First

Birzeit University
Department of Mathematics
math 233.

85/100

First hour exam — Spring 2008

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Question# 1(24%) a):Write the converse ,contrapositive and negation of the following statement

$\overset{P}{\text{If } x^2 = x}$ $\overset{\wedge}{,}$ $\overset{Q}{\text{and } x \neq 0}$ $\overset{\longrightarrow}{\text{then}}$ $\overset{R}{x=1}$

Converse: if $x = 1$ then $x^2 = x$ ~~and~~ $x \neq 0$

Contrapositive: If $x \neq 1$. then $x^2 \neq x$ or $x = 0$.

Contradiction: $x^2 = x$ and $x \neq 0$ and $x \neq 1$

b)Write a useful negation for each of the following statements

$\exists (P \wedge Q)$

1)Some math text books are expensive and not easy to read

~~Some~~ All math text books are cheap or easy to read.

$P \iff Q \wedge R$

2) $x^2 \neq x$ if and only if $x \neq 0$ and $x \neq 1$

there exist $x = 0$ ~~or~~ $x = 1$ such that $x^2 \neq x$

~~$\exists (x = 0 \vee x = 1)(x^2 \neq x)$~~

3) $(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(xy \leq 0)$

$(\forall x \in Z)(\forall y \in Z)(xy > 0)$

c)Which of the following is a toutology, or a contradiction or neither

1) $[(p \Rightarrow q) \wedge q] \Rightarrow \sim p$

neither.

P	Q	~P	[(P⇒q)	[illegible]
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

2) $p \rightarrow (q \vee r) \Leftrightarrow (p \wedge \sim q) \rightarrow r$

toutology

21
17
19
28
85

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Question # 2 (28%): Which of the following statements is true and which is false?Justify your answer

4 1)...F... If $x \in A$, and $A \in B$ then $x \in B$

Counter example : $A = \{1, 2\}$, $B = \{\{1,2\}, 3\}$ $x = 1 \in A$

but $x = 1 \notin B$.

2)...T...$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x \leq y^2)$

4 let $x = 1 \in Z$ and he say $\exists y \in Z$ s.t $x \leq y^2$ so we can find y s.t $x \leq y^2$

let $y = 2 \in Z$, $1 \leq 4$.

4 3)...T...If $A \subseteq B$ and $B \subset C$ then $A \subset C$ suppose that $(A \subseteq B)$ and $(B \subset C)$

let $x \in A$ since $A \subseteq B$ so $x \in B$ (def of $\subseteq$) since $B \subset C$ $x \in C$ which means

$A \subseteq C$ or $A \subset C$ but since $B \subset C$ so $A \subset C$ only.

4 4)...F...$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\forall z \in \mathbb{Z})(xy \leq z)$

let $x = 2 \in Z$, $y = 3 \in Z$ $z = 4 \in Z$, counter example.

$6 \not\leq 4$

5)...T...If $A \cup B = A \cup C$ then $B = C$ suppose $A \cup B = A \cup C$ we need to prove $B =$

~~prove~~ 1) $B \subseteq C$. let $x \in B$ by the def of $(\cup) \Leftrightarrow x \in A \cup B$ since $A \cup B = A \cup C \Leftrightarrow x \in A \cup C$

2) $C \subseteq B$ ~~$x \in C \subseteq B$~~ $\Leftrightarrow \underbrace{x \in A \text{ or } x \in C}_{\text{def of } (\cup)}$ but $x \in B$ not in $A \Leftrightarrow x \in C$

?

4 6)...T...If $A \subset B$ then $B - A \neq \phi$

contradiction : suppose that $A \subset B$, $B - A = \emptyset$ $\emptyset$ nonempty set $\Rightarrow \nexists x \in B - A$

(def of $\emptyset$) $\Rightarrow$ (def of $B - A$) is $(B \cap A')$ $\Rightarrow$ (def of $\cap$) $x \notin B$ and $x \notin A'$ $\Rightarrow$ (by def of comp

$x \in A$!! contradiction with given $(A \subset B)$ (def of $(\subset)$) contradiction the result which in $(x \in A$ and $x \notin B)$.

7)...T...$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z < x + y)$

let $x = 0.5 \in R$, $y = 3 \in R$, $z = \sqrt{2} \in R$.

$\sqrt{2} < 0.5 + 3$

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Question# 3(20%)

X

a)Give an example to show that the following is not true

If A, B , C are sets then $[A-(B\cup C)]\cup[B-(A\cap C)]\subseteq(A\cup B)-C$

let U: universal set is natural number between $(1,8) \Rightarrow 8 \leq N \leq 1$ or $N \in [1,8] = U$. let $A=\{1,2,3\}$, $B=\{4,5\}$, $C=\{7,8\}$

$A' = \{4,5,6,7,8\}$, $B' = \{1,2,3,6,7,8\}$, $C' = \{1,2,3,4,5,6\}$.

$[A-B\cup C]\cup[B-(A\cap C)] \not\subseteq (A\cup B)-C = [A\cap(B'\cap C')]\cup[B\cap(A'\cup C')] \not\subseteq (A\cup B)\cap$

$\boxed{\text{by def. of } (A-B = A\cap B')}$ $B'\cap C' = \{1,2,3\}$, $[A\cap(B'\cap C')] = \{1,2,3\}$,

$A'\cup C' = \{1,2,3,4,5,6,7,8\}$, $B\cap(A'\cup C') = \{4,5\}$, $[A\cap(B'\cap C')]\cup[B\cap(A'\cup C')]$ $= \{1,2,3,4,5\}$. $A\cup B = \{1,2,3,4,5\}$, $[(A\cup B)\cap C'] = \{1,2,3,4,5\}$

b)The sequence $\{a_n\}$ converges to L if and only if for every positive number ε there is an integer N ,such that for all n,

$\sim($ $n>N \rightarrow |a_n-L|<\varepsilon$ $)$

Write down What it means to say that the sequence $\{a_n\}$ does not converge to L

there exist positive number ε forall integer N such that for some n, ~~$n\leq N$~~ $n>N$ and $|a_n-L| \geq \varepsilon$

c) for each $k\in\mathbb{N}$ let

$A_k = [1-\frac{1}{k}, 2+\frac{1}{k})$, find

a) $\bigcup_{k=1}^{\infty} A_k$ $A_1 = [0,3)$ $A_2 = [\frac{1}{2}, 2.5)$ $A_3 = [\frac{2}{3}, \frac{7}{3})$

1 2 3

$\boxed{\bigcup_{k=1}^{\infty} A_k = [0,3)}$.

b) $\bigcap_{k=1}^{\infty} A_k = [1,2)$

c) $\overline{(\bigcup_{k=10}^{\infty} A_k)}$ $A_{10} = [\frac{9}{10}, 2.1)$, $A_{11} = [\frac{10}{11}, \frac{23}{11})$

$\overline{\bigcup_{k=10}^{\infty} A_k} = (-\infty, \frac{9}{10}$ $\cup (2.1,$

0 1 2 3

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$(A\cap B)'$ $(A_1 \cup A_2)' = A_1' \cap A_2'$ $A_1 - A_2$

$= (3)$

N $\frac{9}{10}$ ∞ $A_1' - A_2$. $A_{10} - A_{11}$

Question # 4 (28%):Prove the following

1) If $A \subseteq B$ then $\rho(A) \subseteq \rho(B)$

suppose $A \subseteq B$. we need to prove that $P(A) \subset P(B)$.

let ~~x~~ $X \in P(A) \Rightarrow X \subseteq A$ (by def of P) $\Rightarrow$ but $A \subseteq B$ (given)

so $X \subseteq B \Rightarrow$ so $X \in P(B)$. so $P(A) \subseteq P(B)$.

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2) For any sets A,B,C , $A-(B \cup C)=(A-B) \cap (A-C)$

$A \cap B' \cap A \cap C'$

suppose A, B, C sets we need to prove $A-(B \cup C) = (A-B) \cap (A-C)$

let $X \in A-(B \cup C) \Rightarrow X \in A$ and $X \notin (B \cup C)$ [from property say $A-B = A \cap B'$

$\Rightarrow X \notin B$ and $X \notin C$ (def of $\cup$) $\Rightarrow X \in B'$ and $X \in C'$ (def of complement).

$\Rightarrow X \in A$ and $(X \in B'$ and $X \in C') \Rightarrow X \in A$ and $X \in B'$ and $X \in A$ and $X \in C'$

$\Rightarrow X \in [(A \cap B') \cap (A \cap C')] \Rightarrow X \in [(A-B) \cap (A-C)]$.

3) Let A,B,C be sets such that $A \cup B \neq A \cap C$ prove that A is not a subset of C or B is not a subset of A

Assume that A, B, C sets ~~and $A \cup B \neq A \cap C$~~ by contrapositive :-

Assume $A \subseteq C$ and $B \subseteq A$ we need to prove $A \cup B = A \cap C$

First : $A \cup B \subseteq A \cap C$

let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ (def of $\cup$) . case 1 : $x \in B \Rightarrow x \in A$ (from $(B \subseteq A)$) $\Rightarrow$ ~~x∈...~~

$x \in A \cup B$ # which we assume .

case 2 :- $x \in A \Rightarrow x \in C$ since $(A \subseteq C) \Rightarrow x \in (A \cap C)$ which we need .

second : $A \cap C \subseteq A \cup B$.

let $x \in A \cap C \Rightarrow x \in A$ and $x \in C$ (def of $\cap$) $\Rightarrow$ ~~$x \in C$ since $A \subseteq C$~~

$\Rightarrow x \in A \cup B$. since $x \in A$.

$A \cap C \subseteq A \cup B$ and $A \cup B \subseteq A \cap C$ so $A \cup B = A \cap C$

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