$\frac{97}{100}$

Birzeit University
Department of Mathematics
math 233

First hour exam — Spring 2013

Name : [redacted]

Number: [redacted]

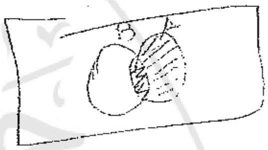
Question#1(15%) Which of the following statements is true and which is false

1)...F...$\{2,\{1,2\}\} \subseteq \{1,\{2\},\{1,2\}\}$

2)...T...$\phi \in \{1,\phi\}$

3)...F...$(\forall x \in \mathbb{R})(x^2 > 0)$ $X=0$

4)...T... for any sets A, B it is true that $(A \cap B) \subseteq B$



5)...T...for any sets A, B it is true that $A = (A - B) \cup (A \cap B)$ A and B'

$(A \cap B') \cup (A \cap B)$

6)...F...If A,B are sets and $x \notin A$ and $A \notin B$ then $x \notin B$

$A + B$

7)...F...$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy > 1)$

$y \in A$ ~~$X \notin A \cup 1$~~

8)...T...$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x + y = 5)$ $A \in B$

9)...F...$\{1\} \subseteq \{\phi, \{1\}\}$ $x \notin A$

10)...F...$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z^2 = xy)$ $\boxed{x}$

1 2 3

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Question #2 a)(9%):Write the converse ,contrapositive and negation of the following statement
If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

converse: - If $A \subseteq C$ then $A \subseteq B$ and $B \subseteq C$

contrapositive: - if $A \nsubseteq C$ then $A \nsubseteq B$ or $B \nsubseteq C$

negation: - ~~$A \subseteq B$ and $B \nsubseteq C$~~ $A \subseteq B$ and $B \subseteq C$ and $A \nsubseteq C$

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b)(6%):Write a useful negation for each of the following statements

1) Some text books are either expensive or not easy to read

all text books are not expensive and easy to read.

2) $(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(\exists z \in \mathbb{Z})(xz \geq y)$

$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\forall z \in \mathbb{Z})(xz < y)$

Question # 3 (10%): Test the validity of the following argument

$P \Rightarrow q$ T

$q \Rightarrow s \wedge t$ T

$s \Rightarrow r \vee \sim t$ T

p T

---

r

$\boxed{1}$ $p \Rightarrow q$ T

$p \equiv T$ then q must be T

$\cancel{p \Rightarrow q\ T}$

$\Rightarrow q \equiv T$

2) $q \Rightarrow s \wedge t$ T

$q = T$

$\Rightarrow s \wedge t = T$

$\Rightarrow s = \cancel{F}T$ and $t = T$

3) $s \Rightarrow r \vee \sim t$ T

$s = T$

$\Rightarrow r \vee \sim t = T$

$\sim t = F$

$\Rightarrow r$ must be T

$\Rightarrow \boxed{r = T}$

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Question#4(10%) Without using truth table prove that the following are equivalent

$(p \to q) \wedge (r \to q)$ is equivalent to $(p \vee r) \to q$

Hint: you can use any of the following $(p \to q) \cong \sim p \vee q$

$\sim(p \wedge q) \cong \sim p \vee \sim q$

$p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$

$(p \to q) \wedge (r \to q)$

$(\sim p \cup q) \wedge (\sim r \cup q)$

$(\sim p \cap \sim r) \cup (q \cap q)$

$\sim (p \cup r) \cup q$

$(p \cup r) \to q$

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Question # 5 (27%):Prove the following $x+y \leq x+y+2\sqrt{x}\sqrt{y}$

1) Prove that if x and y are positive real numbers then $\sqrt{x+y} \leq \sqrt{x}+\sqrt{y}$

By Direct

Suppose $x \geq 0$ and $y \geq 0$

$\Rightarrow x+y \geq 0$

$\Rightarrow 2\sqrt{x}\cdot\sqrt{y} \geq 0$

$\Rightarrow x+y \leq x+y+2\sqrt{x}\sqrt{y}$

$\Rightarrow x+y \leq (\sqrt{x}+\sqrt{y})^2$

$\Rightarrow \sqrt{x+y} \leq |\sqrt{x}+\sqrt{y}|$ [illegible]

$\Rightarrow \sqrt{x+y} \leq \sqrt{x}+\sqrt{y}$

2)Let m,n $\in \mathbb{Z}$ prove that if mn is even then m is even or n is even

By Indirect :-

Suppose m is odd and n is odd

$\Rightarrow \exists K, L \in \mathbb{Z} \; ; \; m = 2K+1 \; , n = 2L+1$

$\Rightarrow mn = (2K+1)(2L+1) = 4KL+2K+2L+1 = 2(2KL+K+L)+1$

$= 2K'+1 \; , \; K' \in \mathbb{Z}$

$\Rightarrow mn$ is odd

3)For any sets A,B prove that If $\rho(A) \subseteq \rho(B)$ *Then* $A \subseteq B$

By Direct

Suppose $\rho(A) \subseteq \rho(B)$ and $x \in A$

$\Rightarrow \{x\} \in \rho(A)$

$\Rightarrow \{x\} \in \rho(B)$ Assume $\rho(A) \subseteq \rho(B)$

$\Rightarrow x \in B$

So $A \subseteq B$

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Question # 6 (15%): Prove or disprove

1) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x \le y^2)$ T

Example $x=0$

$$\boxed{\begin{array}{lll} x = 0 \in \mathbb{Z} & y=0 & 0 \le 0 \\ & y=1 & 0 \le 1 \\ & y=2 & 0 \le 4 \end{array}}$$

2) Some prime numbers are even (T)

example:- 2 is even and prime.

3) If A,B,C are sets and $A \cap B \subseteq A \cap C$ then $B \subseteq C$ (F)

counter example- A = {1, 2, ...}

B = {5, 6, 7}

C = {2, 4}

$A \cap B = \emptyset$

1.5

Question #7(8%) Assume that for each natural number n, $A_n = (\frac{-1}{n}, 2-\frac{1}{n}]$ find

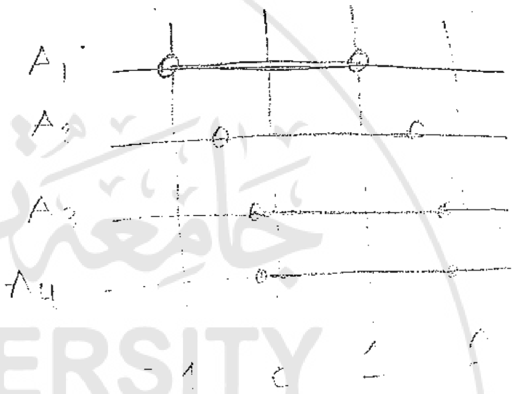
a) $\bigcap_{n\in\mathbb{N}} A_n$ $= [0, 1]$

$A_1 = (-1, 1)$

$A_2 = (-\frac{1}{2}, ...)$

$A_3 = (-\frac{1}{3}, \frac{5}{3})$

b) $\bigcup_{n\in\mathbb{N}} A_n$ $= (-1, 2)$



c) $\bigcup_{n\in\mathbb{N}} \overline{A_n}$ $= \bigcap_{n\in N} A_n = [0, 1]$