55/100

Birzeit University
Department of Mathematics
math 233.

First Hour Exam — spring 2011/2012

Name : [redacted] — Number: [redacted]

Question#1(18%) Which of the following statements is true and which is false

~~1)..F...$\{1,2\}\subseteq\{1,\{2\},\{1,2\}\}$~~ F

2)..F.....$\phi\in\{1,\{\phi\}\}$ F

3)..F.....*If* $A\subseteq\phi$ then $A=\{\phi\}$ F

4)..T...... For any sets A, B , $A-B\subset A$ F

5)..T.....For any sets A, B , $A=(A-B)\cup(A\cap B)$

6)..T.......If A,B are sets and $x\in A$ and $A\in B$ then $x\in B$

7)..T.....$(\forall x\in\mathbb{R})(\forall y\in\mathbb{R})(\exists z\in\mathbb{R})(z^2=xy)$ F

8).........$(\exists x\in\mathbb{Z})(\exists y\in\mathbb{Z})(\forall z\in\mathbb{Z})(xy\le z)$ F

9)..F...If $A\cup B=A\cup C$ then B=C F

10)......... If A$\subseteq$B then $B-A\neq\phi$

$A\subseteq B$ But $A\neq B$

11).........$(\forall x\in\mathbb{R})(\forall y\in\mathbb{R})(\exists z\in\mathbb{R})(z<x+y)$ T

$\exists x\in\mathbb{R}\ \forall y\in\mathbb{R}\ \forall z\in\mathbb{R}\ (z\geq x+y)$

12)..F.....$[(p\Rightarrow q)\wedge q]\Rightarrow\ \sim p$ is a toutology F

$p\Rightarrow q$	p	~p
T	T	F
F	F	F
T	T	T
T	F	T

Question# 2(16%) a): Write the converse, contrapositive and negation of the following statement

If $x^2 = x$ then $x = 0$ or $x = 1$.

$x^2 = x \rightarrow (x = 0 \vee x = 1)$

Converse: if $x = 0 \vee x = 1$ then $x^2 = x$ $(q \rightarrow p)$

$x = 0 \vee x = 1 \longrightarrow x^2 = x$

Contrapositive: if $x \neq 0$ and $x \neq 1 \longrightarrow x^2 \neq x$ $(\sim q \rightarrow \sim p)$

$x \neq 0 \wedge x \neq 1 \longrightarrow x^2 \neq x$

Negation: $x^2 = x$ and $x \neq 0$ and $x \neq 1$ $(p \wedge \sim q)$

$x^2 = x \wedge (x \neq 0 \wedge x \neq 1)$

b) The sequence $\{a_n\}$ converges to L if and only if for every positive number ε there is an integer N, such that for all $n > N$, $|a_n - L| < \varepsilon$

Write down What it means to say that the sequence $\{a_n\}$ does not converge to L

$(\forall \varepsilon > 0)(\exists N \in \mathbb{N}) ; \forall n > N ; |a_n - L| < \varepsilon$

$(\exists \varepsilon > 0)(\forall N \in \mathbb{N}) ; \exists n > N ; |a_n - L| \geq \varepsilon$

Question#3) (9%) For each $k \in \mathbb{N}$ let $A_k = [1 - \frac{1}{k}, 2 - \frac{1}{k})$, find

a) $\bigcup_{k=1}^{\infty} A_k$ $[0, 2)$

$A_1 = [0, 1)$

$A_2 = [\frac{1}{2}, \ldots$

$A_3 = [\frac{2}{3}, \frac{5}{3})$

b) $\bigcap_{k=1}^{\infty} A_k =$ $\emptyset$

c) $\overline{(\bigcup_{k=10}^{\infty} A_k)}$

$A_5 = [1 - \frac{1}{5}, 2 - \frac{1}{5}$

$= \bigcap_{k=10} A^c$

$\bigcap_{k=10}^{\infty} A_{10}' = [0.9, 1.9)$

$\bigcap [0.9, \infty)$

$A' = (-\infty, 0.9) \cup [1.9, \infty)$

Question #4(20%) Prove each of the following statements

a) If a square is odd then it is one more than a multiple of 8.

For example $9 = 1 + 8$, $25 = 1 + 8 \times 3$, $49 = 1 + 8 \times 6$.

b) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x \geq y)$

$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x \leq y)$

$x = 2$

$R \leq -1$

c) $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})(\forall n \in \mathbb{N})(x < ny)$ (F)

$x = 1$

$y = 2$

$2 < n$

d) $\mathrm{P}(A \cap B) = \mathrm{P}(A) \cap \mathrm{P}(B)$

$\boxed{\subseteq}$ Suppos $x \in P(A \cap B)$

$\Rightarrow x \subseteq A \cap B$

$\Rightarrow x \in A \quad x \subseteq B$

$\Rightarrow$

Question #5(12%) Use mathematical induction to prove that $\forall n \in \mathbb{N}$

$$\sum_{i=n}^{i=2n} 2i = 3(n^2+n)$$

1) the stat. is T for $n=1$

$$\sum_{i=1}^{i=2} 2(i) = 3(1+1)$$

$$2(1)+2(2) = 3(2)$$

$$\boxed{\overset{2+4}{6} = 6}$$

2) Suppose the stat. is T for $n=K$ i.e $\forall K \in \mathbb{N}$

$$\sum_{i=K}^{i=2K} 2i = 3(K^2+K)$$

$$= 2K + 2(2K)$$

3) and we show that is T for $n=K+1$;

$$\sum_{i=K+1}^{i=2(K+1)} 2i = 3((K+1)^2+(K+1)).$$

Now: L.H.S

$$\sum_{i=K+1}^{i=2(K+1)} 2i = 2(K+1) + 2(2(K+1))$$

$$\Rightarrow 2K+2 + 2(2K+2)$$

$$2K+2+4K+4.$$

$$\sum_{i=K}^{i=2K} 2i + 6$$

$3(K^2+K)+6$

$3K^2+3K+\cancel{6}\,3+3$

$\cancel{3(K^2+K+2)}\ (3K^2+3)+(3K+3)$

$\cancel{3((K+1)+(K+1))}\ 3((K+1)^2+(K+1))$ 4

Question #6(10%)

Prove that the following two statements are equivalent:

$P \Rightarrow (Q \Rightarrow R)$ and $(P \wedge Q) \Rightarrow R$

In your proof, do not use truth tables. Use the fact that $(A \Rightarrow B) \cong (\sim A \vee B)$ and give a completely algebraic proof.

$P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

$P \rightarrow (Q \rightarrow R)$

$P \rightarrow (\sim Q \vee R)$

$(P \rightarrow \sim Q) \vee (P \rightarrow R)$

$Q \rightarrow \sim P \quad \vee \quad \sim R \rightarrow \sim P$

$\boxed{20}$

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Question #7(15%)

5. Let A;B;C denote arbitrary sets.

(a) Give a careful, step-by-step, proof of the relation
$(A-C)-(B-C) \subseteq A-B$:

suppose $x \in (A-C)-(B-C)$

$x \in (A-C) \wedge x \notin (B-C)$

$x \in A \wedge \cancel{x \notin C} \wedge x \notin B \wedge \cancel{x \in C}$

$x \in A \wedge x \notin B$

$x \in (A-B)$.

$A = \{1,2,3,4,5\}$

$B = \{1,2,\cancel{3},4\}$

$C = \{3,4\}$

$A-C = \{1,2,5\}$

$B-C = \cancel{\emptyset}\ \{1,2\}$

$(A-C)-(B-C) = \{5\}$

$A-B = \{3,5\}$

$\neq$

(b) Show, via a counterexample, that equality need not hold in the above relation; i.e., construct sets A;B;C for which $(A-C)-(B-C) \neq A-B$:

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