

Final Exam

Second Semester 2006/2007

Student Name: Number: Section:
Instructors: Dr. C. Armanios-Omary Dr. K. Altakhman

Question 1 (10 points). Mark each of the following by True or False

- (1) (...) The contrapositive of "If $ab \neq 0$, then $a = 0$ or $b = 0$ " is "If $a \neq 0$ or $b \neq 0$, then $ab \neq 0$ "
- (2) (...) $\forall x$ real number, $\exists y$ real number such that $x - y > 3$.
- (3) (...) If $f : X \rightarrow Y$ is a function, $B \subseteq Y$, $B \neq \phi$, then $f^{-1}(B) \neq \phi$.
- (4) (...) If $f : X \rightarrow Y$, $g : Y \rightarrow Z$ are 1 - 1 functions, then $g \circ f : X \rightarrow Z$ is 1 - 1.
- (5) (...) If S is a relation on \mathbb{Z} defined by $(a, b) \in S$ if $a - b > 0$, then S is transitive.
- (6) (...) If S is a relation on \mathbb{Z} defined by $(a, b) \in S$ if $a \leq b$, then S is symmetric.
- (7) (...) Let S be an equivalence relation on a set X . If $(x, y) \in S$, then $x \in [y]$.
- (8) (...) If $A \cap B$ is an infinite set, then A is an infinite set.
- (9) (...) If $A \cup B$ is an infinite set, then A is infinite.
- (10) (...) Let $f : X \rightarrow Y$ be a function, $B \subseteq Y$, if $x \in f^{-1}(B)$, then $f(x) \in B$.

Question 2 (12 points). (a) Write the negation of the following statement

$$\forall \epsilon > 0, \exists M > 0, \text{ such that, } \forall m, n > M, |m - n| < \epsilon$$

(b) Write the negation of the statement "If $a \mid bc$, then, $a \mid b$ or $a \mid c$."

(c) Write the converse of the statement in (b)

(d) Write the contrapositive of the statement in (b)

Question 3 (6+3 points). (a) Let $A_n = [\frac{1}{n}, \frac{3}{n} + 1)$, $n \in \mathbb{N}$. Then

$$\bigcup_{n \in \mathbb{N}} A_n =$$

$$\bigcap_{n \in \mathbb{N}} A_n =$$

(b) If A, B are disjoint sets, such that $A \subseteq B$. Prove that $A = \phi$

Question 4 (8 points). Use induction to show that, for all $n \in \mathbb{N}$, $2 \mid 3^n - 1$.

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2016

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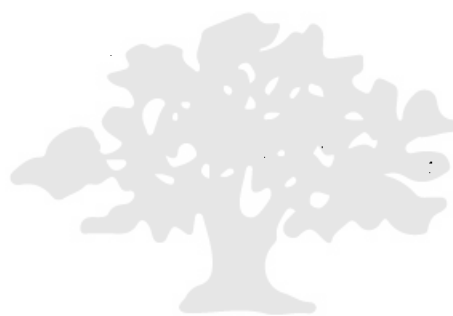
Question 5 (8 points). Let a, b be integers, $a \neq 0$, prove that $\gcd(ka + b, a) = \gcd(b, a)$, for any $k \in \mathbb{Z}$.

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Question 6 (8 points). Let R be an equivalence relation on a set X . Prove that $R \circ R = R$.

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Question 7 (4+4 points). 1. Let $f : A \rightarrow B$, $g : B \rightarrow A$ be functions. If $g \circ f : A \rightarrow A$ is $1 - 1$, prove that f is $1 - 1$.

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin x$, $D = [0, \frac{\pi}{2}]$. Find

$$f(D) =$$

$$f^{-1}(D) =$$

$$f^{-1}(f(D)) =$$

$$f(f^{-1}(D)) =$$

Question 8 (4+4 points). Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(m, n) = 6m + 3n$

1. Is f $1 - 1$? Show your work.

2. Is f onto? Show your work.

Question 9 (3+6+3+5 points). Prove or disprove each of the following

1. If A, B are sets, then $A \times B$ and $A \times B'$ are disjoint.

2. $S = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : mn \geq 0\}$ is an equivalence relation.

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3. If $f : \mathbb{N} \rightarrow \mathbb{N}$ is a 1 - 1 function, then it is onto.

4. If $f : X \rightarrow Y$ is a function, A, B are subsets of X , then $f(A - B) \subseteq f(A) - f(B)$

Question 10 (12 points). 1. Give an example of a relation on a set X that is symmetric and transitive, but not reflexive on X .

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2. Give an example of a function $f : X \rightarrow Y$, $A, B \subseteq X$ and $f(A \cap B) \neq f(A) \cap f(B)$.

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3. Give an example of a set X and a partition of X .

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