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Birzeit University
Department of Mathematics
math 233.

First hour exam
Name: [redacted]

Fall 2008/2009
Number: [redacted]

Question# 1(24%) a):Write the converse ,contrapositive and negation of the following statement

If $x^2 = x$, and $x \neq 0$ then $x=1$

Converse: IF $x = 1$ then $x^2 = x$, and $x \neq 0$ ✓

Contrapositive: IF $x \neq 1$ then $x^2 \neq x$ or $x = 0$ ✓

Negation ~~Contradiction~~: $x^2 = x$ and $x \neq 0$ and $x \neq 1$ ✓

b)Write a useful negation for the following statement

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$ $(A = B) \leftrightarrow (A \subseteq B)$ and $(B \subseteq A)$

it mean that $[(A = B) \to (A \subseteq B)$ and $(B \subseteq A)$ and $((A \subseteq B)$ and $(B \subseteq A) \to A = B)]$

$(A = B$ and $A \nsubseteq B$ or $B \nsubseteq A)$ or $(A \subseteq B$ and $B \subseteq A$ and $A \neq B)$

$A = B$ IF and only IF $A \nsubseteq B$ or $B \nsubseteq A$

$A = B$ IF and only IF (IF $x \in A$ then $x \notin B$) or (IF $x \in B$ then $x \notin A$)

c)Which of the following is a toutology, or a contradiction or neither

1)$[(p \Rightarrow q) \wedge q] \Rightarrow \sim p$

neither

p	q	$p \to q$	$(p \to q) \wedge q$	$\sim q$
T	F	F	F	T
T	T	T	T	F
F	T	T	T	F
F	F	T	F	T

2) $([(p \Rightarrow (q \vee r)] \wedge \sim q) \Rightarrow r$

neither

p	q	$q \vee r$	$p \to (q \vee r)$	$\sim q$	$(p \to q \vee r) \wedge \sim q$
T	F	T		T	T
T	T	T	T	F	F
F	T	T	T	F	F
F	F	F	T	T	

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Question # 2 (35%): Which of the following statements is true and which is false? Justify your answer

1)....F... If $x \in A$, and $A \in P(B)$ then $x \in B$ T by Transitn

let $A = \{1, 2\}$ $B = \{\{1,2\}, 3\}$

$P(B) = \{\varnothing, \{1,2\}, \{3\}, \{\{1,2\}, 3\}\}$ $1 \in A$, but $1 \notin B$

2)....T...$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(y = 2^x)$

Let $x = 2$ $y = 4$

$x \in \mathbb{Z}$, $y \in \mathbb{Z}$

$4 = 2^2$, T = T . so T

$4 = 4$

3)....T...If $A \subset B$ and $B \subseteq C$ then $A \subset C$

let $A \subset B$ and $B \subseteq C$

Let $x \in A$, so $x \in B$ ~~and~~ becase $A \subset B$, $A \neq B$

$x \in B$ and $x \in C$ because $B \subseteq C$, and $A \neq C$ since $A \neq B$

$x \in A$ and $x \in C$ and $A \neq C$

So $A \subset C$

4)....F...$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{Z})(z \leq xy)$ True

~~x = 1 y = 2~~

~~xy = -2~~

~~there is No integer ≤ -2~~

Let $xy = k$ $k-1 \leq k$

Let $z = k-1$ $k-1 < k$ or $k-1 = k$

T or F

T

5)....F...If $A \cap B = A \cap C$ then $B = C$

Let $A = \{1, 2, 3\}$, $B = \{1, 2\}$, $C = \{1, 2, 4\}$

$A \cap B = \{1, 2\}$ $A \cap C = \{1, 2\}$

$A \cap B = A \cap C$ but $B \neq C$

6)....T...If $A \subset B$ then $B - A \neq \phi$

assume $A \subset B$.

Let $x \in A \Rightarrow x \in B$, $A \neq B$ since $A \subset B$ (A greater than B)

$\exists x \in B$ and $x \notin A$

So $(B - A) \neq \varnothing$

7)....T...$(\exists x \in \mathbb{N})(\forall y \in \mathbb{R})(\exists z \in \mathbb{Z})(z < x + y)$

~~Let x = 1 y = -2~~

~~x + y = -2 + 1 = -1~~

~~(z < -1)~~

~~ther is No integer < -1~~

Let $x + y = n$

Let $z = n - 1$

So $n - 1 < n$

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Question# 3(20%)

a)The Group G is called cyclic if and only if there is an element $a \in G$ such that for every $g \in G$, there is an integer n such that $g=a^n$

$G \Leftrightarrow \exists a \in G$ s.t $\forall g \in G$, $\exists n$ $g=a^n$

Write down What it means to say that the Group G is not cyclic

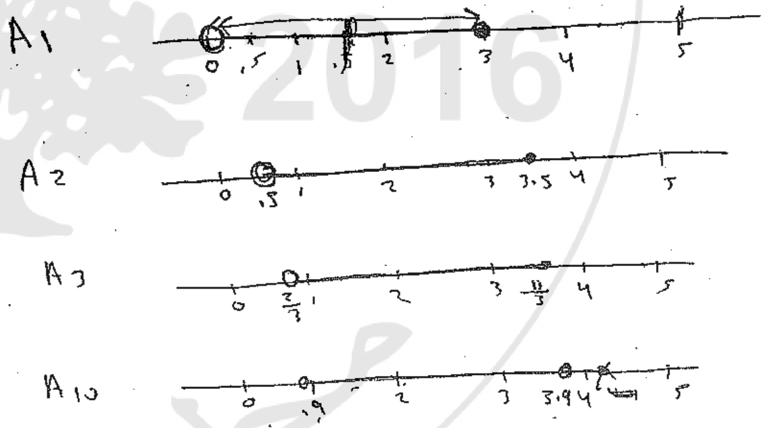
The Group G is Not cyclic IF and only IF For every element $a \in G$ There exist $g \in G$ s.t For all integers n $g \neq a^n$

$A_1 = (0, 3]$

$A_2 = (\frac{1}{2}, \frac{7}{2}]$

b) for each $k \in N$ let

$A_k = (1-\frac{1}{k}, 4-\frac{1}{k}]$, and let $B=(\frac{1}{2},5]$ find $k \in N$



a) $\bigcup_{k=1}^{\infty} A_k$ $(0, 4)$

b) $\bigcap_{k=1}^{\infty} A_k$ $[1, 3]$

c) $\overline{(\bigcup_{k=10}^{\infty} A_k)}$ $= R - (.9, 3.9]$ -2

$= \bigcap_{k=10}^{\infty} \overline{A_k}$

$\bigcap_{10}^{\infty} A_k = (.9, 3.9]$

d) $\bigcup_{k=1}^{\infty} (B \cap A_k)$ $\cancel{\bigcap (B \bigcup A_k) = A \cup (}$ $(0,4) \cap (\frac{1}{2}, 5]$

$(.5, 3.9]$

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Question # 4 (21%):Prove the following

1) If $A \subseteq B$ then $\rho(A) \subseteq \rho(B)$

let $A \subseteq B \Rightarrow x \in A$ and $x \in B$

~~$x \in p(A)$~~ $\Rightarrow \{x\} \in p(A) \Rightarrow \{x\} \in p(B)$ since $A \subseteq B$

let $x \in p(A) \Rightarrow x \subseteq A$ and $x \subseteq B$ sinc $A \subseteq B$

$\Rightarrow x \in p(A)$ and $x \in p(B)$

$\Rightarrow p(A) \subseteq p(B)$

2)If x is a rational number and y is irrational number then x+y is irrational number

by contradiction

Let x rational and y irrational $\xrightarrow{1}$ $x+y$ is rational

$\exists\, m,n \in \mathbb{Z}$ s.t $x = \frac{m}{n}$, $n \neq 0$, $\gcd = 1$

$\left(\frac{m}{n} + y = \frac{L}{r}\right)$, $\frac{m^2}{n^2} + y^2 = \frac{L^2}{r^2}$ $\qquad k, w \in \mathbb{Z}$

$y^2 = \frac{L^2}{r^2} - \frac{m^2}{n^2} = \frac{k^2}{w^2}$!!

a contradiction y is irrational

3)Some prime numbers are even

~~Assume that All prime are~~ odd

~~let k prime number~~

~~$k = 2n+1$~~

~~$(2n+1+2n+1) = 4n+2 \neq 2$~~

~~by A contradiction~~

Assume that All prime number are odd

2 is aprime number and even

So the negation of [All prime number are odd] is tru.

Some prime number are even is true

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$\frac{67}{36}$