Mustafa Jarrar: Lecture Notes in Discrete Mathematics.
Birzeit University, Palestine, 2015

Counting

9.1 Basics of Probability and Counting

9.2 Possibility Trees and the Multiplication Rule

9.3 Counting Elements of Disjoint Sets: Addition Rule

9.5 Counting Subsets of a Set: Combinations

9.6 r-Combinations with Repetition Allowed

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 9 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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9.1 Basics of Probability and Counting

In this lecture:

- Part 1: **Probability and Sample Space**
- Part 2: **Counting in Sub lists**

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Tossing Coins

Tossing two coins and observing whether 0, 1, or 2 heads are obtained.

What are the chances of having 0,1,2 heads?

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What are the chances of having 0,1,2 heads?

2 heads obtained | 1 head obtained | 0 heads obtained

Event	Tally	Frequency (Number of times the event occurred)	Relative Frequency (Fraction of times the event occurred)
2 heads obtained	𝍸 𝍸 |	11	22%
1 head obtained	𝍸 𝍸 𝍸 𝍸 ||	27	54%
0 heads obtained	𝍸 𝍸 ||	12	24%

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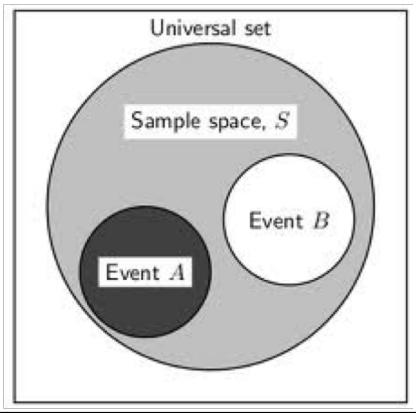
Sample Space

الفضاءالعيني

• Definition

A **sample space** is the set of all possible outcomes of a random process or experiment. An **event** is a subset of a sample space.



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Sample Space

Random Process

العملية العشوائية، تكون فيها مجموعة المخرجات محددة، ولكننا لا نستطيع معرفة اي من المخرجات ستكون النتيجة

Equally Likely Probability Formula

If S is a finite sample space in which all outcomes are equally likely and E is an event in S, then the **probability of E**, denoted $P(E)$, is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the total number of outcomes in } S}.$$

• Notation

For any finite set A, $N(A)$ denotes the number of elements in A.

$$P(E) = \frac{N(E)}{N(S)}.$$

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Probabilities for a Deck of Cards

52 cards

diamonds (♦)
hearts (♥)
clubs (♣)
spades (♠)

a. What is the sample space of outcomes?

➔ the 52 cards in the deck.

b. What is the event that the chosen card is a black face card?

➔ E = {J , Q , K , J , Q , K }

c. What is the probability that the chosen card is a black face card?

$$P(E) = \frac{N(E)}{N(S)} = \frac{6}{52} \cong 11.5\%.$$

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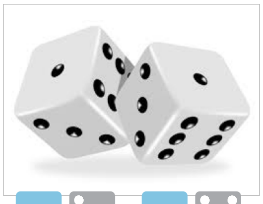
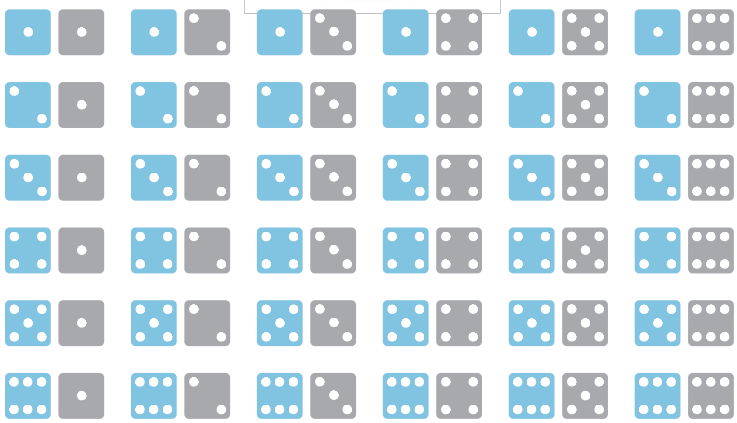
Rolling a Pair of Dice





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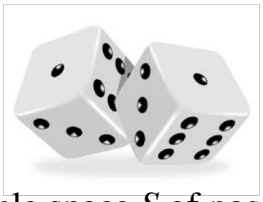
Rolling a Pair of Dice



a. Write the sample space S of possible outcomes.

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}.$

b. write the event E that the numbers showing face up have a sum of 6 and find the probability of this event.

$E = \{15, 24, 33, 42, 51\}.$ $\therefore P(E) = \frac{N(E)}{N(S)} = \frac{5}{36}.$

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Counting the Elements of a List

list:	5	6	7	8	9	10	11	12
	↕	↕	↕	↕	↕	↕	↕	↕
count:	1	2	3	4	5	6	7	8

list:	$m(=m+0)$	$m+1$	$m+2$	$\ldots$	$n\,(=m+(n-m))$
	↕	↕	↕		↕
count:	1	2	3	$\ldots$	$(n-m)+1$

Theorem 9.1.1 The Number of Elements in a List

If m and n are integers and $m \leq n$, then there are $n - m + 1$ integers from m to n inclusive.

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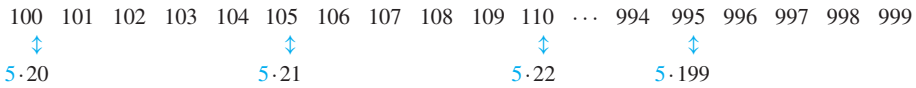
Counting the Elements of a Sublist

a. How many three-digit integers (integers from 100 to 999 inclusive) are divisible by 5?



b. What is the probability that a randomly chosen three-digit integer is divisible by 5?

999 − 100 + 1 = 900.

180/900 = 1/5.

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