation

When a relation *R* is defined *on* a set *A*, the arrow diagram of the relation can be modified so that it becomes a **directed graph.**

For all points *x* and *y* in *A*,

there is an arrow from *x* to *y* $\Leftrightarrow$ $x\ R\ y$ $\Leftrightarrow$ $(x,y)\ R$.



Definition

A relation on a set A is a relation from A to A.

It is important to distinguish clearly between a relation and the set on which it is defined.

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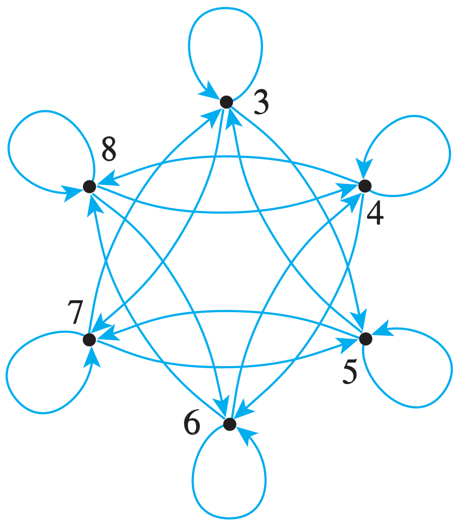
Example

Let $A = \{3, 4, 5, 6, 7, 8\}$ and define a relation *R* on *A* as follows: For all $x, y \in A$, $x\ R\ y \Leftrightarrow 2 \mid (x-y)$.



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N-ary Relations

EnrolledIn(Ali, Dmath) EnrolledIn(Sami, DB)	Binary (2-ary)
Enrollment(Sami, DB, 99)	Ternary (3-ary)
Enrollment(Sami, DB, 99, 2014)	Quaternary (4-ary)
Enrollment(Sami, DB, 99, 2014,F)	5-ary
$R(a_1, a_2, a_3, \ldots\ldots, a_n)$	*n*-ary

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N-ary Relations

- **Definition**

Given sets $A_1, A_2, \ldots, A_n$, an ***n*-ary relation** R on $A_1 \times A_2 \times \cdots \times A_n$ is a subset of $A_1 \times A_2 \times \cdots \times A_n$. The special cases of 2-ary, 3-ary, and 4-ary relations are called **binary, ternary,** and **quaternary relations,** respectively.

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Relational Databases

R on $A1 \times A2 \times A3 \times A4$ as follows:

$(a1, a2, a3, a4) \in R \Leftrightarrow$ a patient with patient ID number $a1$, named $a2$, was admitted on date $a3$, with primary diagnosis $a4$.

Relation

Each row is called **tuple**

Patient

ID	Name	Date	Diagnosis
(011985,	John Schmidt,	020710,	asthma)
(574329,	Tak Kurosawa,	114910,	pneumonia)
(466581,	Mary Lazars,	103910,	appendicitis)
(008352,	Joan Kaplan,	112409,	gastritis)
(011985,	John Schmidt,	021710,	pneumonia)
(244388,	Sarah Wu,	010310,	broken leg)
(778400,	Jamal Baskers,	122709,	appendicitis)

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Relational Databases

R on $A1 \times A2 \times A3 \times A4$ as follows:

$(a1, a2, a3, a4) \in R \Leftrightarrow$ a patient with patient ID number $a1$, named $a2$, was admitted on date $a3$, with primary diagnosis $a4$.

Relation

Each row is called **tuple**

Patient

ID	Name	Date	Diagnosis
(0			
(5			
(4			
(0			
(0			
(2			
(778400,	Jamal Baskers,	122709,	appendicitis)

- Notice that **Tables** in this way are called **Relations**.
- Information stored in this way is called a "**Relational Database**"

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