BIRZEIT UNIVERSITY

Mathematics Department

Student name: **Math 233** **1st Semester 6/07**

Student no.: **Final Exam**

---

1) Let $f : R \to R$ and $a \in R$. Given the statement:

"If $f(x)$ is differentiable at $x = a$ then $f(x)$ is continuous at $x = a$"

Which of the following statements have the same meaning:

(a) If $f(x)$ is continuous at $x = a$ then $f(x)$ is differentiable at $x = a$.

(b) If $f(x)$ is not differentiable at $x = a$ then $f(x)$ is not continuous at $x = a$.

(c) If $f(x)$ is not continuous at $x = a$ then $f(x)$ is not differentiable at $x = a$.

(d) $f(x)$ is not differentiable at $x = a$ or $f(x)$ is continuous at $x = a$.

(e) $f(x)$ is not continuous at $x = a$ or $f(x)$ is differentiable at $x = a$.

(f) $f(x)$ is differentiable at $x = a$ and $f(x)$ is not continuous at $x = a$.

2) Write the negation of the following statement:

$\forall \varepsilon > 0$ $\exists \delta > 0$ such that $\forall x$ $0 < |x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$ is

3) Prove the following statement using contra positive:

If $ab \equiv 0 (\text{mod } 5)$ then $a \equiv 0 (\text{mod} 5)$ or $a \equiv 0 \ (\text{mod} 5)$

4) Let A, B be sets. Prove that if $A \bigcap B = \phi$ then $A - B = A$

5) For every $n \in N$ let $A_n = \left[2 - \frac{1}{n}, 2 + \frac{1}{n}\right)$

(a) Find $\bigcap_{n \in N} A_n$

(b) Find $\bigcup_{n \in N} A_n$

6) (a) Find gcd(952,901)

(b) Find the integers x, y such that $\gcd(952,901) = 952x + 901y$.

7) Use Mathematical induction to prove that $\forall n \in N \quad 3 \backslash n^3 - n$

8) Define the relation $S = \{(x, y) \in ZxZ : x = y \text{ or } x = -y\}$

(a) Prove that S is an equivalence relation.

(b) Find the partition of Z induced by the above equivalence relation.

9) Let $f: ZxZ \to ZxZ$ be defined as $f((x,y)) = (2x, x+y)$

(a) Prove that f is a function.

(b) Is it one? Give full details.

(c) Is it onto? Give full details.

10) Let P be a family of sets. Define the relation on P

$\forall \; A, B \in P \quad A \sim B \Leftrightarrow A$ and B have the same cardinality. Prove that this relation is an equivalence relation

(11) Mark each of the following statement True a False:

__________ $(p \wedge q) \wedge (\sim p)$

__________ If $A \subseteq B$ $\quad a \in B$ $\quad$ then $\quad a \in A \cap B$.

__________ $\forall n \in N \quad n^2 + n$ $\quad$ is even.

__________ $\{x, y\} = \{a\} \Leftrightarrow x = y = a$.

__________ $\forall b \neq 0 \quad b \in Z \qquad \gcd(0,b) = |b|$.

__________ $\exists p$ $\quad$ prime that is greater than all other primes.

__________ If $a^2 \equiv 4 (\bmod 5)$ $\quad$ then $\quad a \equiv 2 (\bmod 5)$.

__________ The relation on z defined as $\forall a,b \in Z \quad$ a ~ b ⇔ a - b > 0 is reflexive.

__________ $f : X \rightarrow Y$ is a function. If $f(x) \in f(A)$ where $A \subseteq X$ then $x \in A$ then

__________ N is a finite set.

12) Prove or disprove. Give full details.

(a) $\sqrt{x+y} = \sqrt{x} + \sqrt{y} \qquad \forall x,y \in$ N

(b) Let $A = \{x \in R : \quad x < 5\}$ and $B = \{x \in R : x^2 < 25\}$ then $B \subseteq A$

(c) Let a,b be nonzero integers. If $a \backslash b$ and $b \backslash a$ then $a = \pm b$

(d) The relation $S = \{(x, y) \in ZxZ : \gcd(x, y) = 1\}$ is an equivalence relation.

(e) $f : Z \to Z$ defined by $f(x) = 8x^3 + 1$ is a one to one, onto function.(Prove or disprove Algebraicaly)

(f) Z is countably infinite.