

Birzeit University
Department of Mathematics
math 233.

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First Hour Exam

spring 2011/2012

Name : ~~XXXXXXXXXX~~

Number: ~~XXXXXX~~

T, R

Question#1(18%) Which of the following statements is true and which is false

1) ~~F~~ $\{(1,2)\} \in \{1, \{2\}, \{1,2\}\}$ F

2) ~~F~~ $\emptyset \in \{1, \{\emptyset\}\}$ F

3) ~~F~~ If $A \subseteq \emptyset$ then $A = \{\emptyset\}$ F

4) ~~T~~ For any sets A, B, $A - B \subset A$ F

5) ~~T~~ For any sets A, B, $A = (A - B) \cup (A \cap B)$ F

6) ~~T~~ If A, B are sets and $x \in A$ and $A \in B$ then $x \in B$ F

7) ~~T~~ $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z^2 = xy)$ F

8) ~~T~~ $(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\forall z \in \mathbb{Z})(xy \leq z)$ F

9) ~~T~~ If $A \cup B = A \cup C$ then $B = C$ F

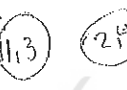
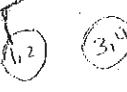
10) ~~T~~ If $A \subset B$ then $B - A \neq \emptyset$ F

11) ~~T~~ $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z < x + y)$ F

12) ~~F~~ $[(p \Rightarrow q) \wedge q] \Rightarrow \sim p$ is a tautology F



$(A \cap B) \cup (A - B)$
 $A \cap (A \cup B) = A$
 $A \cup (A \cap B) = A \cup B$



p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

Question# 2(16%) a): Write the converse, contrapositive and negation of the following statement

If $x^2 = x$ then $x=0$ or $x=1$. $x^2 = x \rightarrow (x=0 \vee x=1)$

Converse: If $x=0 \vee x=1$ then $x^2 = x$ ($q \rightarrow p$)

$$x=0 \vee x=1 \rightarrow x^2 = x$$

Contrapositive: if $x \neq 0$ and $x \neq 1 \rightarrow x^2 \neq x$ ($\neg q \rightarrow \neg p$)

$$x \neq 0 \wedge x \neq 1 \rightarrow x^2 \neq x$$

Negation:

$$x^2 = x \text{ and } x \neq 0 \text{ and } x \neq 1 \quad (p \wedge \neg q)$$

$$x^2 = x \wedge (x \neq 0 \wedge x \neq 1)$$

b) The sequence $\{a_n\}$ converges to L if and only if for every positive number ϵ

there is an integer N, such that for all $n > N$, $|a_n - L| < \epsilon$

Write down what it means to say that the sequence $\{a_n\}$ does not converge to L

$\exists \epsilon > 0 \forall N \in \mathbb{N} \exists n > N \text{ such that } |a_n - L| \geq \epsilon$

Question#3) (9%) For each $k \in \mathbb{N}$ let $A_k = [1 - \frac{1}{k}, 2 - \frac{1}{k})$, find

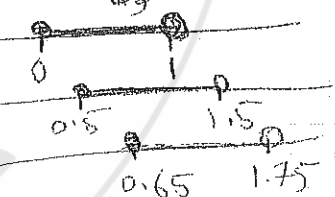
a) $\bigcup_{k=1}^{\infty} A_k$

$$[0, 2)$$

$$A_1 = [0, 1)$$

$$A_2 = [\frac{1}{2}, \frac{3}{2})$$

$$A_3 = [\frac{2}{3}, \frac{5}{3})$$



b) $\bigcap_{k=1}^{\infty} A_k = \emptyset$



c) $\overline{\bigcup_{k=1}^{\infty} A_k}$

$$= \bigcap_{k=1}^{\infty} \overline{A_k} = \bigcap_{k=1}^{\infty} (-\infty, 1 - \frac{1}{k}) = (-\infty, 0.9)$$

$$A_5 = [1 - \frac{1}{5}, 2 - \frac{1}{5}) = [0.8, 1.8)$$

$$\bigcap_{k=10}^{\infty} A_k = [0.9, 1.9)$$

$$A = (-\infty, 0.9) \cup [1.9, \infty)$$

Question #4(20%) Prove each of the following statements

a) If a square is odd then it is one more than a multiple of 8.

For example $9 = 1 + 8$, $25 = 1 + 8 \times 3$, $49 = 1 + 8 \times 6$.

~~any odd number and it's not one more than a multiple of 8~~

~~$n \in \mathbb{Z} \rightarrow \exists k \in \mathbb{Z} \text{ s.t. } n = 2k+1, \text{ for } n \in \mathbb{Z}$~~

~~$\Rightarrow n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$~~

b) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x > y)$

~~$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x \leq y)$~~

~~$x = 2, y = -1$~~

c) $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})(\forall n \in \mathbb{N})(x < ny)$

~~F~~

~~$x = 1, y = 2$~~

d) $P(A \cap B) = P(A) \cap P(B)$

~~$2 < 4$~~

\square Suppos $x \in P(A \cap B)$
 $\Rightarrow x \in A \cap B$
 $\Rightarrow x \in A \quad x \in B$
 \Rightarrow

Question #5(12%) Use mathematical induction to prove that $\forall n \in \mathbb{N}$

$$\sum_{i=n}^{2n} 2i = 3(n^2 + n)$$

$$\sum_{i=n}^{2n} 2i = 2n + 4n = 6n$$

1) the stat. is T for $n=1$

2) the statement is true for $n=1$

$$\sum_{i=1}^{2} 2i = 3(1+1)$$

$$6n = 3(n^2 + n)$$

$$6 = 3(1+1)$$

$$2(1) + 2(2) = 3(2)$$

$$\boxed{2+4 = 6} \quad \checkmark$$

3) suppose the statement is true for $n=k$

2) Suppose the stat. is T for $n=k$ i.e. $\forall k \in \mathbb{N}$

$$\sum_{i=k}^{2k} 2i = 3(k^2 + k)$$

$$= 2k + 2(2k)$$

4) the statement is true for $n=k+1$

3) and we show that for

$$n = k+1$$

$$\sum_{i=k+1}^{2(k+1)} 2i = 3(k+1)^2 + (k+1)$$

Now L.H.S

$$\sum_{i=k+1}^{2(k+1)} 2i = 2(k+1) + 2(2(k+1))$$

$$3(k^2 + k) + 6$$

$$3k^2 + 3k + 3 + 3$$

$$\Rightarrow 2k + 2 + 2(2k + 2)$$

$$3(k^2 + k + 1) = (3k^2 + 3) + (3k + 3)$$

$$2k + 2 + 4k + 4$$

$$3(k+1)(k+1) = 3(k+1)^2 + 6$$

$$\sum_{i=k}^{2k} 2i + 6$$

Question #6(10%)

Prove that the following two statements are equivalent:

$$P \Rightarrow (Q \Leftrightarrow R) \text{ and } (P \wedge Q) \Rightarrow R$$

In your proof, do NOT use truth tables. Use the fact that

$$(A \Rightarrow B) \equiv (\sim A \vee B) \text{ and give a completely algebraic proof.}$$

$$P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$$

$$P \rightarrow (Q \rightarrow R)$$

$$P \rightarrow (\sim Q \vee R) \equiv (P \wedge \sim Q) \rightarrow R$$

$$P \rightarrow (\sim Q \vee R)$$

$$\sim(P \wedge \sim Q) \vee R \equiv (\sim P \vee Q) \vee R$$

$$(\sim P \vee Q) \vee (P \rightarrow R)$$

$$Q \rightarrow \sim P \vee \sim R \rightarrow \sim P$$

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Question #7(15%)

5. Let A;B;C denote arbitrary sets.

(a) Give a careful, step-by-step, proof of the relation $(A-C)-(B-C) \subseteq A-B$:

~~Suppose~~ suppose $x \in (A-C)-(B-C)$

$$x \in (A-C) \wedge x \notin (B-C)$$

$$x \in A \wedge x \notin C \wedge x \notin B \wedge x \in C$$

$$x \in A \wedge x \notin B \wedge x \in C$$

$$x \in (A-B)$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 2, 4\}$$

$$C = \{3, 4\}$$

$$A-C = \{1, 2, 5\}$$

$$B-C = \{1, 2\}$$

$$A-B = \{3, 5\}$$

$$(A-C)-(B-C) = \{5\}$$

\neq

(b) Show, via a counterexample, that equality need not hold in the above relation; i.e., construct sets A;B;C for which $(A-C)-(B-C) \neq A-B$: