80/100

Birzeit University
Department of Mathematics
math 233

Second hour exam | Spring 2012

Name : [redacted] | Number: [redacted]

section: 1

Questio#1(36%) Prove or disprove each of the following statements

a)If $A \neq \emptyset$ and $A\times B = A\times C$ then $B=C$

The statement is true.

proof: (Direct) let $A \neq \phi$ and $A\times B = A\times C$, $a\in A, b\in B, c\in C$

$\Rightarrow (a,b)\in A\times B$ || $(a,c)\in A\times C$

$\Rightarrow (a,b) = (a,c)$

$\Rightarrow b = c$

$\Rightarrow b\in B, \in C$

$\Rightarrow B=C$.

$\subseteq$ suppose $A\times B = A\times C$ and $b\in B$. Let $a\in A$ $(a,b)\in A\times B$

$\Rightarrow (a,b)\in A\times C$ (since $A\times B = A\times C$) $\Rightarrow a\in A$ and $b\in C$

$\supseteq$ suppose $A\times B = A\times C$ and $c\in C$. let $a\in A$; $(a,c)\in A\times C$

$\Rightarrow (a,c)\in A\times B$

$\Rightarrow a\in A$ and $c\in B$

b)If f and g are functions then $f\cup g$ is a function

The statement is false.

counter example:

$f = \{(1,2),(3,4)\}$ $\Rightarrow$ function

$g = \{(1,5),(4,6)\}$ $\Rightarrow$ function

$f\cup g = \{(1,2),(1,5),(3,4),(4,6)\}$ $\Rightarrow$ not function

because $f\cup g(1) = 2$ or 5 (not one value)

c) let $x\in A \Rightarrow (x,x)\in R$, $(x,x)\in S$

$\Rightarrow (x,x)\in R\cup S$ $\Rightarrow$ ref.

Let $(x,y)\in R \vee (x,y)\in S \Rightarrow (y,x)\in R \vee (y,x)\in S$

$\Rightarrow (y,x)\in R\cup S$ $\Rightarrow$ sym.

transitive ؟ و لماذا في الأسفل

c)If R and S are equivalence relations on A then $R\cup S$ is an equivalence relations on A

The statement is true.

~~R, S eq. relation on A $\Rightarrow$ R∪S is eq. relation~~

~~Proof: let R~~

$A = \{1,2,3\}$

$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$

$S = \{(1,1),(2,2),(3,3),(3,1),(1,3)\}$

$R\cup S = \{(1,1),(2,2),(3,3),(1,2),(2,1),(3,1),(1,3)\}$

$(2,1)\in R$, $(1,3)\in R$

$(2,3)\notin R$

(because R and S are equivalence relation (ref, sym))

~~let $x\in A \Rightarrow (x,x)\in R$, $(x,x)\in S$ $\Rightarrow (x,x)\in R\cup S$ $\Rightarrow R\cup S$ is ref.~~

~~$(y,x)\in R$, $(w,z)\in S \Rightarrow (y,x)\in R\cup S$, $(w,z)\in R\cup S \Rightarrow R\cup S$ is sym.~~

Let $(x,w),(w,y)\in R$, $(x,w),(w,y)\in S$ $\Rightarrow (x,y)\in R \vee (x,y)\in S$ (R, S eq. relation)

$\therefore R\cup S$ is ... $\Rightarrow (x,y)\in R\cup S \Rightarrow$ trans.

d) If f^{-1} is a function then f is one to one function

The statement is false.

Counter example

$f^{-1} = \{(1,1), (2,1), (3,4)\}$

$f = \{(1,1), (1,2), (4,3)\}$

f is not a function and not 1-1.

$(x,x) \in R$, $(x,x) \in S$

e) If R and S equivalence relations on A then $R \circ S$ is an equivalence relations on A

The statement is true.

Proof:

$(x,y) \in R \circ S \Rightarrow \exists w \in A;\ (x,w) \in R \wedge (w,y) \in S$

$\Rightarrow (w,x) \in R \wedge (y,w) \in S$

$\Rightarrow (y,x) \in R \circ S \Rightarrow$ Sym.

Let $x \in A \Rightarrow (R \circ S)(x) = R(S(x)) = R(x) \Rightarrow x$

$\Rightarrow (x,x) \in R \circ S \Rightarrow$ refl.

$A = \{1,2,3\}$

$R = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$

$S = \{(1,1), (2,2), (3,3), (2,1), (1,2)\}$

$R \circ S = \{(1,1), (1,3), (2,2), (3,3), (3,1), (2,3), (1,2)\}$

$(2,3) \in R \circ S$ but $(3,2) \notin R \circ S$

f) If R is a relations on A and $R \circ R \subseteq R$ then R is transitive relations

The statement is true

Let $(x,y) \in R$, $(x,y) \in R \circ R \Rightarrow \exists w \in Dom(R);\ (x,w) \in R \wedge (w,y) \in R$

Suppose $R \circ R \subseteq R$ and suppose $(x,y) \in R$ and $(y,z) \in R$

$(x,z) \in R \circ R$

$\Rightarrow (x,z) \in R$ (since $R \circ R \subseteq R$) $\Rightarrow$ R is transitive

Question#2(14%) Let A ={ a,b,c} , define R on X=P(A) as follows

R={(C,D) ∈ X×X : number of elements of C= number of elements of D}

a)Show that R is an equivalence relation onX

let C ∈ X ⇒ (C,C) ∈ X×X ⇒ R is ref.

let (C,D) ∈ X×X ⇒ C ∈ X ∧ D ∈ X, # of element C = # of element D ⇒ D ∈ X ∧ C ∈ X, # of element D = # of element C ⇒ (D,C) ∈ X×X ⇒ R is sym.

let (C,D) ∈ R ∧ (D,F) ∈ R ⇒ C ∈ X ∧ D ∈ X ∧ D ∈ X ∧ F ∈ X (# of element C = # of element D) ∧ (# of element D = # of element F) ⇒ # of element C = # of element F ⇒ (C,F) ∈ R ⇒ trans.

∴ R is equivalence relation on X

b) find [{b,c}]

[{b,c}] = { {a,b}, {a,c}, {b,c} }

c) Find the partion induced by the equivalence relation R

{ {a}, {b}, {c} }

3 → { {a,b,c} } ⇒ R = X×X

Question #3(10%) Let A={1,2,3,4} and let P={{1,2},{3},{2,4}}

Find if possiple an equivalence relation R on A such that

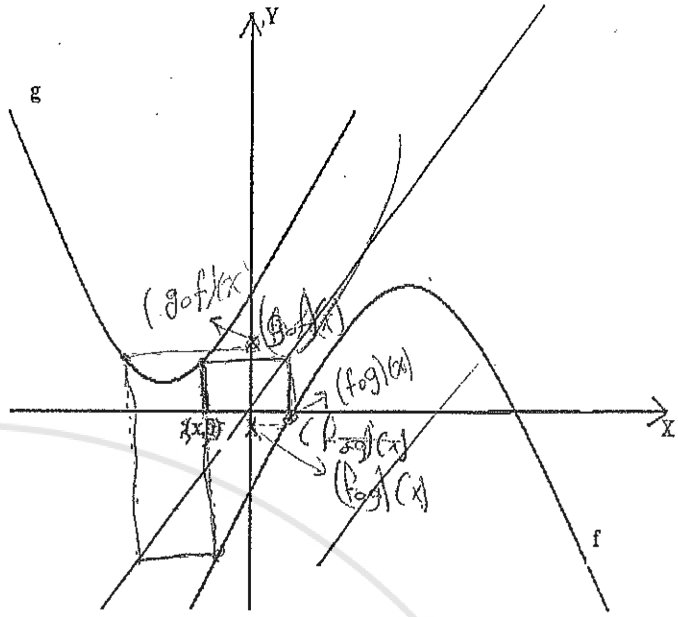
A/R = P justify your answer

P is not partition because {1,2} ∩ {2,4} = {2} but {1,2} ≠ {2,4}

⇒ P is not equ. Relation.

Question #4(10%) Use the graph of the functions f, and g to indicate on the graph the value of $(f \circ g)(x)$ and $(g \circ f)(x)$

$f(g(x))$



Question #5(15%) Let A, B, C be nonempty set and let $f: A \rightarrow B$, $g: B \rightarrow C$ be functions.

a) Show that if $g \circ f$ is onto then $g: B \rightarrow C$ is onto.

$(g \circ f)$ let $(a,c) \in g \circ f \Rightarrow \exists b \in B; (a,b) \in f \wedge (b,c) \in g$

$\forall c \in C; g(f(a)) = c$

$f \circ g$ is onto

$\Rightarrow \forall c \in C; g(b) = c$

$\Rightarrow g$ is onto.

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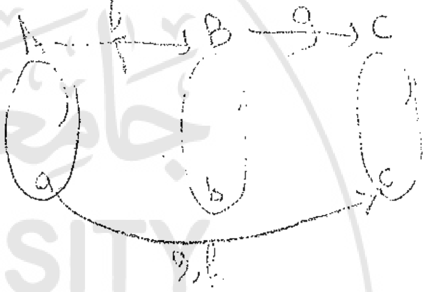


Suppose $g \circ f$ is onto and let $c \in C$

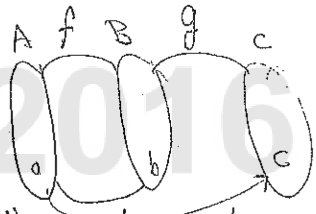
$\Rightarrow \exists a \in A; g \circ f(a) = c$

$\Rightarrow g(f(a)) = c$

$\exists b \in B; f(a) = b$

$\Rightarrow g(f(a)) = g(b) = c$

$\Rightarrow g$ is onto.



Suppose $(g \circ f)$ is onto and suppose $c \in C$.

$\Rightarrow \exists a \in A; (a,c) \in f \circ g$.

$\Rightarrow f(g(a)) = c$

c) Show that the converse of (a) is not true

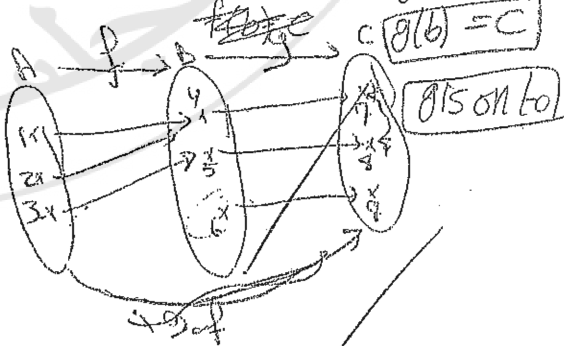
Converse :- if $g: B \rightarrow C$ is onto then $g \circ f$ is onto $\Rightarrow \exists b \in B; f(a) = b$, $g(f(a)) =$ c $g(b) = c$ g is onto

$f = \{(1,4), (2,4), (3,5)\}$

$g = \{(4,7), (5,8), (6,9)\}$

$g \circ f = \{(1,7), (2,7), (3,8)\}$

$y = 9 \notin Rang(g \circ f)$.



Converse : if g is onto then $g \circ f$ is onto.

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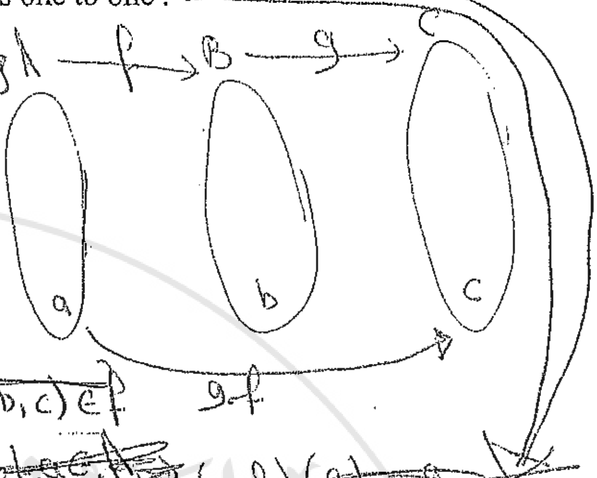
Question #6(15%) Let A, B, C be nonempty set and

let $f : A \to B$, $g : B \to C$ be functions.

a) Show that if f, g are one to one functions then $g \circ f : A \to C$ is one to one.



~~let $(a, c) \in g \circ f \Rightarrow \exists b \in B; (a,b) \in f \wedge (b,c) \in g$~~

suppose f, g is 1-1

~~$\Rightarrow g(f(a)) = g(b)$~~

and suppose $g \circ f(x) = g \circ f(y)$ ~~$\Rightarrow f(a_1) \neq f(a_2)$~~

$g(f(x)) = g(f(y))$

g is 1-1 $\Rightarrow f(x) = f(y)$ f is 1-1 $\Rightarrow x = y$

~~let $(a, c) \in g \circ f \Rightarrow \exists b \in B; (a,b) \in g \wedge (b,c) \in f$~~

let $\Rightarrow (g \circ f)(a_1) = (g \circ f)(a_2)$

$\Rightarrow g(f(a_1)) = g(f(a_2))$

~~g is 1-1 $f(a_1) = f(a_2)$~~

g is 1-1 $f(a_1) = f(a_2)$

~~f is 1-1 $a_1 = a_2 \Rightarrow g \circ f$ is 1-1~~

a) let $(g \circ f)(a_1) = (g \circ f)(a_2)$

$\Rightarrow g(f(a_1)) = g(f(a_2))$

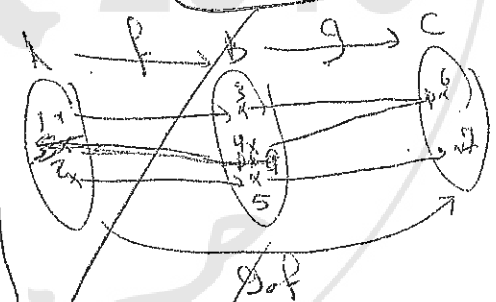
g is 1-1 $\Rightarrow f(a_1) = f(a_2)$

f is 1-1 $\Rightarrow a_1 = a_2$

$\therefore \Rightarrow g \circ f$ is 1-1

c) Show that the converse of (a) is not true

converse: If $g \circ f$ is 1-1 then f, g is 1-1



$f = \{(1,3), (2,5)\}$

$g = \{(3,6), (4,6), (5,7)\}$

$g \circ f = \{(1,6), (2,7)\}$

$g \circ f$ is 1-1

but g is not one to one.

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