حلول لبعض الاسئلة المختارة من شابتر 6

ملاحظة: الحل عبارة عن صور وليس نص مكتوب. واذا لم يوجد رقم السؤال فوق الصورة، فتكون الصورة(الحل) مكملة للصورة(للحل) السابقة.

على سبيل المثال سؤال رقم 8 من سكشن 1، له صورتان.

6.1

6.1.3

Let $R, S,$ and T be defined as:

$R = \{x \in \mathbf{Z} \mid x \text{ is divisible by } 2\}$
$S = \{y \in \mathbf{Z} \mid y \text{ is divisible by } 3\}$
$T = \{z \in \mathbf{Z} \mid z \text{ is divisible by } 6\}$

a.

The objective is to determine whether $R \subseteq T$.

For $2 \in \mathbf{Z}$, 2 is divisible by 2 and 2 is not divisible by 6.

That is, $2 \in R$ and $2 \notin T$.

Therefore, $R \not\subseteq T$.

Comment

Step **2** of 3

b.

The objective is to determine whether $T \subseteq R$.

Let $z \in T$.

Then by the definition of T, z is divisible by 6.

That is, $z = 6k$, for some integer k.

As $6 = 2 \cdot 3$, so $z = 2(3k)$.

Implies that z is divisible by 2 as $3 \in \mathbf{Z}, k \in \mathbf{Z} \Rightarrow 3k \in \mathbf{Z}$.

Thus, $z \in R$.

Therefore, $z \in T \Rightarrow z \in R$ follows that $T \subseteq R$.

Comment

Step **3** of 3

c.

The objective is to determine whether $T \subseteq S$.

Let $z \in T$.

Then by the definition of T, z is divisible by 6.

That is, $z = 6k$, for some integer k.

As $6 = 3 \cdot 2$, so $z = 3(2k)$.

Implies that z is divisible by 3 as $2 \in \mathbf{Z}, k \in \mathbf{Z} \Rightarrow 2k \in \mathbf{Z}$.

Thus, $z \in S$.

Therefore, $z \in T \Rightarrow z \in S$ follows that $T \subseteq S$.

6.1

6.1.4

Let $A = \{ n \in Z | n = 5r$ for some integer $r\}$ and $B = \{ m \in Z | m = 20s$ for some integer $s\}$.

(a)

Here $A \not\subset B$.

Explanation: Because there are elements of *A* that are not in *B*.

For example, 5 is in *A* because $5 = 5 \cdot 1$.

But 5 is not in *B*. because if it were, then $5 = 20s$ for some integer *s*.

Which would imply that $s = \frac{1}{4}$ and this contradict to fact that *s* is an integer.

Comment

Step 2 of 2

(b)

Here $B \subseteq A$.

Explanation: To explain $B \subseteq A$, we must show that every element of *B* is in *A*.

Suppose *m* is any element of *B*.

Then $m = 20s$ for some integer *s*.

Let $r = 4s$. Then *r* is an integer.

Because product of two integers is also an integer.

Now,

$$5r = 5 \cdot (4s)$$
$$= 20s$$
$$= m$$

Thus, *m* satisfies the condition for being in *A*.

Hence, every element of *B* is in *A*.

6.1

6.1.8

Let A, B be the subsets of universal set U.

(a)

The objective is to write the set $\{x \in U \mid x \in A \text{ and } x \in B\}$ in words.

Consider the set,

$X = \{x \in U \mid x \in A \text{ and } x \in B\}$

Write the set in words as below,

The set X contains all the elements of U, which are also the elements of both the sets A and B.

By the definition of intersection of the sets, the given set is same as $A \cap B$.

Therefore, the shorthand notation of the set is $\boxed{\{x \in U \mid x \in A \text{ and } x \in B\} = A \cap B}$.

Comment

Step 2 of 4

(b)

The objective is to write the set $\{x \in U \mid x \in A \text{ or } x \in B\}$ in words.

Consider the set,

$X = \{x \in U \mid x \in A \text{ or } x \in B\}$

Write the set in words as below,

The set X contains all the elements of U which are the elements of both the sets A and B or either the set A or the set B.

By the definition of union of the sets, the given set is same as $A \cup B$.

Therefore, the shorthand notation of the set is $\boxed{\{x \in U \mid x \in A \text{ or } x \in B\} = A \cup B}$.

6.1

(c)

The objective is to write the set $\{x \in U \mid x \in A \text{ and } x \notin B\}$ in words.

Consider the set,

$$X = \{x \in U \mid x \in A \text{ and } x \notin B\}$$

Write the set in words as below,

The set X contains all the elements of U which are the elements of the sets A and not an element of set B.

By the definition of difference of sets, the given set is same as $A - B$.

Therefore, the shorthand notation of the set is $\boxed{\{x \in U \mid x \in A \text{ and } x \notin B\} = A - B}$.

Comment

Step 4 of 4

(d)

The objective is to write the set $\{x \in U \mid x \notin A\}$ in words.

Consider the set,

$$X = \{x \in U \mid x \notin A\}$$

Write the set in words as below,

The set X contains all the elements of U which are not elements of the set A.

By the definition of complement of set, the given set is same as A^c.

Therefore, the shorthand notation of the set is $\boxed{\{x \in U \mid x \notin A\} = A^c}$.

6.1

6.1.10

(a) $A \cup B = \{1,3,5,6,7,9\}$

Comment

Step **2** of 8

(b) $A \cap B = \{3,9\}$

Comment

Step **3** of 8

(c) $A \cup C = \{1,2,3,4,5,6,7,8,9\}$

Comment

Step **4** of 8

(d) $A \cap C = \phi$

6.1

(e) $A - B = \{1,5,7\}$

Comment

Step 6 of 8

(f) $B - A = \{6\}$

Comment

Step 7 of 8

(g) $B \cup C = \{2,3,4,6,8,9\}$

Comment

Step 8 of 8

(h) $B \cap C = \{6\}$

6.1

6.1.12

The objective is to find $A \cup B$.

The union of A and B is:

$$\begin{aligned} A \cup B &= \{x \in \mathbf{R} \mid -3 \le x \le 0\} \cup \{x \in \mathbf{R} \mid -1 < x < 2\} \\ &= \{x \in \mathbf{R} \mid -3 \le x < 2\} \end{aligned}$$

Therefore, the interval for $A \cup B$ is $\boxed{[-3,2)}$.

Comment

Step 2 of 10

(b)

The objective is to find $A \cap B$.

The intersection of A and B is:

$$\begin{aligned} A \cap B &= \{x \in \mathbf{R} \mid -3 \le x \le 0\} \cap \{x \in \mathbf{R} \mid -1 < x < 2\} \\ &= \{x \in \mathbf{R} \mid -1 < x \le 0\} \end{aligned}$$

Therefore, the interval for $A \cap B$ is $\boxed{(-1,0]}$.

Comment

Step 3 of 10

(c)

The objective is to find A^c.

The complement of A is:

$$\begin{aligned} A^c &= \{x \in \mathbf{R} \mid \text{ it is not the case that } x \in [-3,0]\} \\ &= \{x \in \mathbf{R} \mid \text{it is not the case that } x \ge -3 \text{ and } x \le 0\} \\ &= \{x \in \mathbf{R} \mid x < -3 \text{ or } x > 0\} \\ &= (-\infty,-3) \cup (0,\infty) \end{aligned}$$

Therefore, the interval for A^c is $\boxed{(-\infty,-3) \cup (0,\infty)}$.

Comment

Step 4 of 10

(d)

The objective is to find $A \cup C$.

The union of A and C is:

$$\begin{aligned} A \cup C &= \{x \in \mathbf{R} \mid -3 \le x \le 0\} \cup \{x \in \mathbf{R} \mid 6 < x \le 8\} \\ &\{x \in \mathbf{R} \mid -3 \le x \le 0 \text{ or } 6 < x \le 8\} \\ &= [-3,0] \cup (6,8] \end{aligned}$$

Therefore, the interval for $A \cup C$ is $\boxed{[-3,0] \cup (6,8]}$.

6.1

(e)

The objective is to find $A \cap C$.

The intersection of A and C is:

$$\begin{aligned} A \cap C &= \{x \in \mathbf{R} \mid -3 \le x \le 0\} \cap \{x \in \mathbf{R} \mid 6 < x \le 8\} \\ &= \{x \in \mathbf{R} \mid -3 \le x \le 0 \text{ and } 6 < x \le 8\} \\ &= \phi \quad \begin{bmatrix} \text{since } -3 \le x \le 0 \text{ and } 6 < x \le 8 \\ \text{have no common elements} \end{bmatrix} \end{aligned}$$

Therefore, the interval for $A \cap C$ is $\boxed{\phi}$.

Comment

Step 6 of 10

(f)

The objective is to find B^c.

The complement of B is:

$$\begin{aligned} B^c &= \{x \in \mathbf{R} \mid \text{ it is not the case that } x \in (-1, 2)\} \\ &= \{x \in \mathbf{R} \mid \text{it is not the case that } x > -1 \text{ and } x < 2\} \\ &= \{x \in \mathbf{R} \mid x \le -1 \text{ or } x \ge 2\} \\ &= (-\infty, -1] \cup [2, \infty) \end{aligned}$$

Therefore, the interval for B^c is $\boxed{(-\infty, -1] \cup [2, \infty)}$.

Comment

Step 7 of 10

(g)

The objective is to find $A^c \cap B^c$.

The intersection of A^c and B^c, is:

$$\begin{aligned} A^c \cap B^c &= [(-\infty, -3) \cup (0, \infty)] \cap [(-\infty, -1] \cup [2, \infty)] \\ &= (-\infty, -3) \cup [2, \infty) \end{aligned}$$

Therefore, the interval for $A^c \cap B^c$ is $\boxed{(-\infty, -3) \cup [2, \infty)}$.

6.1

(i)

The objective is to find $(A\cap B)^c$.

The complement of $A\cap B$ is:

$$\begin{aligned}(A\cap B)^c &= \{x\in\mathbf{R} \mid \text{ it is not the case that } x\in(-1,0]\}\\ &= \{x\in\mathbf{R} \mid \text{it is not the case that } x>-1 \text{ and } x\le 0\}\\ &= \{x\in\mathbf{R} \mid x\le -1 \text{ or } x>0\}\\ &= (-\infty,-1]\cup(0,\infty)\end{aligned}$$

Therefore, the interval for $(A\cap B)^c$ is $\boxed{(-\infty,-1]\cup(0,\infty)}$.

Comment

Step **10** of 10

(j)

The objective is to find $(A\cup B)^c$.

The complement of $A\cup B$ is:

$$\begin{aligned}(A\cup B)^c &= \{x\in\mathbf{R} \mid \text{ it is not the case that } x\in[-3,2)\}\\ &= \{x\in\mathbf{R} \mid \text{it is not the case that } x\ge -3 \text{ and } x<2\}\\ &= \{x\in\mathbf{R} \mid x<-3 \text{ or } x\ge 2\}\\ &= (-\infty,-3)\cup[2,\infty)\end{aligned}$$

Therefore, the interval for $(A\cup B)^c$ is $\boxed{(-\infty,-3)\cup[2,\infty)}$.

6.1

6.1.19

$A_1 = \{1, 1^2\} = \{1\}$
$A_2 = \{2, 2^2\} = \{2, 4\}$
$A_3 = \{3, 3^2\} = \{3, 9\}$
$A_4 = \{4, 4^2\} = \{4, 16\}$

Comment

Step **2** of 4

(a)

$$A_1 \cup A_2 \cup A_3 \cup A_4 = \{1\} \cup \{2,4\} \cup \{3,9\} \cup \{4,16\}$$
$$= \{1,2,3,4,9,16\}$$

Comment

Step **3** of 4

(b)

$$A_1 \cap A_2 \cap A_3 \cap A_4 = \{1\} \cap \{2,4\} \cap \{3,9\} \cap \{4,16\}$$
$$= \phi$$

Comment

Step **4** of 4

(c) $A_1, A_2, A_3 \text{and} A_4$ are not mutually disjoint because $A_2 \cap A_4 = \{4\} \neq \phi$

6.1

6.1.20

(a)

Consider the set,

$B_1 \cup B_2 \cup B_3 \cup B_4$.

The objective is to determine the set $B_1 \cup B_2 \cup B_3 \cup B_4$ from the above defined sets.

From the definition of set union,

$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$.

So, the set $B_1 \cup B_2 \cup B_3 \cup B_4$ can be defined as follows:

$$\begin{aligned} B_1 \cup B_2 \cup B_3 \cup B_4 &= \{x \in \mathbf{R} \mid x \in [0,1] \text{ or } x \in [0,2] \text{ or } x \in [0,3] \text{ or } x \in [0,4]\} \\ &= \{x \in \mathbf{R} \mid x \in [0,4]\} \\ &= \{x \in \mathbf{R} \mid 0 \le x \le 4\} \end{aligned}$$

Thus, the required result is,

$\boxed{B_1 \cup B_2 \cup B_3 \cup B_4 = \{x \in \mathbf{R} \mid 0 \le x \le 4\} \text{ or } [0,4]}$.

Comment

Step 2 of 3

(b)

Consider the set,

$B_1 \cap B_2 \cap B_3 \cap B_4$.

The objective is to determine the set $B_1 \cap B_2 \cap B_3 \cap B_4$ from the above defined sets.

From the definition of set intersection,

$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$.

So, the set $B_1 \cap B_2 \cap B_3 \cap B_4$ can be defined as follows:

$$\begin{aligned} B_1 \cap B_2 \cap B_3 \cap B_4 &= \{x \in \mathbf{R} \mid x \in [0,1] \text{ and } x \in [0,2] \text{ and } x \in [0,3] \text{ and } x \in [0,4]\} \\ &= \{x \in \mathbf{R} \mid x \in [0,1]\} \\ &= \{x \in \mathbf{R} \mid 0 \le x \le 1\} \end{aligned}$$

Thus, the required result is,

$\boxed{B_1 \cap B_2 \cap B_3 \cap B_4 = \{x \in \mathbf{R} \mid 0 \le x \le 1\} \text{ or } [0,1]}$.

Comment

Step 3 of 3

(c)

$\boxed{\text{No}}$.

A collection of sets are called mutually disjoint, if the intersection of any two sets must be an empty set.

It is clear that the sets B_1, B_2, B_3, and B_4 have elements common in pairwise, so they are not mutually disjoint.

6.1

6.1.29

Let $\mathbf{R}$ be the set of all real numbers.

Define the set $\mathbf{R}^+$ is the set which contains all the positive real numbers and the set $\mathbf{R}^-$ contains all the negative real numbers.

The objective is to check that the set $\{\mathbf{R}^+, \mathbf{R}^-, \{0\}\}$ is partition of real number set $\mathbf{R}$.

Partition of the set:

Let the set $\{A_1, A_2, \ldots A_n\}$ be the subsets of A. The set $\{A_1, A_2, \ldots A_n\}$ is said to be partition of A, if each the elements are mutually disjoint and $A_1 \cup A_2 \cup \ldots \cup A_n = A$.

Comment

Step 2 of 3

Check that whether the set $\{\mathbf{R}^+, \mathbf{R}^-, \{0\}\}$ is mutually disjoint set or not.

By the definition of the sets $\mathbf{R}^+$ and $\mathbf{R}^-$, there are no common elements in both the sets $\mathbf{R}^+$ and $\mathbf{R}^-$.

Thus, $\mathbf{R}^+ \cap \mathbf{R}^- = \varnothing$

By the definition of the set $\mathbf{R}^+$, the set does not contain the element 0.

Thus, $\mathbf{R}^+ \cap \{0\} = \varnothing$

By the definition of the set $\mathbf{R}^-$, the set does not contain the element 0.

Thus, $\mathbf{R}^- \cap \{0\} = \varnothing$

By the definition of mutually disjoint, the set $\{\mathbf{R}^+, \mathbf{R}^-, \{0\}\}$ is mutually disjoint set.

Comment

Step 3 of 3

As the real number set contains negative numbers, zero and positive numbers so the real numbers set can be write as union of sets $\mathbf{R}^-, \{0\}$, and $\mathbf{R}^+$.

Therefore, $\mathbf{R} = \mathbf{R}^- \cup \{0\} \cup \mathbf{R}^+$

The set $\{\mathbf{R}^+, \mathbf{R}^-, \{0\}\}$ satisfies all the conditions of the partition of the set.

Hence, the set $\{\mathbf{R}^+, \mathbf{R}^-, \{0\}\}$ is a partition of the set.

6.1

6.1.31

Consider the sets $A = \{1,2\}$ and $B = \{2,3\}$.

Power set of set A:

Let A be any set. The set $P(A)$ contains all the subsets of the set A and the set is called as power set of A

(a)

Objective is to find the power set $P(A \cap B)$.

By the definition of intersection of sets, $A \cap B = \{2\}$

The power set of $A \cap B$ set is set of all subsets of the set $\{2\}$.

$$P(A \cap B) = \{\phi, \{2\}\}$$

Therefore, the power set of $A \cap B$ is $\boxed{P(A \cap B) = \{\phi, \{2\}\}}$

Comment

Step 2 of 4

(b)

Objective is to find the power set $P(A)$.

The power set of A set is set of all subsets of the set $\{1,2\}$.

$$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$$

Therefore, the power set of A is $\boxed{P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}}$.

6.1

(c)

Objective is to find the power set $P\ (A \cup B)$.

By the definition of intersection of sets, $A \cup B = \{1,2,3\}$

The power set of $A \cup B$ set is set of all subsets of the set $\{1,2,3\}$.

$$P\ (A \cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Therefore, the power set of $A \cup B$ is,

$$\boxed{P\ (A \cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}}$$

Comment

Step **4** of 4

(d)

Objective is to find the power set $P\ (A \times B)$.

By the definition of cross product of sets,

$$A \times B = \{(1,2),(1,3),(2,2),(2,3)\}$$

The power set of $A \times B$ set is set of all subsets of the set $\{(1,2),(1,3),(2,2),(2,3)\}$.

$$P\ (A \times B) = \left\{\begin{array}{l} \phi, \{(1,2)\}, \{(1,3)\}, \{(2,2)\}, \{(2,3)\}, \{(1,2),(1,3)\}, \\ \{(1,2),(2,2)\}, \{(1,2),(2,3)\}, \{(1,3),(2,2)\}, \\ \{(1,3),(2,3)\}, \{(2,2),(2,3)\}, \{(1,2),(1,3),(2,2)\}, \\ \{(1,2),(1,3),(2,3)\}, \{(1,2),(2,2),(2,3)\}, \\ \{(1,3),(2,2),(2,3)\}, \{(1,2),(1,3),(2,2),(2,3)\} \end{array}\right\}$$

Therefore, the power set of $A \times B$ is,

$$\boxed{P\ (A \times B) = \left\{\begin{array}{l} \phi, \{(1,2)\}, \{(1,3)\}, \{(2,2)\}, \{(2,3)\}, \{(1,2),(1,3)\}, \\ \{(1,2),(2,2)\}, \{(1,2),(2,3)\}, \{(1,3),(2,2)\}, \\ \{(1,3),(2,3)\}, \{(2,2),(2,3)\}, \{(1,2),(1,3),(2,2)\}, \\ \{(1,2),(1,3),(2,3)\}, \{(1,2),(2,2),(2,3)\}, \\ \{(1,3),(2,2),(2,3)\}, \{(1,2),(1,3),(2,2),(2,3)\} \end{array}\right\}}$$

6.1

6.1.33

(a)

Objective is to find the power set $P(\varnothing)$.

Power set of set A:

Let A be any set. The set $P(A)$ contains all the subsets of the set A and the set is called as power set of A

The power set of $\varnothing$ set is set of all subsets of the set $\varnothing$.

The only subset of the empty set is the set itself, thus

$P(\varnothing)=\{\varnothing\}$

Therefore, the power set of $\varnothing$ is $\boxed{P(\varnothing)=\{\varnothing\}}$

Comment

Step 2 of 3

(b)

Objective is to find the power set $P(P(\varnothing))$.

The power set of $P(\varnothing)$ set is set of all subsets of the set $P(\varnothing)=\{\varnothing\}$.

$P(P(\varnothing))=\{\varnothing,\{\varnothing\}\}$

Therefore, the power set of A is $\boxed{P(P(\varnothing))=\{\varnothing,\{\varnothing\}\}}$.

Comment

Step 3 of 3

(c)

Objective is to find the power set $P(P(P(\varnothing)))$.

The power set of $P(P(\varnothing))$ set is set of all subsets of the set $P(P(\varnothing))=\{\varnothing,\{\varnothing\}\}$.

$P(P(P(\varnothing)))=\{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}$

Therefore, the power set of A is $\boxed{P(P(P(\varnothing)))=\{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}}$.

6.1

6.1.35

Given $A = \{a, b\}$, $B = \{1, 2\}$ and $C = \{2, 3\}$

Comment

Step 2 of 5

a)

$B \cup C = \{1, 2, 3\}$

$A \times (B \cup C) = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Comment

Step 3 of 5

b)

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$A \times C = \{(a, 2), (a, 3), (b, 2), (b, 3)\}$

$(A \times B) \cup (A \times C) = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Comment

Step 4 of 5

c)

$B \cap C = \{2\}$

$A \times (B \cap C) = \{(a, 2), (b, 2)\}$

Comment

Step 5 of 5

d) $(A \times B) \cap (A \times C) = \{(a, 2), (b, 2)\}$

6.2

6.2.9

Let A, B and C are any three sets.

Prove that $(A-B)\cup(C-B)=(A\cup C)-B.$

First prove that $(A-B)\cup(C-B)\subseteq(A\cup C)-B.$

Suppose there is an element x in $(A-B)\cup(C-B)$, by definition of union,
$x\in A-B$ or $x\in C-B.$

Case I:

If $x\in A-B$ then by set difference $x\in A$ and $x\notin B.$

Since $x\in A$, by definition of union it follows that $x\in A\cup C.$

Thus, $x\in A\cup C$ and $x\notin B$, the definition of set difference implies that $x\in(A\cup C)-B.$

Hence, $x\in A-B\Rightarrow x\in(A\cup C)-B.$

Case II:

If $x\in C-B$ then by set difference $x\in C$ and $x\notin B.$

Since $x\in C$, by definition of union it follows that $x\in A\cup C.$

Thus, $x\in A\cup C$ and $x\notin B$, the definition of set difference implies that $x\in(A\cup C)-B.$

Hence, $x\in C-B\Rightarrow x\in(A\cup C)-B.$

Therefore, both the cases yields $x\in(A\cup C)-B.$

Therefore, $(A-B)\cup(C-B)\subseteq(A\cup C)-B.$

Comment

Step **2** of 2

Now prove that $(A\cup C)-B\subseteq(A-B)\cup(C-B).$

Suppose there is an element x in $(A\cup C)-B$, by definition of set difference,
$x\in A\cup C$ and $x\notin B.$

Since $x\in A\cup C$, by definition of union $x\in A$ or $x\in C.$

Case I:

If $x\in A$ and $x\notin B$ then the definition of set difference implies that $x\in A-B.$

Case II:

If $x\in C$ and $x\notin B$ then the definition of set difference implies that $x\in C-B.$

From case I and II and by definition of union it follows that $x\in(A-B)\cup(C-B).$

Therefore, $(A\cup C)-B\subseteq(A-B)\cup(C-B).$

Now both set containments are proved.

Hence, by definition of set equality, conclude that $(A-B)\cup(C-B)=(A\cup C)-B.$

6.2

6.2.11

Let A and B be any sets.

The objective is to prove that $A \cup (A \cap B) = A$ for all sets A and B.

To show $A \cup (A \cap B) = A$, it is enough to show that $A \cup (A \cap B) \subseteq A$ and $A \subseteq A \cup (A \cap B)$.

Comment

Step 2 of 3

Show that $A \cup (A \cap B) \subseteq A$.

That is $\forall x$, if $x \in A \cup (A \cap B)$, then $x \in A$

Let $x \in A \cup (A \cap B)$

By the definition of union, $x \in A$ or $x \in A \cap B$

By the definition of intersection, $x \in A$ or $[x \in A \text{ and } x \in B]$

This implies that, $x \in A$

Therefore, $A \cup (A \cap B) \subseteq A$

Comment

Step 3 of 3

Show that $A \subseteq A \cup (A \cap B)$.

That is $\forall x$, if $x \in A$, then $x \in A \cup (A \cap B)$

Let $x \in A$

By the definition of union, $x \in A \cup (A \cap B)$ because $x \in A$.

Therefore, $A \subseteq A \cup (A \cap B)$

Hence, from both the proofs, the required result is $\boxed{A \cup (A \cap B) = A}$.

6.2

6.2.14

The objective is to prove that $A \cup C \subseteq B \cup C$ if $A \subseteq B$ for all the sets A, B and C.

Suppose A, B and C are sets and $A \subseteq B$.

Let $x \in A \cup C$, then by definition of union either $x \in A$ or $x \in C$.

Case (i):

Suppose that $x \in A$.

Then, $x \in B$ since $A \subseteq B$.

Thus, it is true that $x \in B$ or $x \in C$.

Hence, by the definition of union $x \in B \cup C$.

Comment

Step 2 of 2

Case (ii):

Suppose that $x \in C$.

Then, by the definition of union $x \in B \cup C$.

Thus, in both the cases, $x \in B \cup C$.

Therefore, $\boxed{A \cup C \subseteq B \cup C}$ if $A \subseteq B$ for all the sets A, B and C.

6.2.16

Let $A, B,$ and C be any sets.

The objective is to prove that if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

To show $A \subseteq B \cap C$, it is enough to show that $\forall x,$ if $x \in A,$ then $x \in B \cap C$.

Let $x \in A$ and $A \subseteq B, A \subseteq C$.

By the definition of subset, $x \in B$ and $x \in C$.

By the definition of intersection, $x \in B \cap C$.

Therefore, for any x, if $x \in A$ then $x \in B \cap C$.

By the definition of subset, $A \subseteq B \cap C$.

Therefore, the result $\boxed{A \subseteq B \cap C}$ is true for all sets $A, B,$ and C with $A \subseteq B$ and $A \subseteq C$.

6.2

6.2.19

The objective is to prove that for all sets A, B and C, $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

To prove that $A\times(B\cap C)=(A\times B)\cap(A\times C)$, prove that

(i) $A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ and

(ii) $(A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$

Comment

Step 2 of 5

Prove that $A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$.

Suppose $(x,y)\in A\times(B\cap C)$.

Then, $x\in A$ and $y\in B\cap C$.

By definition of intersection, $y\in B$ and $y\in C$.

Comment

Step 3 of 5

As $x\in A$ and $y\in B$, $(x,y)\in A\times B$.

Also, As $x\in A$ and $y\in C$, $(x,y)\in A\times C$.

By definition of intersection, since $(x,y)\in A\times B$ and $(x,y)\in A\times C$,

$(x,y)\in(A\times B)\cap(A\times C)$

Therefore, $A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ ....(1)

6.2

Prove that $(A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$.

Suppose $(x,y)\in(A\times B)\cap(A\times C)$.

By definition of intersection,

$(x,y)\in(A\times B)$ and $(x,y)\in(A\times C)$

Now,

$(x,y)\in(A\times B) \Rightarrow x\in A$ and $y\in B$

and $(x,y)\in(A\times C) \Rightarrow x\in A$ and $y\in C$

Comment

Step 5 of 5

As $y\in B$ and $y\in C$, $y\in B\cap C$ (by definition if intersection)

Also, $x\in A$.

Therefore,

$(x,y)\in A\times(B\cap C)$

Hence, $(A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$. ......(2)

From (1) and (2), $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

6.2

6.2.31

Consider the statement as,

"For all sets A and B, if $B \subseteq A^c$ then $A \cap B = \phi$."

The objective is to prove the above statement is true.

Suppose that $A \cap B \neq \phi$.

Let $x \in A \cap B$ for some element say x.

Then, by the definition of intersection, $x \in A$ and $x \in B$. ...... (1)

Comment

Step 2 of 2

Given that $B \subseteq A^c$ and $x \in B$.

Then, by the definition of subset, $x \in A^c$.

By the definition of compliment, $x \notin A$. ...... (2)

From, (1) and (2), $x \in A$ and $x \notin A$.

This is a contradiction,

Therefore, the assumption that $A \cap B \neq \phi$ is false.

Hence, the statement "For all sets A and B, if $B \subseteq A^c$ then $A \cap B = \phi$." is true.

6.2

6.2.36

(a)

Let A and B be two sets.

Prove that $(A-B)\cup(B-A)\cup(A\cap B)=A\cup B.$

First we prove that $(A-B)\cup(B-A)\cup(A\cap B)\subseteq A\cup B.$

Observe the following conditions:

(1) If $x\in A-B$ that implies $x\in A$ and $x\notin B.$

(2) If $x\in B-A$ that implies $x\in B$ and $x\notin A.$

(3) If $x\in A\cap B$ that implies $x\in A$ and $x\in B.$

The set $A\cup B$ contains all the elements of the set A and of the set $B.$

So if an element x satisfies any one of the above three conditions then x must be in the union of A and $B.$

Therefore, $(A-B)\cup(B-A)\cup(A\cap B)\subseteq A\cup B$ .... (1)

Similarly, prove that $A\cup B\subseteq(A-B)\cup(B-A)\cup(A\cap B).$

If $x\in A\cup B$ then it must be in one of the sets A and $B.$

Then it satisfies exactly one of the above three conditions.

That implies that $x\in(A-B)\cup(B-A)\cup(A\cap B).$

Therefore, $A\cup B\subseteq(A-B)\cup(B-A)\cup(A\cap B)$ .... (2)

From (1) and (2) conclude that $(A-B)\cup(B-A)\cup(A\cap B)=A\cup B.$

Comment

Step 2 of 2

(b)

Show that the sets $(A-B),(B-A)$ and $(A\cap B)$ are mutually disjoint.

In order to prove these three sets are mutually disjoint, show that the intersection of any two of them is the empty set.

Consider the three conditions in the part (a).

Notice that, the three conditions are mutually exclusive and so no two of them can be satisfied at the same time.

That implies that, the intersection of any two of the sets doesn't contain any element.

Therefore, the intersection of any two of the sets is the empty set.

Hence, conclude that the sets $(A-B),(B-A)$ and $(A\cap B)$ are mutually disjoint.

6.3

6.3.4

Consider the statement as,

"For all sets A, B and C, if $B \cap C \subseteq A$ then $(A-B)\cap(A-C)=\phi$."

The objective is to find a counter example to show that the above statement is false.

Consider that all sets are subsets of a universal set U.

Counterexample:

Let U be a universal set defined as,

$$U=\{1,2,3,4,5,6\}$$

Suppose for some sets A and B defined as,

$$A=\{1,4\}, B=\{1,2,3\} \text{ and } C=\{1,5,6\}.$$

$$\begin{aligned} B\cap C &= \{1,2,3\}\cap\{1,5,6\} \\ &= \{1\} \\ &\subset A \qquad \text{......}(1) \end{aligned}$$

Comment

Step 2 of 2

Now, consider the value of $(A-B)\cap(A-C)$.

$$\begin{aligned} (A-B)\cap(A-C) &= \{4\}\cap\{4\} \\ &= \{4\} \\ &\neq \phi \quad \text{......}(2) \end{aligned}$$

From (1) and (2), though $B \cap C \subseteq A$, it is not necessary that $(A-B)\cap(A-C)=\phi$ is true.

Therefore, the statement "For all sets A, B and C, if $B \cap C \subseteq A$ then $(A-B)\cap(A-C)=\phi$."

is false.

6.3

6.3.7

Consider the following statement,

"For all sets $A, B,$ and $C, (A-B)\cap(C-B)=A-(B\cup C)$."

The objective is to prove the above statement if it is true else provide a counter example.

The above statement is **false**.

Observe the following counter example,

Let $A=\{x,y,z\}, B=\{y,z,w,q\}, C=\{x,z,p\}$ be the subsets of a universal set U.

Then,

$$\begin{aligned} A-B &= \{x,y,z\}-\{y,z,w,q\} \\ &= \{x\} \end{aligned}$$

$$\begin{aligned} C-B &= \{x,z,p\}-\{y,z,w,q\} \\ &= \{x,p\} \end{aligned}$$

Thus, $(A-B)\cap(C-B)=\{x\}\cap\{x,p\}=\{x\}$ ...... (1)

Comment

Step 2 of 2

Now,

$$\begin{aligned} (B\cup C) &= \{y,z,w,q\}\cup\{x,z,p\} \\ &= \{x,y,z,w,p,q\} \end{aligned}$$

$$\begin{aligned} A-(B\cup C) &= \{x,y,z\}-\{x,y,z,w,p,q\} \\ &= \phi \qquad \text{......}(2) \end{aligned}$$

From (1) and (2), it is clear that,

$$\boxed{(A-B)\cap(C-B)\neq A-(B\cup C)}.$$

6.3

6.3.10

Consider the statement,

If $A \subseteq B$, then $A \cap B^c = \varnothing$.

The objective is to verify whether the statement is true or not.

Recall, the **definition for Unions, and, Intersections**,

Suppose *A*, and, *B* are two subsets of a universal set *U*.

1. For a set *A* to be a subset of a set *B* is denoted by $A \subseteq B$, and, is defined by,

$A \subseteq B : \{\forall x, \text{if } x \in A \text{ then } x \in B\}$.

2. The Complement of *A* is denoted by A^c, and, is defined by,

$A^c = \{x \in U \mid x \notin A\}$.

3. The intersection of *A*, and, *B* is denoted by $A \cap B$, and, is defined by,

$A \cap B = \{x \in U \mid x \in A, \text{ and, } x \in B\}$.

Comment

Step 2 of 2

Prove that whether the statement is true or false as,

Let *A*, *B*, be any sets and given that $A \subseteq B$.

To prove, $A \cap B^c = \varnothing$.

Conversely assume that $A \cap B^c \neq \varnothing$.

That is, there exists an element $x \in A \cap B^c$

$x \in A \cap B^c$

$\Rightarrow x \in A \text{ and } x \in B^c$ Intersection of two sets

$\Rightarrow x \in A \text{ and } x \notin B$ **Complement of a set**

$\Rightarrow x \in A \text{ and } x \notin A$ **Since $A \subseteq B$**

Since $x \in A \text{ and } x \notin A$ which is not possible.

Therefore, which is contradiction to our assumption $A \cap B^c \neq \varnothing$.

Thus, $A \cap B^c = \varnothing$

That is, For all sets *A* and *B*, if $A \subseteq B$ then $A \cap B^c = \varnothing$.

Hence, the given statement is **True**.

6.3

6.3.12

Consider the statement,

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

For all sets A, B and C.

Here all sets are subsets of a universal set U.

The objective is to verify whether the statement is true or not.

Comment

Step 2 of 4

Recall, the **definition for Differences**, **and, Intersections**,

Suppose A and B are two subsets of a universal set U.

1. The difference of B minus A is denoted by $B - A$, and, is defined by,

$$B - A = \{x \in U \mid x \in B, \text{ and } x \notin A\}.$$

2. The intersection of A, and B is denoted by $A \cap B$, and is defined by,

$$A \cap B = \{x \in U \mid x \in A, \text{ and } x \in B\}.$$

6.3

Let A, B, and C be any sets.

Consider the left hand side:

$A \cap (B - C)$

Let $x \in A \cap (B - C)$

$\Rightarrow x \in A$ and $x \in (B - C)$ Intersection of two sets

$\Rightarrow x \in A$ and $x \in B$ and $x \notin C$ Difference of two sets

$\Rightarrow x \in (A \cap B)$ and $x \notin (A \cap C)$ Intersection of two sets

$\Rightarrow x \in (A \cap B) - (A \cap C)$ Difference of two sets

Thus,

$A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ...... (1)

Comment

Step 4 of 4

Consider the right hand side:

$(A \cap B) - (A \cap C)$

Let $y \in (A \cap B) - (A \cap C)$

$\Rightarrow y \in (A \cap B)$ and $y \notin (A \cap C)$ Difference of two sets

$\Rightarrow y \in A$ and $y \in B$ and $y \notin C$ Intersection of two sets

$\Rightarrow y \in A$ and $y \in (B - C)$ Difference of two sets

$\Rightarrow y \in A \cap (B - C)$ Intersection of two sets

Thus,

$(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ...... (2)

From (1) and (2),

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Therefore, the given statement is **True**.

6.3

6.3.15

Assume that $A \cap C = B \cap C$ and $A \cup C = B \cup C$. for all sets $A, B,$ and C.

Thus, the statement $A = B$ is true.

Proof: To prove $A = B$, we must show that $A \subseteq B$ and $B \subseteq A$.

Now, there are two cases: either $x \in C$ or $x \notin C$.

Case 1: When $x \in C$.

First to show $A \subseteq B$.

Assume that $x \in A$. Then we must show that $x \in B$ to prove $A \subseteq B$.

Since $x \in A$ and $x \in C$. Then $x \in A \cap C$.

Now, $A \cap C = B \cap C$.

Which implies that $A \cap C \subseteq B \cap C \quad ...(1)$

and $B \cap C \subseteq A \cap C. \quad ...(2)$

Because $x \in A \cap C$.

Thus, $x \in B \cap C$. (from 1.)

Which implies that $x \in B$ and $x \in C$.

Now, since $x \in B$ implies that $A \subseteq B. \quad ...(a)$

Comment

Step 2 of 4

Now, to show $B \subseteq A$.

Assume that $x \in B$. Then we must show that $x \in A$ to prove $B \subseteq A$.

Since $x \in B$ and $x \in C$. Then $x \in B \cap C$.

Now, $A \cap C = B \cap C$.

Which implies that $A \cap C \subseteq B \cap C \quad ...(1)$

and $B \cap C \subseteq A \cap C. \quad ...(2)$

Because $x \in B \cap C$.

Thus, $x \in A \cap C$. (from 2.)

Which implies that $x \in A$ and $x \in C$.

Now, since $x \in A$ implies that $B \subseteq A. \quad ...(b)$

Now from *a* and *b*, it is clear that $A = B$.

Hence proved the desired result.

6.3

Case 2: When $x \notin C$.

First to show $A \subseteq B$.

Assume that $x \in A$. Then we must show that $x \in B$ to prove $A \subseteq B$.

Since $x \in A$ and $x \notin C$. Then $x \in A \cup C$.

Now, $A \cup C = B \cup C$.

Which implies that $A \cup C \subseteq B \cup C$ $\quad ...(1)$

and $B \cup C \subseteq A \cup C$. $\quad ...(2)$

Because $x \in A \cup C$.

Thus, $x \in B \cup C$. (from 1.)

Which implies that $x \in B$ or $x \in C$.

But $x \notin C$.

Thus $x \in B$ implies that $A \subseteq B$. $\quad ...(a)$

Comment

Step 4 of 4

Now, to show $B \subseteq A$.

Assume that $x \in B$. Then we must show that $x \in A$ to prove $B \subseteq A$.

Since $x \in B$ and $x \notin C$. Then $x \in B \cup C$.

Now, $A \cup C = B \cup C$.

Which implies that $A \cup C \subseteq B \cup C$ $\quad ...(1)$

and $B \cup C \subseteq A \cup C$. $\quad ...(2)$

Because $x \in B \cup C$.

Thus, $x \in A \cup C$. (from 2.)

Which implies that $x \in A$ or $x \in C$.

But $x \notin C$.

Thus $x \in A$ implies that $B \subseteq A$. $\quad ...(b)$

Now from *a* and *b*, it is clear that $A = B$.

Hence proved the desired result.

6.3

6.3.20

Let A and B be subsets of a universal set U.

The objective is to prove whether the following statement is true or not else provide a counter example.

For all sets A and B,

$\mathscr{P}(A\cap B) = \mathscr{P}(A) \quad \mathscr{P}(B).$

Comment

Step 2 of 3

Let $X \in \mathscr{P}(A\cap B)$.

By the definition of power set,

$X \subseteq A \cap B$.

From the definition of intersection,

$X \subseteq A$ and $X \subseteq B$.

Thus, by the definition of a power set, $X \in \mathscr{P}(A)$ and $X \in \mathscr{P}(B)$.

Therefore,

$X \in \mathscr{P}(A) \cap \mathscr{P}(B)$.

Hence,

$\mathscr{P}(A\cap B) \subseteq \mathscr{P}(A) \quad \mathscr{P}(B)$ ...... (1)

6.3

Conversely,

Let $X \in \mathcal{P}(A) \cap \mathcal{P}(B)$.

By the definition of intersection,

$X \in \mathcal{P}(A)$ and $X \in \mathcal{P}(B)$.

Thus, by the definition of a power set,

$X \subseteq A$ and $X \subseteq B$..

Since, $X \subseteq A$, all the elements of X are also the elements of A.

Since, $X \subseteq B$, all the elements of X are also the elements of B.

Thus,

$X \subseteq A \cap B$.

Hence, by the definition of a power set,

$X \in \mathcal{P}(A \cap B)$.

Therefore,

$\mathcal{P}(A) \quad \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$ ...... (2).

Hence, from (1) and (2),

$\boxed{\mathcal{P}(A \cap B) = \mathcal{P}(A) \quad \mathcal{P}(B)}$.

6.3

6.3.32

Consider the following statement as,

"For all sets A and B, $(A-B)\cup(A\cap B)=A.$"

Here, A and B are subsets of universal set.

The objective is to provide an algebraic proof for the above statement by mentioning appropriate properties for each step of the proof.

Comment

Step 2 of 2

Consider the left-hand side part of the equation as,

$(A-B)\cup(A\cap B)=(A\cap B^c)\cup(A\cap B)$ By the set difference law

$=A\cap(B^c\cup B)$ By the distributive law

$=A\cap(B\cup B^c)$ By the commutative law for union

$=A\cap(U)$ By the compliment law for union.

$=A$ By the identity law for intersection

Therefore, $\boxed{(A-B)\cup(A\cap B)=A.}$

6.3

6.3.39

$(A-B)\cup(B-A)=(A\cap B^c)\cup(B\cap A^c)$ by set difference

$=\{A\cup(B\cap A^c)\}\cap\{B^c\cup(B\cap A^c)\}$ by the distributive law

$=\{(A\cup B)\cap(A\cup A^c)\}\cap\{(B^c\cup B)\cap(B^c\cup A^c)\}$

by the distributive law

$=\{(A\cup B)\cap U\}\cap\{U\cap(B^c\cup A^c)\}$

by the complement laws

$=(A\cup B)\cap(B\cap A)^c$ by De Morgan's law

$=(A\cup B)-(A\cap B)$ by the set difference law

$\therefore (A-B)\cup(B-A)=(A\cup B)-(A\cap B)$

6.3

6.3.45.b

(b)

Derive the stated property: $(A-B)\cup(B-C)=(A\cup B)-(B\cap C)$ in the method of algebraic argument.

Suppose A, B, and C are any three sets. Then, consider the set expression.

$(A-B)\cup(B-C)$	
$=(A\cap B^c)\cup(B\cap C^c)$	Using set difference law twice
$=((A\cap B^c)\cup B)\cap((A\cap B^c)\cup C^c)$	Using distributive law
$=(B\cup(A\cap B^c))\cap((A\cap B^c)\cup C^c)$	Applying commutative law on '$\cup$'
$=((B\cup A)\cap(B\cup B^c))\cap((A\cap B^c)\cup C^c)$	Using distributive law
$=((B\cup A)\cap U)\cap((A\cap B^c)\cup C^c)$	Using complement law for '$\cup$'
$=(B\cup A)\cap((A\cap B^c)\cup C^c)$	By the identity law for '$\cap$'
$=(A\cup B)\cap((A\cap B^c)\cup C^c)$	Applying commutative law on '$\cup$'
$=((A\cup B)\cap(A\cap B^c))\cup((A\cup B)\cap C^c)$	Using distributive law
$=(((A\cup B)\cap A)\cap B^c)\cup((A\cup B)\cap C^c)$	Applying associative law on '$\cap$'
$=((A\cap(A\cup B))\cap B^c)\cup((A\cup B)\cap C^c)$	Applying commutative law on '$\cap$'
$=(A\cap B^c)\cup((A\cup B)\cap C^c)$	By the absorption law
$=((A\cap B^c)\cup\varnothing)\cup((A\cup B)\cap C^c)$	By the identity law for '$\cup$'
$=((A\cap B^c)\cup(B\cap B^c))\cup((A\cup B)\cap C^c)$	Using complement law for '$\cap$'
$=((B^c\cap A)\cup(B^c\cap B))\cup((A\cup B)\cap C^c)$	Applying commutative law on '$\cap$'
$=(B^c\cap(A\cup B))\cup((A\cup B)\cap C^c)$	Using distributive law
$=((A\cup B)\cap B^c)\cup((A\cup B)\cap C^c)$	Applying commutative law on '$\cap$'
$=(A\cup B)\cap(B^c\cup C^c)$	Using distributive law
$=(A\cup B)\cap(B\cap C)^c$	Using De Morgan's law
$=(A\cup B)-(B\cap C)$	Using set difference law

Therefore, it can be concluded that

$(A-B)\cup(B-C)=(A\cup B)-(B\cap C)$.

6.4

6.4.5

Let B be the Boolean algebra, with the operations, addition "+" and multiplication " $\cdot$".

Suppose a and b are any elements of B.

Then, by the Commutative law for addition "+",

$$(a \cdot b) + a = a + (a \cdot b)$$

By the Distributive law for addition "+" over multiplication " $\cdot$",

$$= (a + a) \cdot (a + b)$$

As B be the Boolean algebra, for all a in B, $a + a = a$.

$$= a \cdot (a + b)$$

By the Distributive law for multiplication " $\cdot$" over addition "+",

$$= a \cdot a + a \cdot b$$

As B is the Boolean algebra, for all a in B, $a \cdot a = a$.

$$= a + a \cdot b$$

By the Identity law for multiplication " $\cdot$",

$$= a \cdot 1 + a \cdot b$$

By the Distributive law for multiplication " $\cdot$" over addition "+",

$$= a \cdot (1 + b)$$

By the Commutative law for addition "+",

$$= a \cdot (b + 1)$$

As B is the Boolean algebra, for all b in B, $b + 1 = 1$.

$$= a \cdot 1$$

By the Identity law for multiplication " $\cdot$",

$$= a$$

Therefore, for all a and b in B, $\boxed{(a \cdot b) + a = a}$.

6.4

6.4.7.b

(b)

Let B be the Boolean algebra, with the operations, addition "+" and multiplication " $\cdot$ ".

Suppose 1 and $1'$ are the identity elements of B with respect to multiplication " $\cdot$ ".

To show that there is only identity element of B with respect to multiplication " $\cdot$ ", it is enough to show that $1 = 1'$.

The Identity law for multiplication " $\cdot$ " tells that, for all $a \in B$,

$a \cdot 1 = a$ and $a \cdot 1' = a$

It follows that,

$a \cdot 1 = a \cdot 1'$

Add $\overline{a}$ to both sides,

$\overline{a} + (a \cdot 1) = \overline{a} + (a \cdot 1')$

By the Distributive law for addition "+"over multiplication " $\cdot$ ",

$(\overline{a} + a) \cdot (\overline{a} + 1) = (\overline{a} + a) \cdot (\overline{a} + 1')$

By the Commutative law for addition"+",

$(a + \overline{a}) \cdot (\overline{a} + 1) = (a + \overline{a}) \cdot (\overline{a} + 1')$

By the Complement law for addition "+",

$1 \cdot (\overline{a} + 1) = 1' \cdot (\overline{a} + 1')$

By the Universal bound law for addition "+",

$1 \cdot 1 = 1' \cdot 1'$

By the Identity law for multiplication " $\cdot$ ",

$1 = 1'$

Thus, there exists only one element of B that is an identity for multiplication" $\cdot$ ".

6.4

6.4.10

Let B be the Boolean algebra with operations addition "+", and multiplication " $\cdot$".

Suppose $x, y,$ and z are any elements of B.

Consider the statement,

$x+y=x+z$.

As B be the Boolean algebra and for all a and b in B, $(a+b)\cdot a=a$.

By using the above fact that, y can be written as,

$y=(y+x)\cdot y$

$=(x+y)\cdot y$ By the Commutative law for addition "+".

$=y\cdot(x+y)$ By the Commutative law for multiplication ".".

$=y\cdot(x+z)$ By hypothesis, $x+y=x+z$.

$=(y\cdot x)+(y\cdot z)$ By the Distributive law for multiplication "$\cdot$", over addition "+".

$y=(x\cdot y)+(z\cdot y)$ By the Commutative law for addition ".".

Comment

Step 2 of 2

Consider the statement,

$x\cdot y=x\cdot z$.

From the above proof,

$y=(x\cdot y)+(z\cdot y)$

$y=(x\cdot z)+(z\cdot y)$ By hypothesis, $x\cdot y=x\cdot z$.

$=(z\cdot x)+(z\cdot y)$ By the Commutative law for multiplication ".".

$=z\cdot(x+y)$ By the Distributive law for multiplication "$\cdot$", over addition "+".

$=z\cdot(x+z)$ By hypothesis, $x+y=x+z$.

$=z\cdot(z+x)$ By the Commutative law for addition "+".

$=z$ By using the fact $(a+b)\cdot a=a$.

Therefore, for all $x, y,$ and z in B, if $x+y=x+z$, and $x\cdot y=x\cdot z$, then $\boxed{y=z}$.