Birzeit University
Mathematics Department
Math 233

Final Exam | Second Semester 2006/2007

Student Name: .... Number:.................Section:.....
Instructors: Dr. C. Armanios-Omary Dr. K. Altakhman

Question 1 *(10 points).* **Mark each of the following by True or False**

(1) (...) The contrapositive of "If $ab \neq 0$, then $a = 0$ or $b = 0$" is "If $a \neq 0$ or $b \neq 0$, then $ab \neq 0$"

(2) (...) $\forall x$ real number, $\exists y$ real number such that $x - y > 3$.

(3) (...) If $f : X \to Y$ is a function, $B \subseteq Y$, $B \neq \phi$, then $f^{-1}(B) \neq \phi$.

(4) (...) If $f : X \to Y$, $g : Y \to Z$ are $1-1$ functions, then $g \circ f : X \to Z$ is $1-1$.

(5) (...) If S is a relation on $\mathbb{Z}$ defined by $(a, b) \in S$ if $a - b > 0$, then S is transitive.

(6) (...) If S is a relation on $\mathbb{Z}$ defined by $(a, b) \in S$ if $a \leq b$, then S is symmetric.

(7) (...) Let S be an equivalence relation on a set X. If $(x, y) \in S$,then $x \in [y]$.

(8) (...) If $A \cap B$ is an infinite set, then A is an infinite set.

(9) (...) If $A \cup B$ is an infinite set, then A is infinite.

(10) (...) Let $f : X \to Y$ be a function, $B \subseteq Y$, if $x \in f^{-1}(B)$, then $f(x) \in B$.

Question 2 *(12 points).* (a) Write the negation of the following statement

$$\forall \varepsilon > 0, \exists M > 0, \text{ such that, } \forall m, n > M, |\, m - n \,| < \varepsilon$$

(b) Write the negation of the statement "If $a \mid bc$, then, $a \mid b$ or $a \mid c$."

(c) Write the converse of the statement in (b)

(d) Write the contrapositive of the statement in (b)

Question 3 *(6+3 points)*. (a) Let $A_n = [\frac{1}{n}, \frac{3}{n} + 1)$, $n \in \mathbb{N}$. Then

$$\bigcup_{n\in\mathbb{N}} A_n =$$

$$\bigcap_{n\in\mathbb{N}} A_n =$$

(b) If A, B are disjoint sets, such that $A \subseteq B$. Prove that $A = \phi$

Question 4 *(8 points)*. Use induction to show that, for all $n \in \mathbb{N}$, $2 \mid 3^n - 1$.

Question 5 *(8 points)*. Let a, b be integers, $a \neq 0$, prove that $\gcd(ka + b, a) = \gcd(b, a)$, for any $k \in \mathbb{Z}$.

Question 6 *(8 points)*. Let R be an equivalence relation on a set X. Prove that $R \circ R = R$.

Question 7 (4+4 points). 1. Let $f : A \to B$, $g : B \to A$ be functions. If $g \circ f : A \to A$ is $1-1$, prove that f is $1-1$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x$, $D = [0, \frac{\pi}{2}]$. Find

$f(D) =$

$f^{-1}(D) =$

$f^{-1}(f(D)) =$

$f(f^{-1}(D)) =$.

Question 8 (4+4 points). Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined by $f(m,n) = 6m + 3n$

1. Is f $1-1$?. Show your work.

2. Is f onto? Show your work.

Question 9 *(3+6+3+5 points)*. Prove or disprove each of the following

1. If A, B are sets, then $A \times B$ and $A \times B'$ are disjoint.

2. $S = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : mn \geq 0\}$ is an equivalence relation.

3. If $f : \mathbb{N} \to \mathbb{N}$ is a $1-1$ function, then it is onto.

4. If $f : X \to Y$ is a function, A, B are subsets of X, then $f(A - B) \subseteq f(A) - f(B)$

Question 10 *(12 points).* 1. Give an example of a relation on a set X that is symmetric and transitive, but not reflexive on X.

2. Give an example of a function $f : X \to Y$, $A, B \subseteq X$ and $f(A \cap B) \neq f(A) \cap f(B)$.

3. Give an example of a set X and a partition of X.

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