BIRZEIT UNIVERSITY

Mathematics Department

Student r | **Math 233** | **2nd Semester 09/2010**

Student n | **First Hour Exam** | 98

(10%) Q#1 Which of the following statements are True and which are False:

F The negation of the statement "$\forall x \in Z \quad \exists y \in Z$ Such that x divides y" is "$\exists x \in Z$ such that $\forall y \in Z$ y divides x".

T If $p \wedge q$ is a true statement, then $p \vee q$ is true.

T The truth table of the statement $(p \wedge q) \vee (r \wedge s)$ has 16 rows.

T If $\sqrt{x^2} = x$ then $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$.

F Let $A = \{1,2,3,\ldots,10\}$ then $\forall x \in A \quad \exists y \in A$ such that $x + y < 14$.

$y \neq 10$ $A+y$ $y = 10$

F For every set A, $\phi \in A$.

T If $A \subseteq B$ and $C \subseteq D$ then $A \cap C \subseteq B \cap D$.

$x \in A$ and $x \in C \rightarrow x \in B$ and $x \in$

T $\{x \in R / x \neq x\} \subseteq \{x \in R / x + 8 = x\}$.

T $(\bigcup \{A_\alpha / \alpha \in \Lambda\})' = \bigcap \{A'_\alpha / \alpha \in \Lambda\}$

F $\{a,b\} = \{a\} \Leftrightarrow a = 0$.

(12%) Q#2 (a) Let $A_x = \{3, x+1\} \quad \forall x \in R$ $\quad A_0 = \{3,1\}, A_{-1} = \{3,0\}, A_1 = \{3,2\}$

$\bigcup_{x \in R} A_x = R$

$\bigcap_{x \in R} A_x = \{3\}$

(b) Let $A_n = \{5\} \cup \left[\frac{1}{n}, 3n+1\right)$ (interval) $\forall n \in N$. $\quad A_1 = \{5\}$

$\bigcup_{n \in N} A_n = (0, \infty)$

$\bigcap_{n \in N} A_n = [1, 4) \cap \{5\}$

(15%) Q#3) Use mathematical induction to prove that $\forall n \in N \quad \frac{1}{1.2}+\frac{1}{2.3}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

step(1): check $n=1$ $\quad \frac{1}{1.2}=\frac{1}{1+1} \Rightarrow \frac{1}{2}=\frac{1}{2}$ ✓

step(2): assume the for $n=k \Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+ \dots +\frac{1}{k(k+1)}=\frac{k}{k+1}$

step(3): we will prove true for $n=k+1 \Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+ \dots +\frac{1}{(k+1)(k+2)}=\frac{(k+1)}{(k+2)}$

$\longrightarrow \frac{1}{1.2}+\frac{1}{2.3}+\dots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$

$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)(k+1)}{(k+1)(k+2)}=\frac{(k+1)}{k+2}$

so, by PMI $\frac{1}{1.2}+\frac{1}{2.3}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$ for $\forall n \in N$.

(15%)Q#4) Prove the validity of this argument. Write full details.

1. $A \rightarrow (C \rightarrow B)$
2. $\sim D \vee A$
3. $\dfrac{C}{D \rightarrow B}$

4- $D \rightarrow A$ ✓ contrapositive (2)

5- $D \rightarrow (C \rightarrow B)$ ✓ (1) + (4) by theorem $(A \rightarrow B) \wedge (B \rightarrow C) \longrightarrow A \rightarrow C$

(6) - $D \rightarrow (\sim C \vee B)$ ✓ (5) contrapositive

(7) - $\sim D \vee (\sim C \vee B)$ ✓ (6) contrapositive

(8) - $\sim D \vee B \vee \sim C$ ✓ (7) commutative law

(9) - $\sim D \vee B$ ✓ (3)+(8) def (∨)

(10) $D \rightarrow B$ ✓ def of (→)

30

(14%) Q#5)(a) Prove that If $A-B\subseteq C$ then $A-C\subseteq B$.

by theorm if $C\subseteq B$ and $A\subseteq C$ then $A\subseteq B$

$A-B \subseteq C$ $\rightarrow (x\in A \wedge x\notin B \rightarrow x\in C)$

$\rightarrow x\in A \wedge x\in B' \rightarrow x\in C$ why??

$\rightarrow x\notin C \rightarrow x\notin A \vee x\notin B'$ contrap.

$\rightarrow x\notin C \rightarrow (x\in A \rightarrow x\in B)$?? -4

$\rightarrow x\notin C \rightarrow (x\in A \rightarrow x\in B)$

because $(C'\cap A)\subseteq (C\cup A')$ $x\in(C\cup A) \rightarrow x\in B$ $x\in(A-C)\subseteq B$

(b) Prove that $A-A\cap B'=A\cap B$.

$A-A\cap B' = A\cap(A'\cup B)$ How??

$=(A\cap A')\cup(A\cap B)$ -2

$= \varnothing\cup(A\cap B)$

$= A\cap B$

(10%) Q#6) " The product of every two irrational number is irrational"

(a) Write the above statement in the conditional form

$\forall x,y\in$ irrational s.t $xy\in$ irrational

$\forall x,y$ irrational $\rightarrow xy$ irrational

(b) Write the converse statement.

If $xy\in$ irrational $\rightarrow \forall x,y\in$ irrational and -1/2

(c) Write the contrapositive statement.

If $xy\notin$ irrational $\rightarrow \exists x,y\notin$ irrational -1/2

xy rational $\rightarrow$ $x \wedge y$ - - -

(d) Write the negation of the statement.

$\exists x,y$ irrationals $xy\notin$ irrational but xy rational -1

(e) Write the negation of the following statement:

For every $\varepsilon>0$, there exists $\delta>0$ such that

$|f(x)-f(c)|<\varepsilon$ for every x $0<|x-c|<\delta$

$\sim(\forall\varepsilon>0, \exists\delta>0$ s.t $|f(x)-f(c)|<\varepsilon$ $\forall x$ $0<|x-c|<\delta)$

$\exists\varepsilon>0$, s.t. $\forall\delta>0$ $|f(x)-f(c)|\geq\varepsilon$, $\exists x$ $0\geq|x-c|\geq\delta$

for some -1/2

15.5

(25%) Q#7) Prove or disprove: If the statement is true prove it, if not give a counter example.

(a) For every $n \in N$ $\quad n^2 - n + 5$ is odd.

by Direct proof :-

cas (1) :- n is even number

$\exists k \in Z \Rightarrow n = 2k$

$(2k)^2 - (2k) + 5 = 4k^2 - 2k + 5 = 4k^2 - 2k + 4 + 1$

$= 2(2k^2 - k + 2) + 1 \Rightarrow 2k^2 - k + 2 \in Z$

so if n is even $\rightarrow n^2 - n + 5$ is odd

cas (2) :- n is odd number

$\exists k \in Z \rightarrow n = 2k+1$

$(2k+1)^2 - (2k+1) + 5 = 4k^2 + 4k + 1 - 2k - 1 + 5 = 4k^2 + 2k + 4 + 1 = 2(2k^2 + k + 2) + 1$ is an odd number

so, $\forall n \in N$ $n^2 - n + 5$ is odd number

(b) For each real number x, there exists a real number y such that $xy = 1$.

the Statement is not true

let $y = 0 \Rightarrow xy \neq 1$, $0 \in R$

$xy = 0$

(c) If $a, b \in N$ and $n = ab$ then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

$a, b \in N$ $\quad n = ab \longrightarrow a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

* by contra positive:-

$a > \sqrt{n}$ and $b > \sqrt{n} \rightarrow n \neq ab$

$n = (\sqrt{n})(\sqrt{n}) > a\sqrt{n} > ab$

$n > a\sqrt{n} > ab \Rightarrow n > ab$

$n \neq ab$

so by contrapositive if $a, b \in N$ and $ab = n$ then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$ is true.

(d) If $A \cap B \subseteq C'$ and $A \cup C \subseteq B$ then $A \cap C = \phi$

$A \cap B \subseteq C' \wedge A \cup C \subseteq B \longrightarrow A \cap C = \phi$

-4

$\cancel{x \in (A \cap B \subseteq C' \wedge A \cup C \subseteq B) \rightarrow x \in (A \cap B) \wedge x \in C'}$

$\cancel{\rightarrow x \in A \wedge x \in B \wedge x \in C'}$

$\cancel{\rightarrow x \in (A \cap C') \wedge x \in B}$

$\cancel{\rightarrow x \in (A \cap C')}$. $\rightarrow$ if $x \in (A \cap C') \rightarrow$

$x \notin (A \cap C)$

$(A \cap C') \cap (A \cap C) = A \cap A \cap C \cap C'$

$= A \cap \phi$

$= \phi$

16

$A' = \{1, 3\}$

(e) $A - B = B - A$ for any sets A,B

$B' = \{2, 3\}$

$U = \{1, 2, 3, 5\}$

$A = \{2, 5\}$

$B = \{5, 1\}$

$A - B = \{2\}$

$B - A = \{1\}$

So, $A - B \neq B - A$

Bonus(10%) Use mathematical induction to prove that $\forall n \in N$.

$$\left(1+\frac{1}{2}\right)^n \geq 1+\frac{n}{2}$$

Step(1): check $n=1 \Rightarrow (1+\frac{1}{2})^1 \geq 1+\frac{1}{2}$

$1\frac{1}{2} \geq 1\frac{1}{2}$

Step(2): assume true for $n=k$

$(1+\frac{1}{2})^k \geq 1+\frac{k}{2}$

Step(3): we will prove true for $n=k+1$

$(1+\frac{1}{2})^{k+1} \geq 1+\frac{k+1}{2}$

$(1+\frac{1}{2})^{k+1} = (1+\frac{1}{2})^k (1+\frac{1}{2}) \geq (1+\frac{k}{2})(1+\frac{1}{2})$

$(1+\frac{1}{2})^{k+1} \geq 1+\frac{k}{2}+\frac{1}{2}+\frac{k}{4}$

$(1+\frac{1}{2})^{k+1} \geq 1+\frac{k+1}{2}+\frac{k}{4}$ $\qquad \frac{k}{4} > 0$

$(1+\frac{1}{2})^{k+1} \geq 1+\frac{k+1}{2}+\frac{k}{4} \geq 1+\frac{k+1}{2}$

$(1+\frac{1}{2})^{k+1} \geq 1+\frac{k+1}{2}$

So, by PMI $\forall n \in N$ $(1+\frac{1}{2})^n \geq 1+\frac{n}{2}$

15