IMAGE ENHANCEMENT FREQUENCY DOMAIN

Aziz M. Qaroush

Birzeit University

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Outline

- Background
- Fundamentals of Filtering in Frequency Domain
- Steps for Filtering in Frequency Domain
- Smoothing in Frequency Domain
- Sharpening in Frequency Domain
- Selective Filtering
- Homomorphic Filtering

Fourier Series

- Any function that periodically repeats itself can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficients. This sum is called a Fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nw_0 t + b_n \sin nw_0 t$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} g(t)\,dt$$

$$a_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} g(t) \cos nw_0 t \; dt$$

$$b_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} g(t) \sin nw_0 t \; dt$$

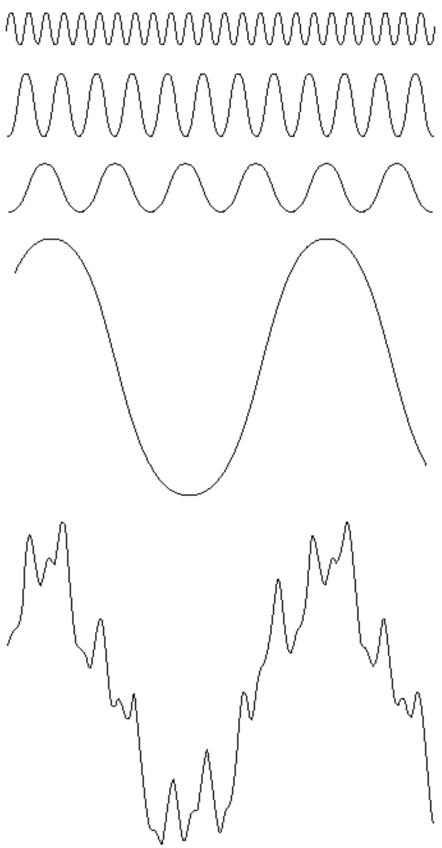


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Fourier Transform

- A function that is not periodic but the area under its curve is finite can be expressed as the integral of sines and/or cosines multiplied by a weighing function. The formulation in this case is Fourier transform.

$$\Im\{f(t)\} = F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt$$

- We can reconstruct f(t) back using

$$\Im^{-1}\{F(\mu)\} = f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{j2\pi\mu t}\, d\mu$$

The Fourier Transform and its Inverse

- The Fourier transform F(u) :

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

- The inverse Fourier transform f(x) :

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

- Two variables Fourier transform F(u, v) :

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy$$

- The inverse transform f(x, y) :

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du dv$$

Fourier Transform 1D - Example

- The Fourier Transform of One Continuous Variable

$$F(\mu) = AW\frac{\sin(\pi\mu W)}{\pi\mu W} = AW\,\mathrm{sinc}(\pi\mu W)$$

- Usually we work with the magnitude of F(u)

$$|F(\mu)| = \left|AW\frac{\sin(\pi\mu W)}{\pi\mu W}\right| = AW\,|\mathrm{sinc}(\pi\mu W)|$$

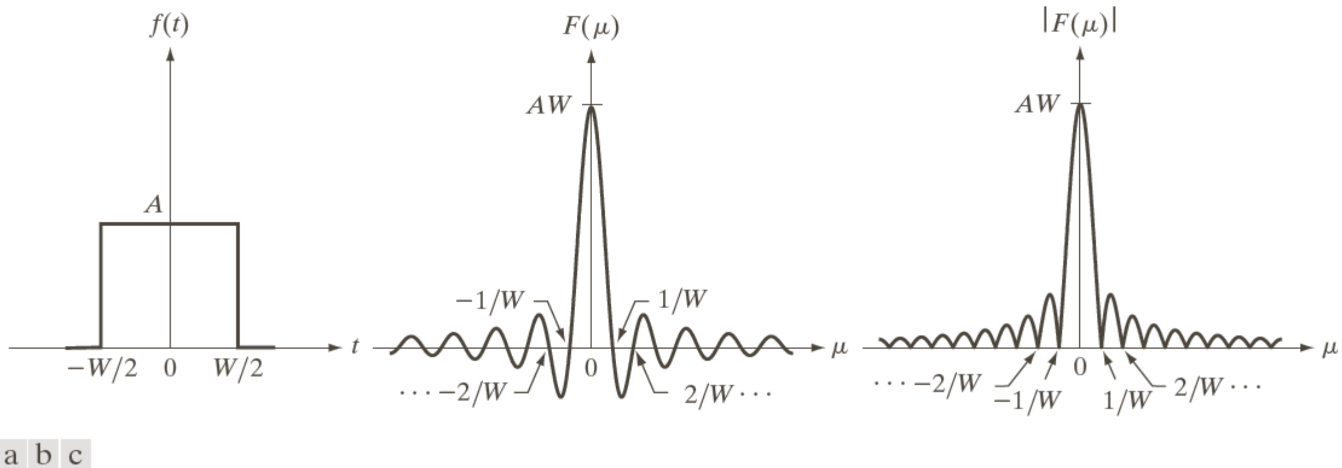


FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Fourier Transform 2D - Example

$$F(\mu,\upsilon) = ATZ\left|\frac{\sin(\pi\mu T)}{\pi\mu T}\right|\left|\frac{\sin(\pi\upsilon Z)}{\pi\upsilon Z}\right|$$

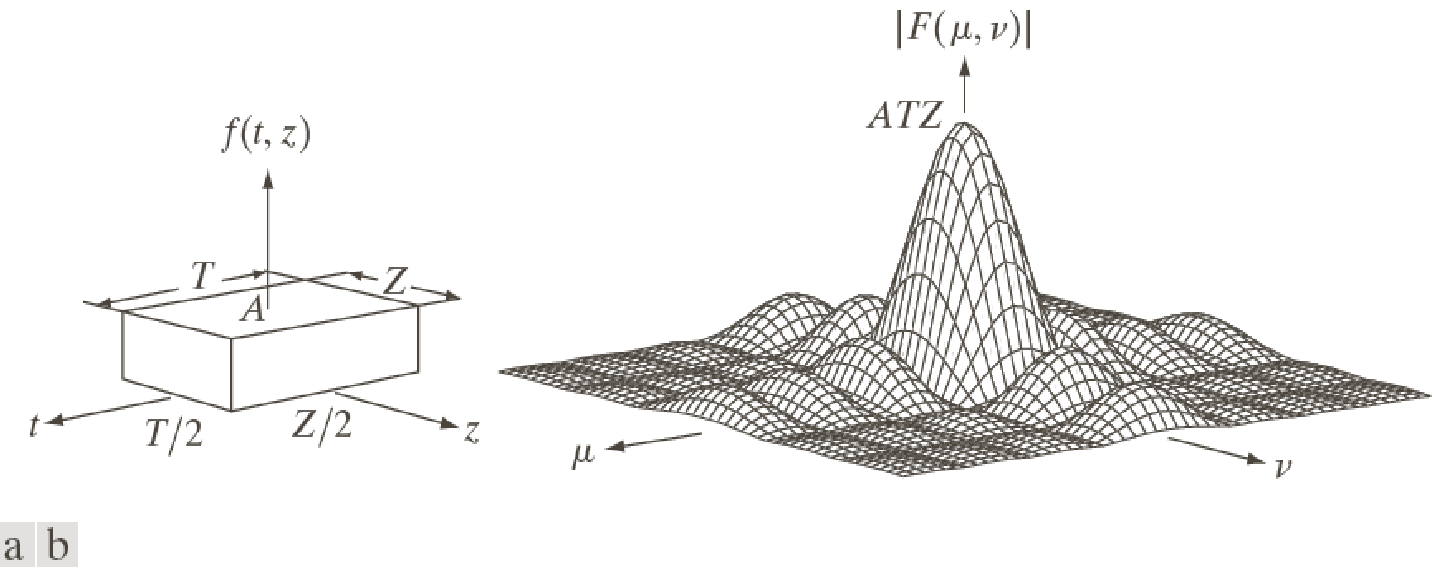


FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t-axis, so the spectrum is more "contracted" along the μ-axis. Compare with Fig. 4.4.

The one dimensional DFT and its Inverse

- The discrete Fourier Transform F(u) :

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad for \;\; u = 0,1,\cdots,M-1$$

- The inverse DFT :

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad for \;\; x = 0,1,\cdots,M-1$$

- Apply euler’s formula :

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]$$

The two dimensional DFT and its Inverse

- The discrete fourier transform of a function f(x,y) of size M x N :

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

- The inverse Fourier transform :

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

Some Properties of the 2-D DFT - Periodicity

- It is more convenient for processing and display to shift the spectrum to the middle of domain by multiplying the sampled function by $(-1)^{x+y}$

$$f(x,y)e^{j2\pi(\mu_0 x/M+\upsilon_0 y/N)} \Leftrightarrow F(\mu-\mu_0,\upsilon-\upsilon_0)$$

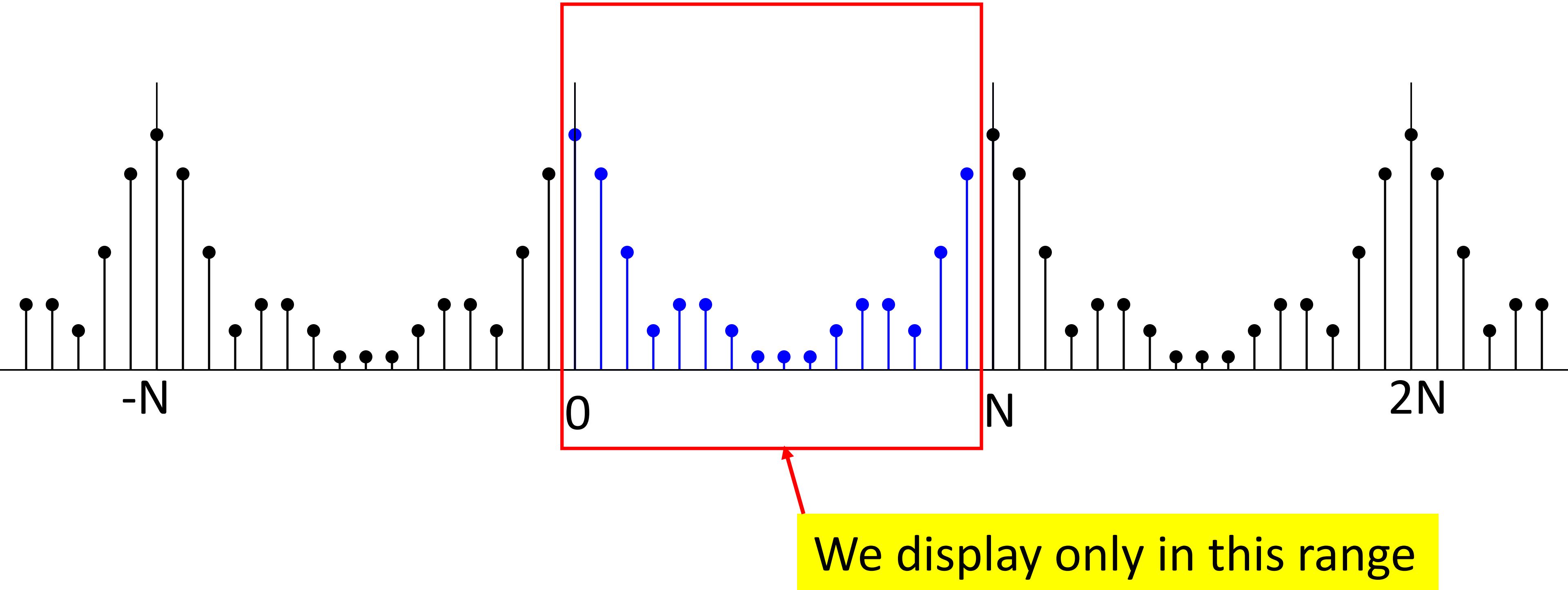
$$\text{if we let } \mu_0 = M/2 \text{ and } \upsilon_0 = N/2$$

$$f(x,y)e^{j\pi(x+y)} \Leftrightarrow F(\mu-M/2,\upsilon-N/2)$$

$$f(x,y)(-1)^{(x+y)} \Leftrightarrow F(\mu-M/2,\upsilon-N/2)$$

Some Properties of the 2-D DFT - Periodicity

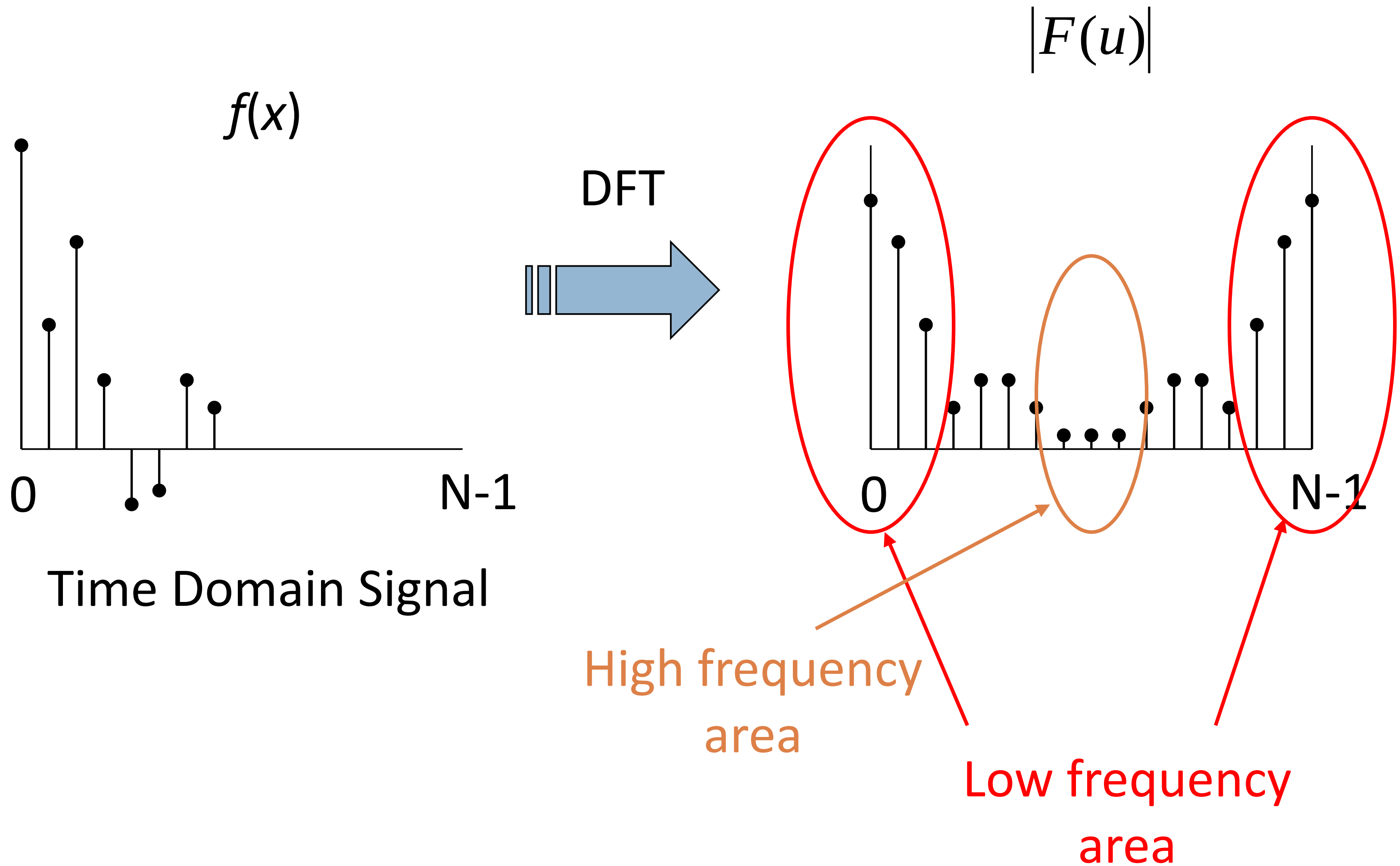
From DFT: $$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}$$



DFT repeats itself every N points (Period = N) but we usually display it for n = 0 ,..., N-1

Some Properties of the 2-D DFT - Periodicity

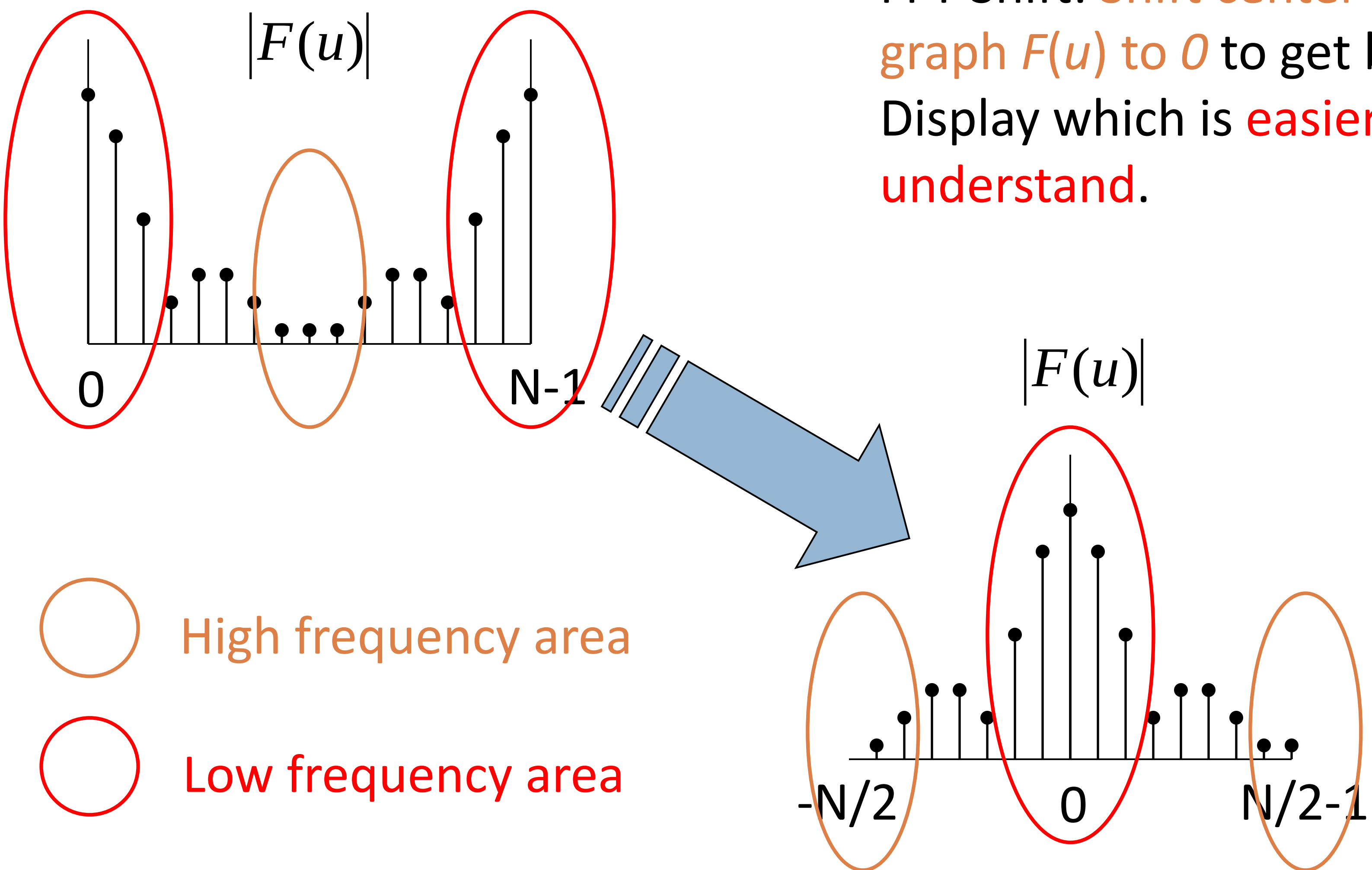
- **Conventional Display for 1-D DFT**



The graph $F(u)$ is not easy to understand !

Some Properties of the 2-D DFT - Periodicity

Conventional Display for 1-D DFT

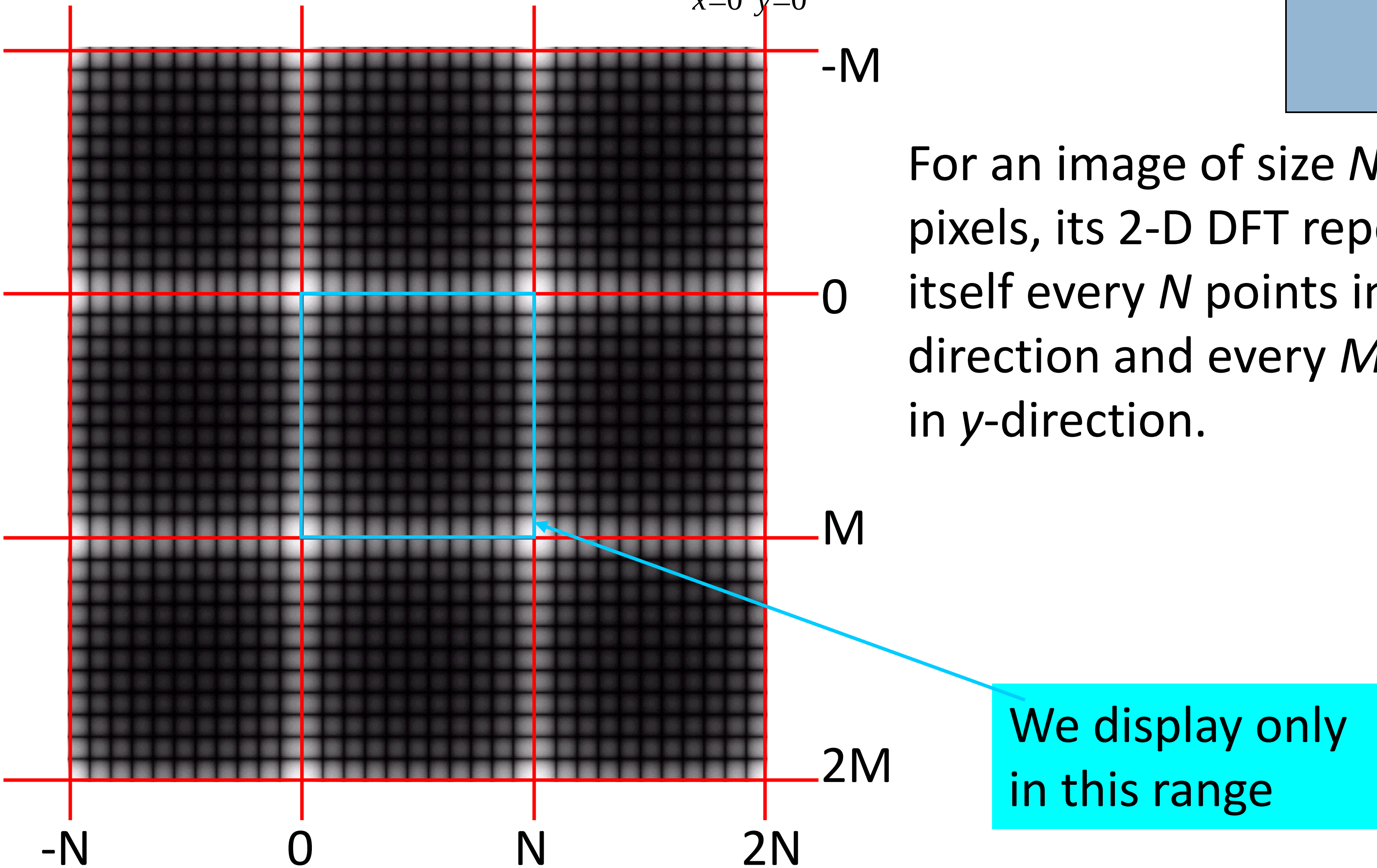


FFT Shift: Shift center of the graph *F*(*u*) to *0* to get better Display which is easier to understand.

Periodicity of 2-D DFT

2-D DFT: $$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

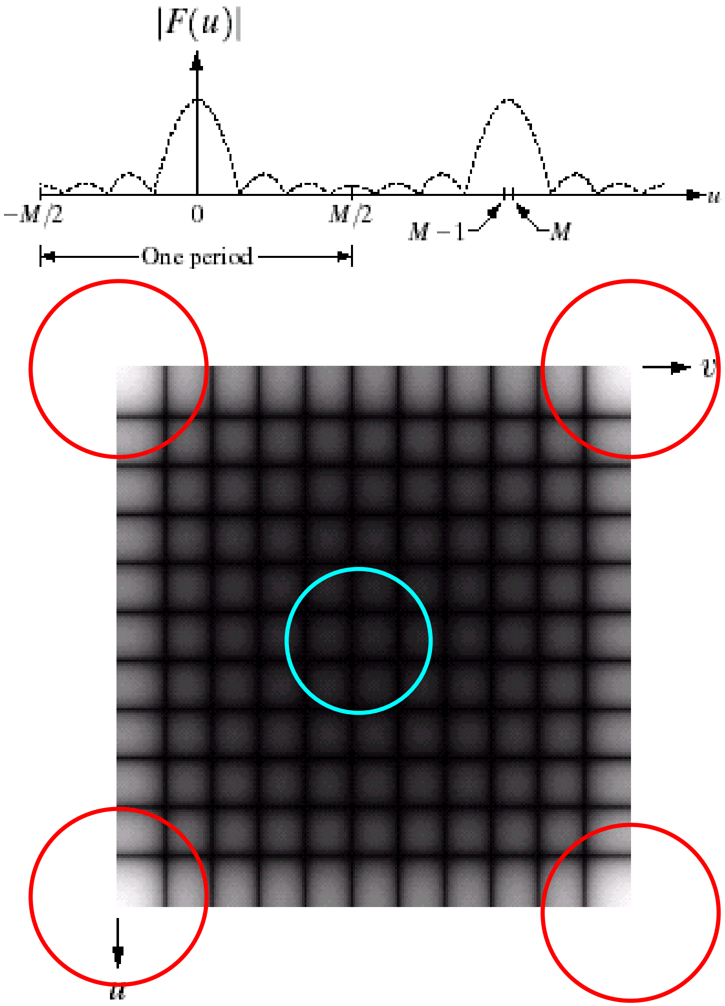
$g(x,y)$



For an image of size NxM pixels, its 2-D DFT repeats itself every N points in x-direction and every M points in y-direction.

Conventional Display for 2-D DFT

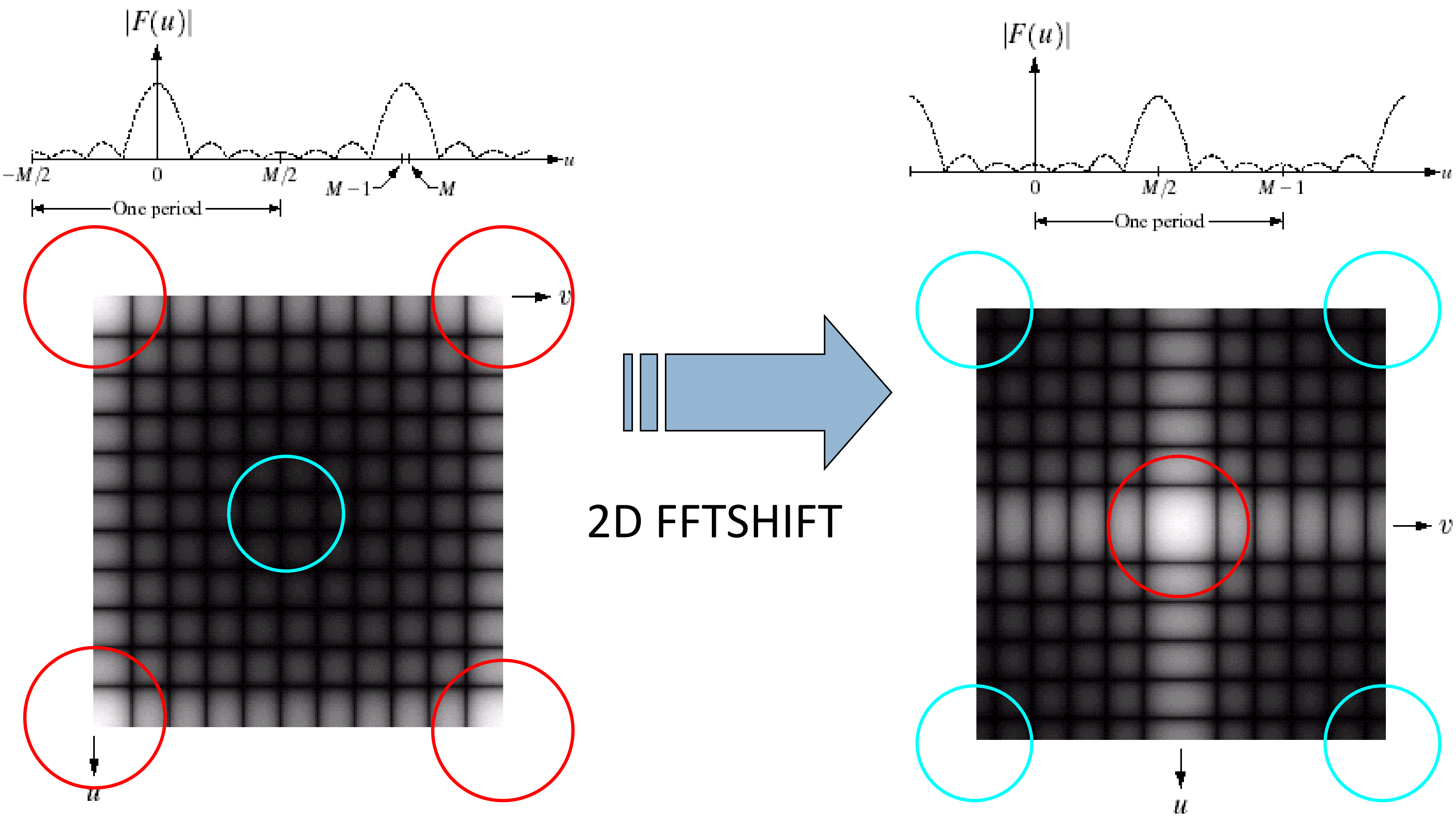
$F(u,v)$ has low frequency areas at corners of the image while high frequency areas are at the center of the image which is inconvenient to interpret.



High frequency area

Low frequency area

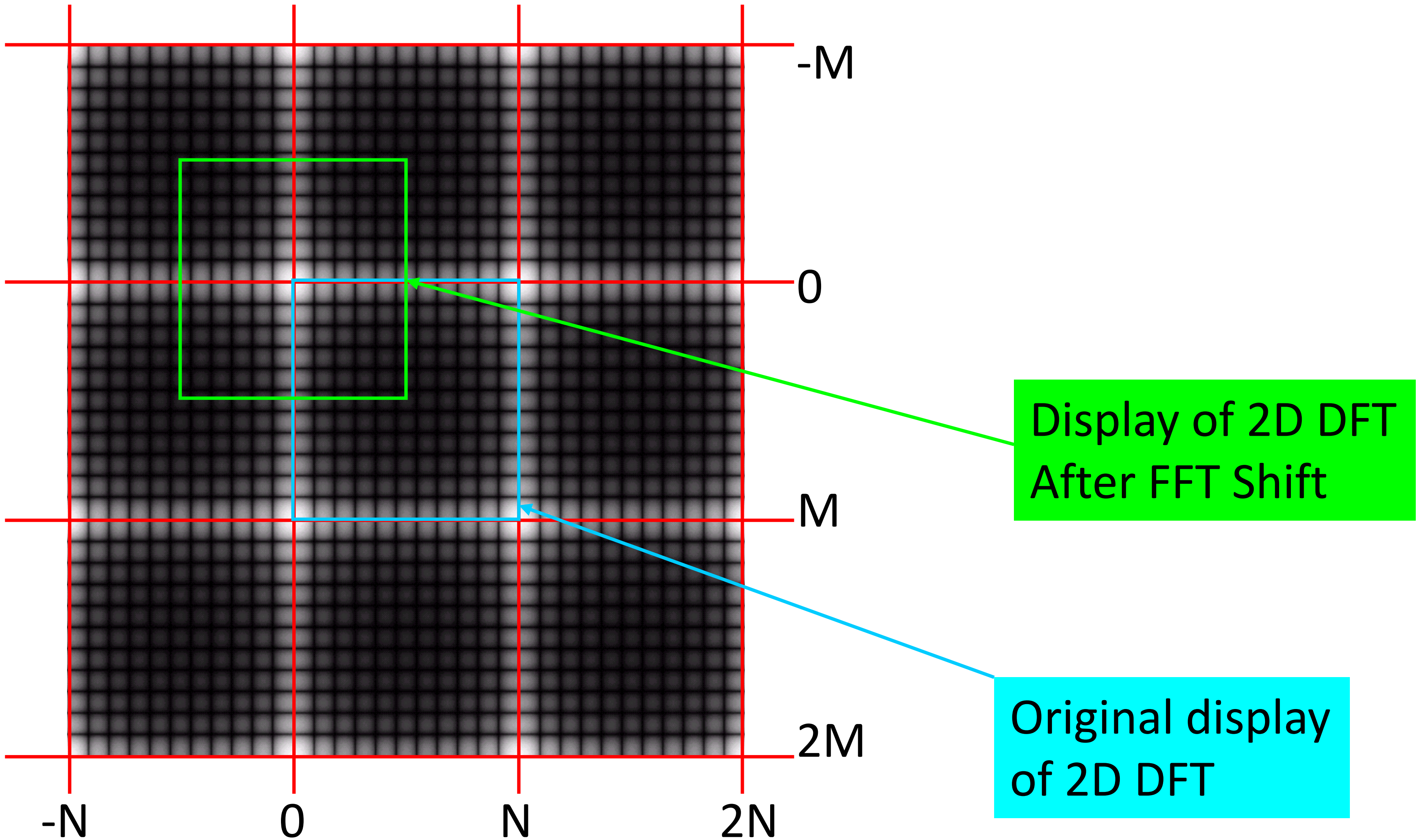
2-D FFT Shift : Better Display of 2-D DFT



High frequency area

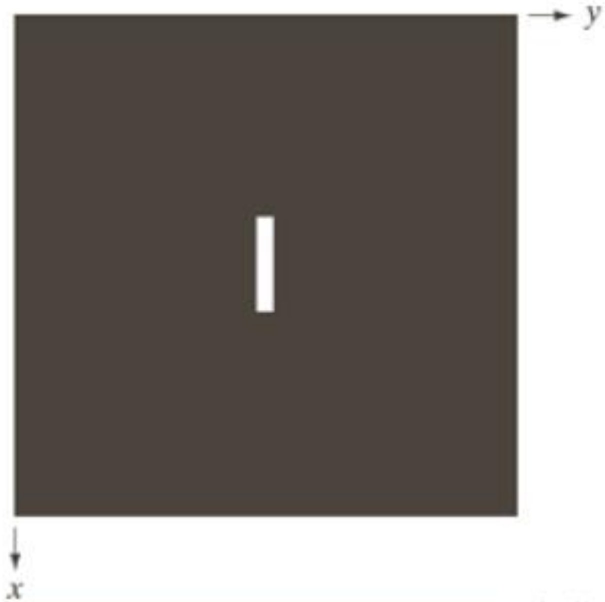
Low frequency area

2-D FFT Shift (cont.) : How it works

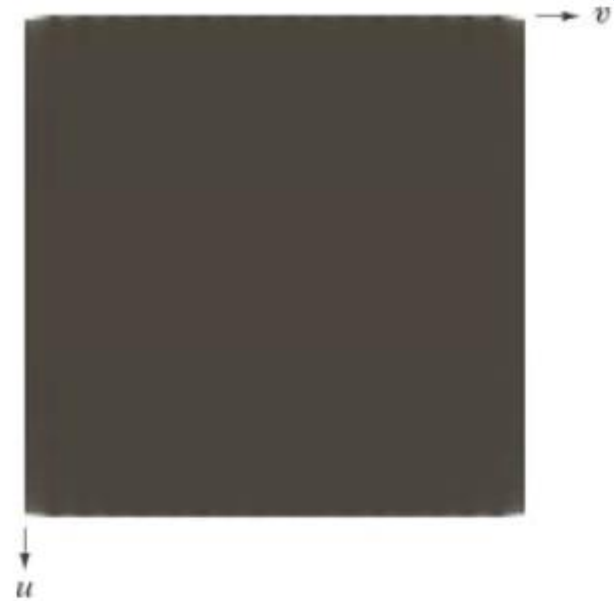


Example of 2-D DFT

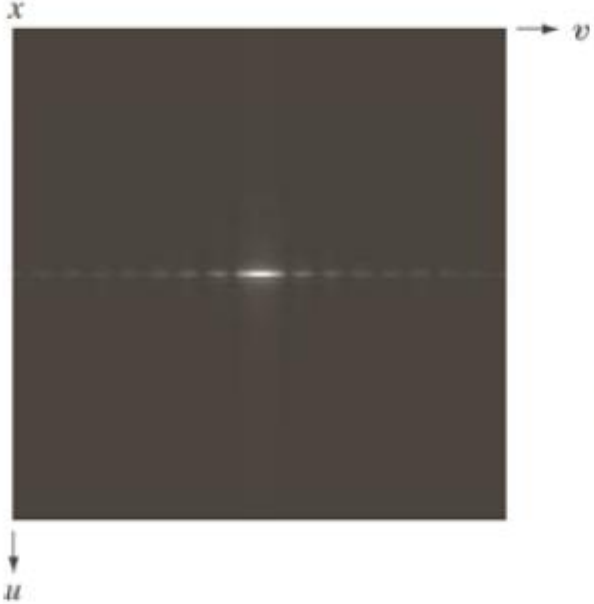
Image



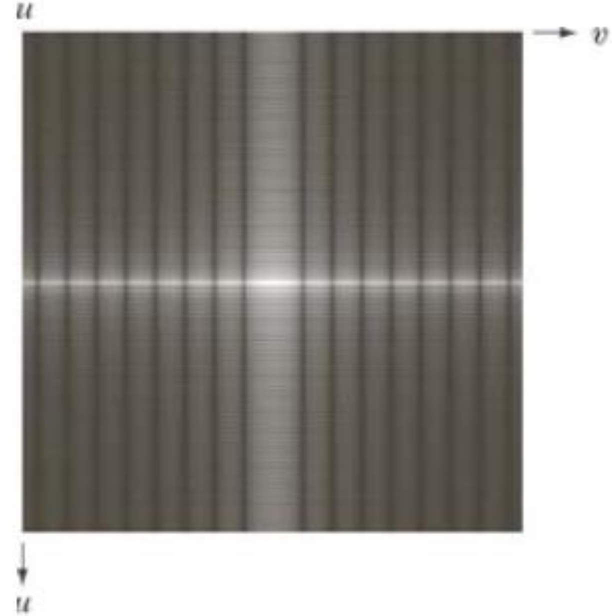
Spectrum
without
shifting



Spectrum
after
shifting



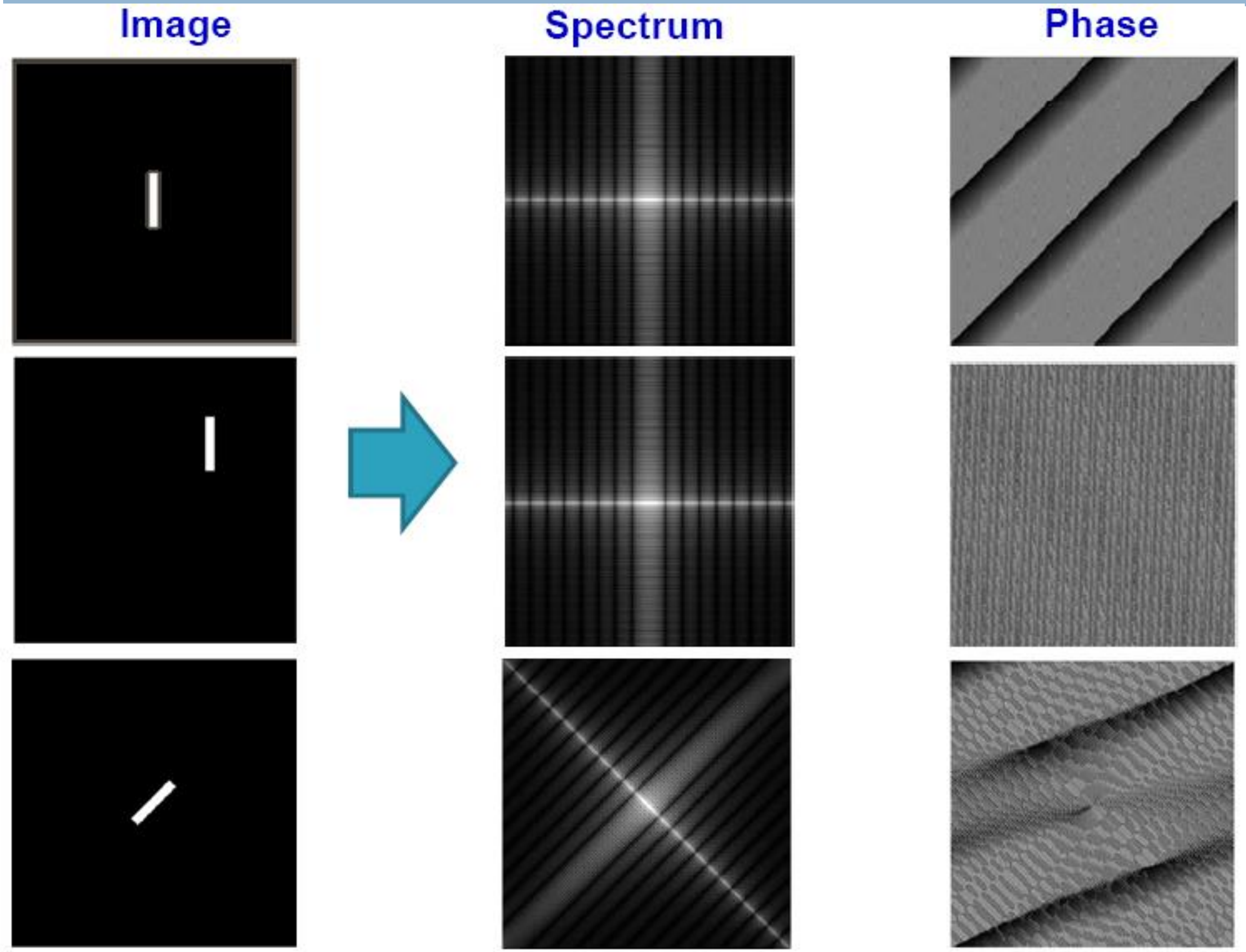
Spectrum
After shifting
and log
transformation



Some Properties of the 2-D DFT

- **Fourier Spectrum and Phase Angle**
  - The 2-D DFT is complex in general and usually is expressed as two separate functions in the frequency domain
    - The Magnitude $$|F(u,v)| = [R^2(x,y) + I^2(x,y)]^{1/2}$$
    - Phase $$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$
    - The power spectrum $$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$
    - The magnitude of F(0,0) (dc component) is proportional to the average value of f(x,y) and is typically the largest component in the spectrum

Some Properties of the 2-D DFT



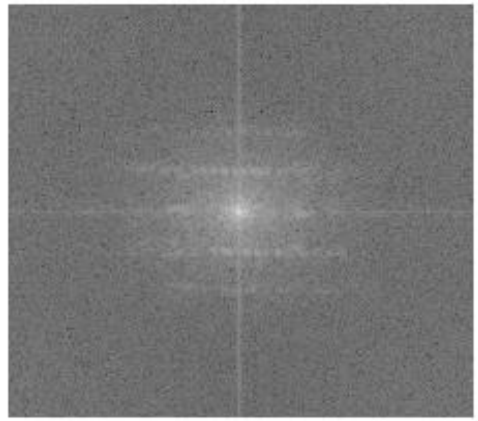
Some Properties of the 2-D DFT

- **Fourier Spectrum and Phase Angle**
  - The components of the spectrum of the DFT reflects the amplitudes of the sinusoids that combine to represent the images.
  - A large amplitude at any frequency implies greater prominence of that frequency in the image, and vice versa.
  - The phase of the spectrum is a measure of the displacement of various sinusoids with respect to their origin.
  - We can say that the magnitude of the DFT is an array whose components determine the intensities in the image while the phase angle carry the information about where discernable objects are located

Some Properties of the 2-D DFT

- Fourier Spectrum and Phase Angle

Image



Magnitude of Spectrum

Phase of Spectrum

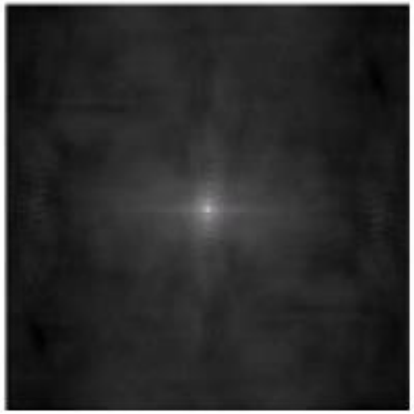
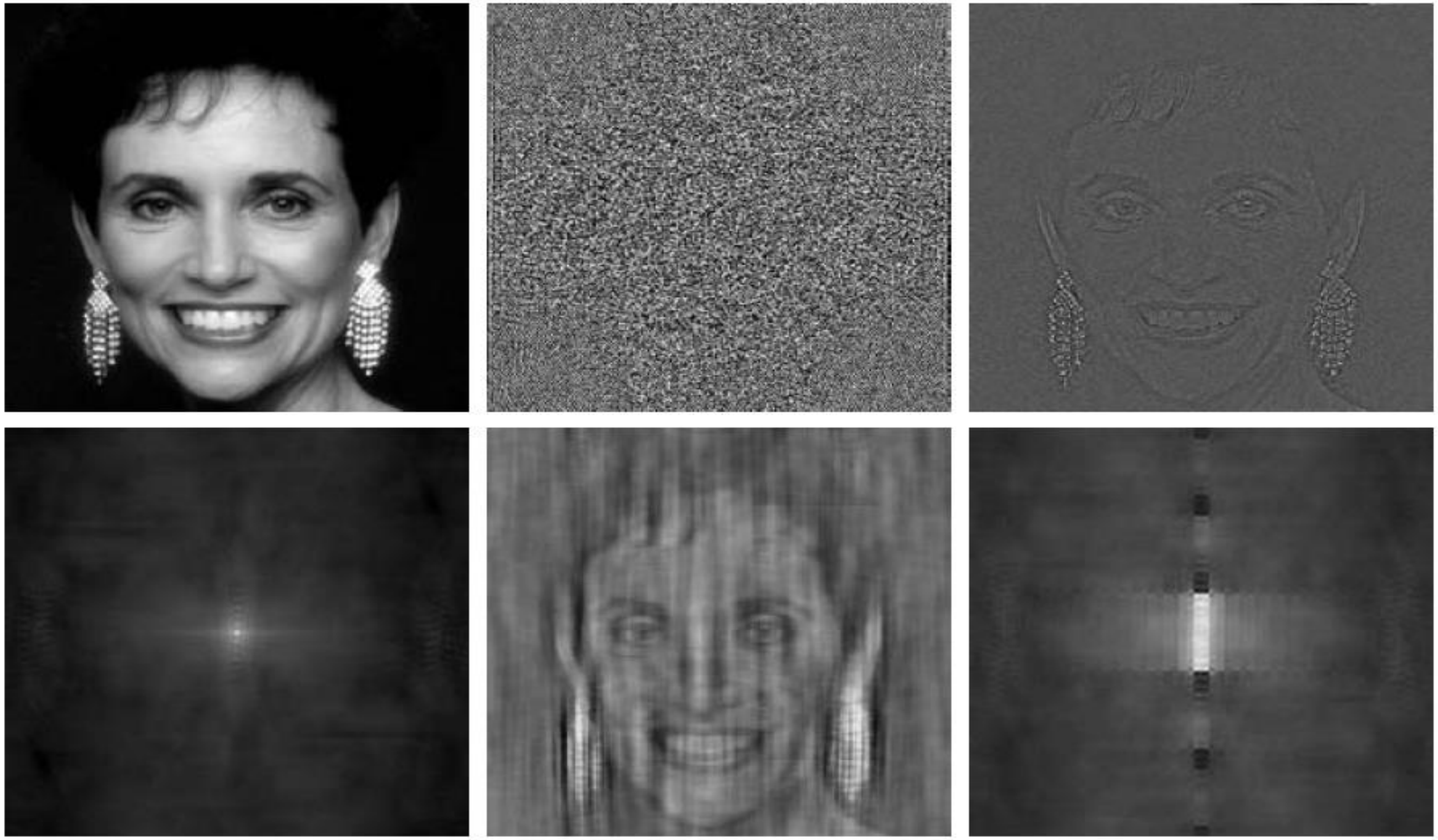


Image Reconstructed
Using Magnitude only

Image Reconstructed
Using Phase only

Some Properties of the 2-D DFT

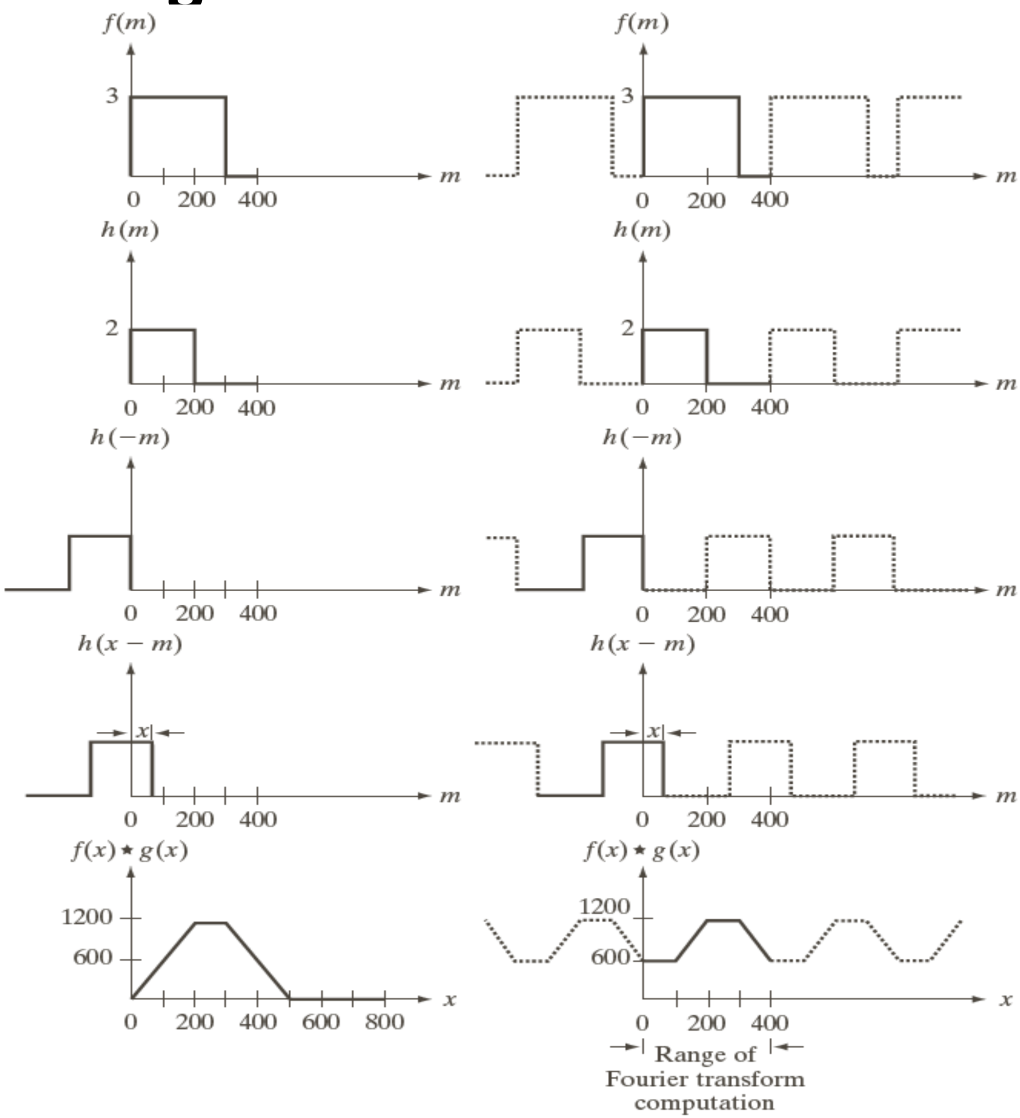


| a | b | c |
| d | e | f |

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

Some Properties of the 2-D DFT

- Zero padding



a	f
b	g
c	h
d	i
e	j

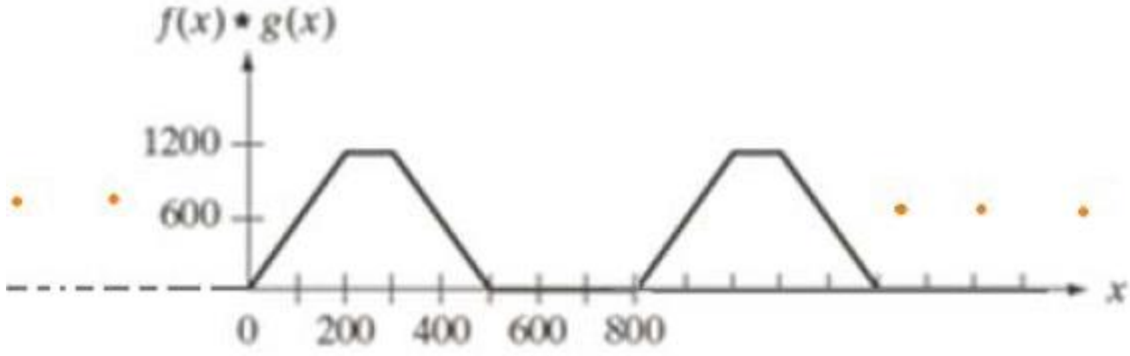
FIGURE 4.28 Left column: convolution of two discrete functions obtained using the approach discussed in Section 3.4.2. The result in (e) is correct. Right column: Convolution of the same functions, but taking into account the periodicity implied by the DFT. Note in (j) how data from adjacent periods produce wraparound error, yielding an incorrect convolution result. To obtain the correct result, function padding must be used.

Some Properties of the 2-D DFT

- **Zero padding**
  - A simple solution is to zero-pad both functions f(x,y) and h(x,y) such they are of equal size P that satisfies

$$P \geq A + B - 1$$

    - where A and B are the size of the original functions *B*
  - If we perform convolution after zero-padding the result will be periodic and each period contains the desired result



Some Properties of the 2-D DFT

Zero padding

- For two 2-D functions f(x,y) and h(x,y) with sizes AxB and CxD, zero-padding should be performed in both directions such that the new size is PxQ

$$P \geq A + C - 1 \quad and \quad Q \geq B + D - 1$$

- If the two function/arrays/images are of the same size, padding is achieved by

$$P \geq 2M - 1 \quad and \quad Q \geq 2N - 1$$

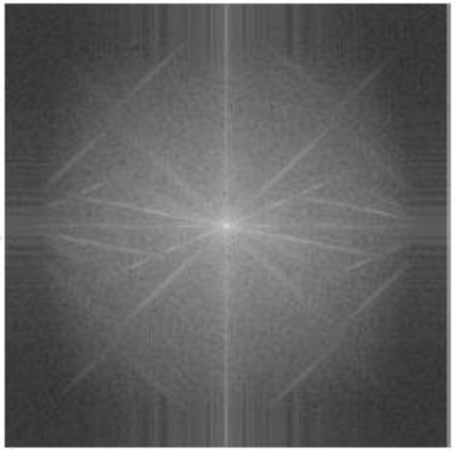
- The padded zeros are to be removed once we have the final result

Some Properties of the 2-D DFT

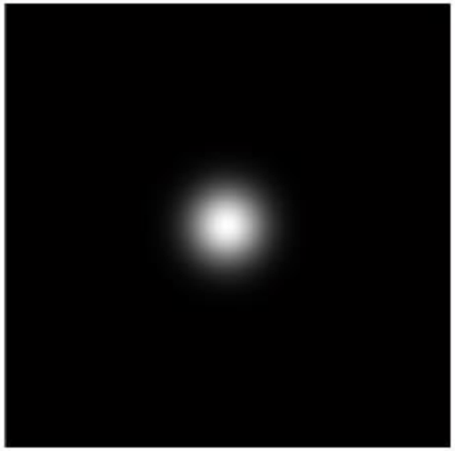
- Zero padding - Example

Original

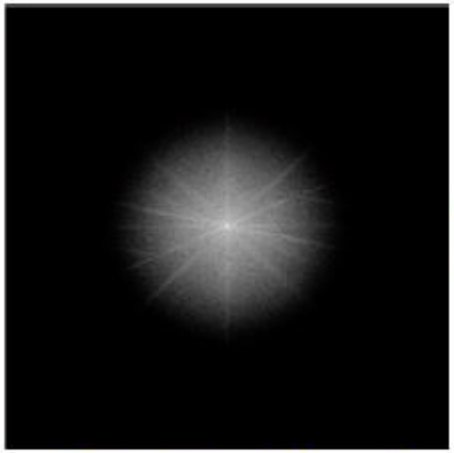
DFT



|F(u,v)|



H(u,v)



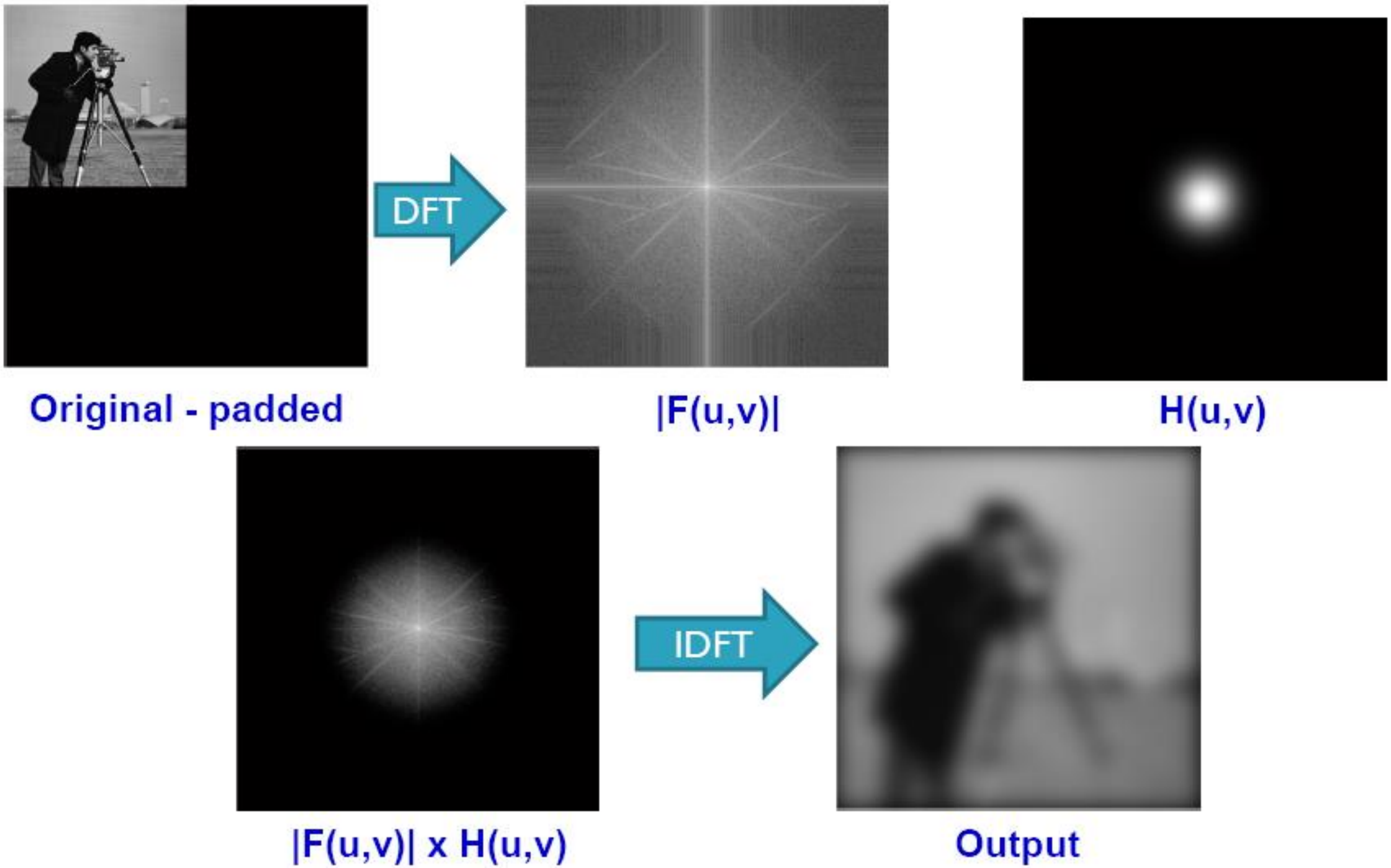
|F(u,v)| x H(u,v)

IDFT

Output

Some Properties of the 2-D DFT

- Zero padding - Example



Some Properties of the 2-D DFT

- **Zero padding – Example**
  - Note
    - More blurring in the image that was padded
    - Blurring near the image borders is symmetric in the padded case

Output without padding

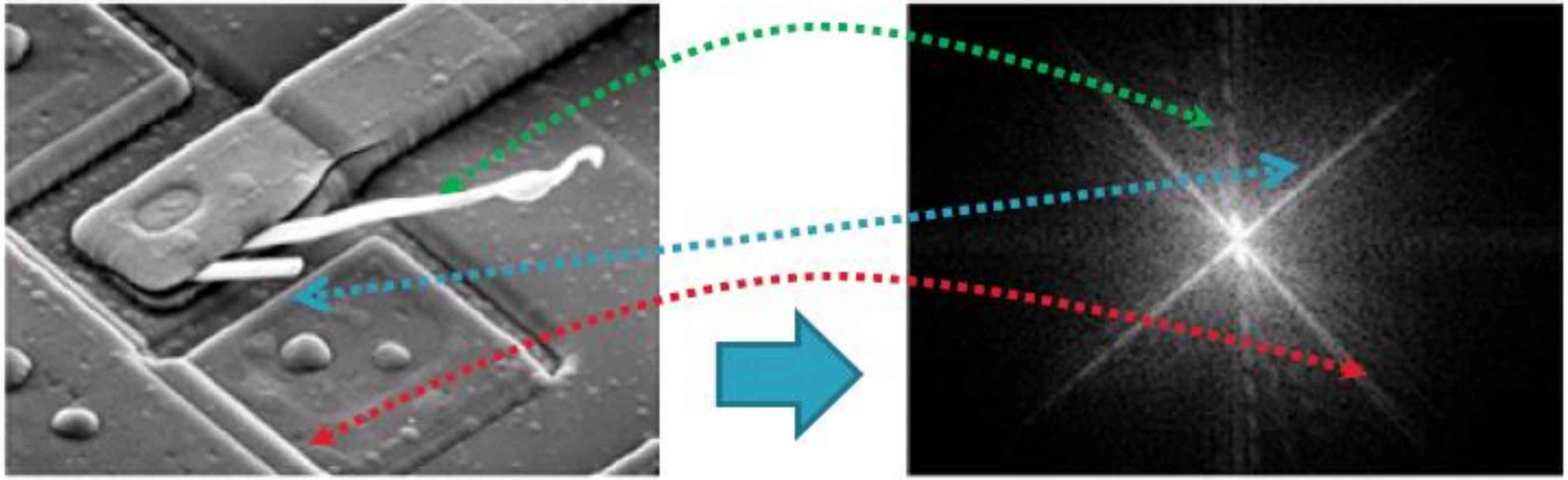
Output with padding

Fundamentals of Frequency Filtering

- When the image is represented in frequency domain using Fourier transform, it is almost impossible to make direct association between the pixel values and their transform
- However, the following generalization can be made regarding the relation between the two representations
  - Since frequency is related to spatial rates of change, we can associate frequencies in the Fourier transform with pattern of intensity variations in the image
  - For example
    - The slowest frequency component (u=v=0) is associated with average intensity (DC component)
    - Smooth intensity variations correspond to low frequencies such as walls or cloudless sky
    - High frequencies correspond to higher levels or abrupt intenisty variations such as edges and noise

Fundamentals of Frequency Filtering

- **Example**



- **Note**
  - The correspondence between the strong edges at angles 45 and -45 and the presence of high intensity frequencies in the same direction
  - A little variation that is off the vertical access in the frequency domain which is associated by the oxide protrusion variation in the vertical axis in the image

Fundamentals of Frequency Filtering

- Generally, image filtering in the frequency domain consists primarily of the following steps
  1) Computing the Fourier transform of the image
  2) Modifying the magnitude of image spectrum using specific operation
  3) Taking the inverse Fourier transform of the result
- The basic filtering operation can be expressed as

$$g(x,y) = \Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right]$$

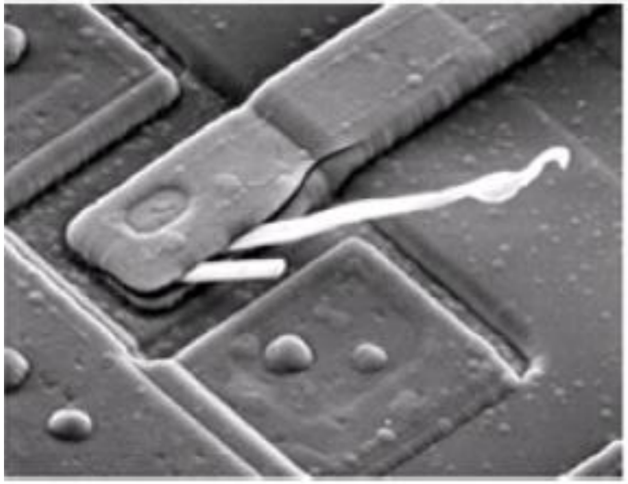
  where H(u,v) is called the filter function and it is of same size as F(u,v)
- Filtering is performed usually by using real filter functions H(u,v) as we don't want to modify the structure of the image which is contained in the phase of F(u,v)

Fundamentals of Frequency Filtering

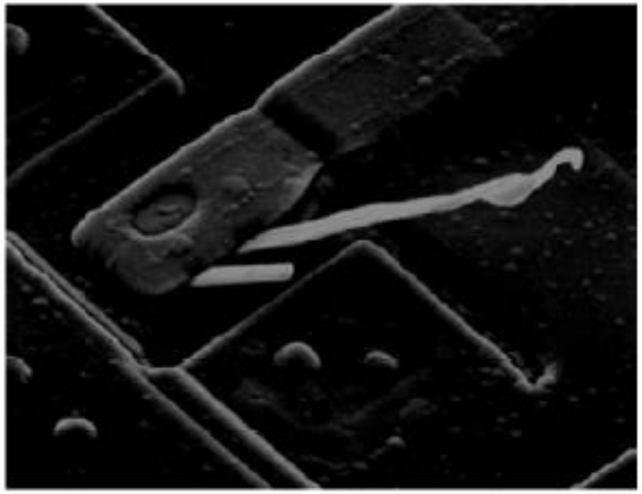
- One simple filter is

$$H(\mu,\upsilon)=\begin{cases}0\ , & \mu=\upsilon=0\\ 1\ , & otherwise\end{cases}$$

- Simply this filter sets the average intensity value (the dc value) to zero. We should expect the output image to be darker than the original

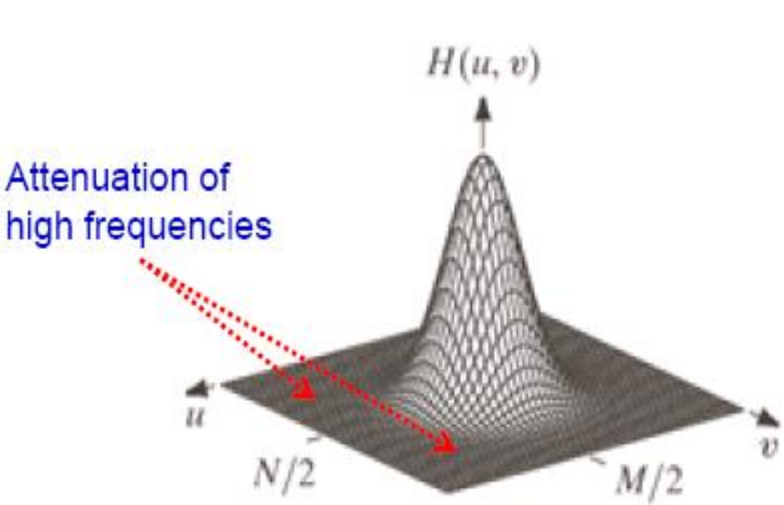


$$\Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right]$$

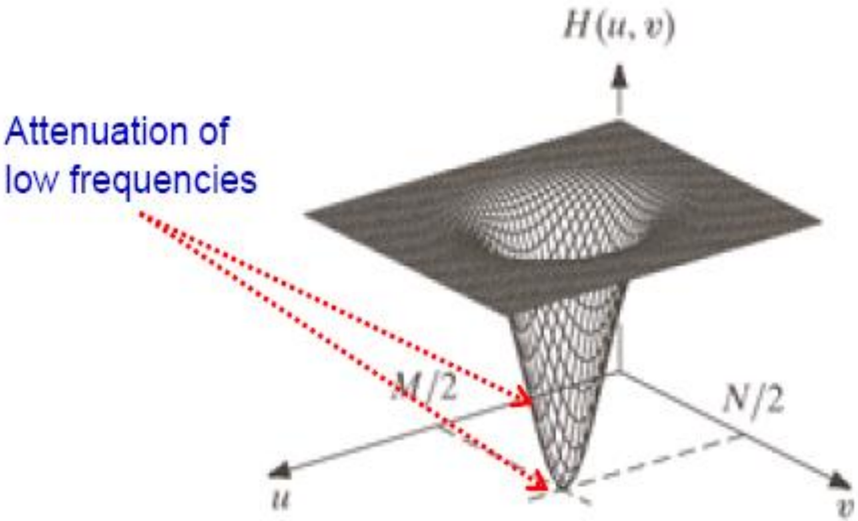


Fundamentals of Frequency Filtering

- **Common filter functions**
  - Lowpass filter: *it is used to attenuate high frequency components of the image. Thus, the output image appears blurred*
  - Highpass filter: *it is exactly the opposite of lowpass filter, i.e. it attenuates low frequency components. It enhances sharp details in the image but reduces image contrast*



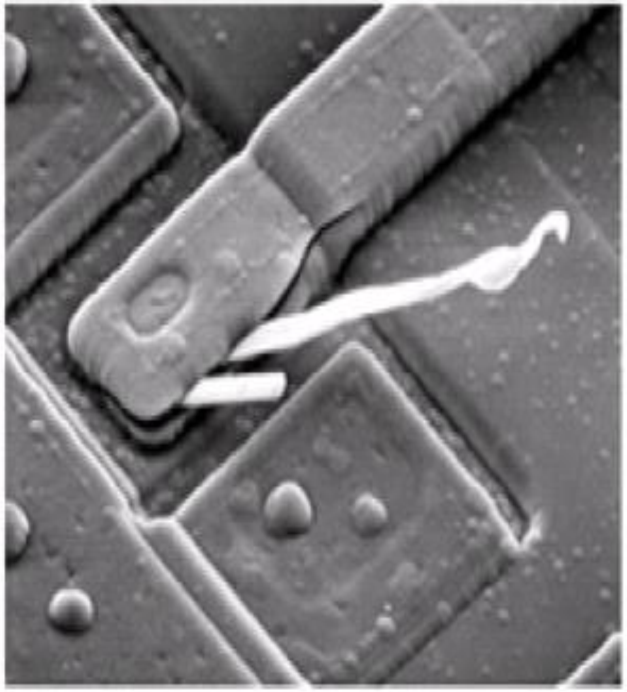
Lowpass Filter



Highpass Filter

Fundamentals of Frequency Filtering

- **Common filter functions**
  - Example



Original

Lowpass Filtered

Highpass Filtered

Fundamentals of Frequency Filtering

- **Filter Function Design Considerations**
  - **Effect on the phase:** *the filtering equation*

$$g(x,y)=\Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right]$$

can be written as

$$g(x,y)=\Im^{-1}\left[H(\mu,\upsilon)Real(F(\mu,\upsilon))+jH(\mu,\upsilon)imag(F(\mu,\upsilon))\right]$$

*Thus, if H(u,v) is real (we call it **zero-phase-shift filter**) , then the phase of F(u,v) is not changed. This is an essential requirement since as we saw earlier, the phase of the spectrum specifies the structure of the image objects*

- *So it's important to preserve the phase of the original image when filtering*

Fundamentals of Frequency Filtering

- **Filter Function Design Considerations**
  - **Effect of changing the phase - example**

Original

Image reconstructed by multiplying the phase by 0.5 without changing |F(u,v)|

Image reconstructed by multiplying the phase by 0.25 without changing |F(u,v)|

Fundamentals of Frequency Filtering

- **Filter Function Design Considerations**
  - **Padding filters with Zeros:** *the filtering equation*

$$g(x,y)=\Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right]$$

implies convolution in the spatial domain and requires H(u,v) to be of the same size as F(u,v). So, do we pad the filter in the spatial domain or in the frequency domain to avoid wraparound errors ??

  - **Answer:** *we usually zero-pad the original image such that its size is at least 2Mx2N then we specify the desired filter in the frequency domain with the same size as the padded image*
  - **Why?** *We are concerned about the filter shape in the frequency domain. If we define the filter in the frequency domain, find its IDFT, pad it with zeros, and then computing the DFT of the padded filter, the padded filter in the frequency domain is not exactly the same as the original unpadded filter*

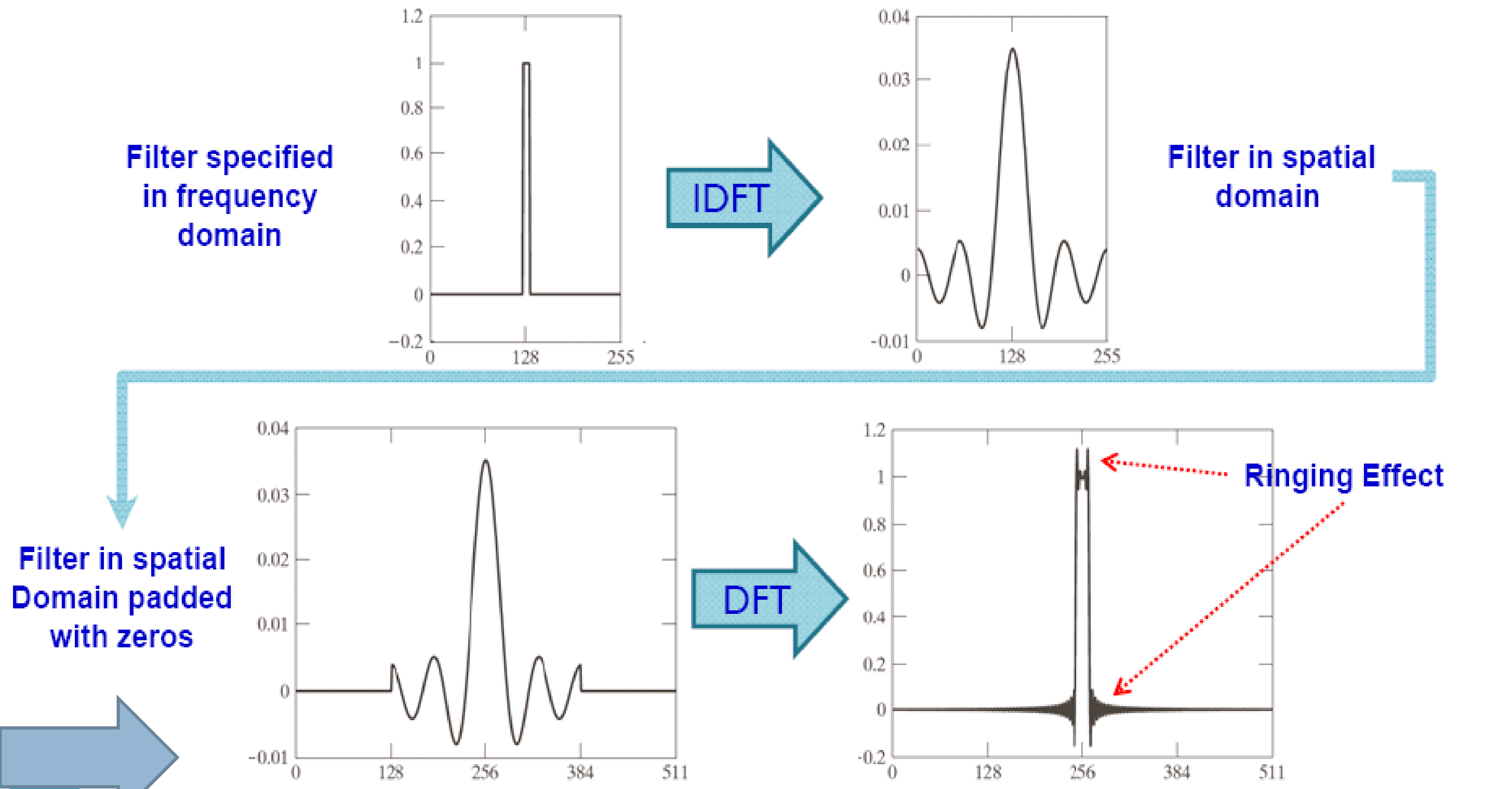
Fundamentals of Frequency Filtering

a b c

FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

Fundamentals of Frequency Filtering

- **Filter Function Design Considerations**
  - **Padding filters with Zeros – 1D Example**



Fundamentals of Frequency Filtering

- **Steps for Filtering in the Frequency Domain**

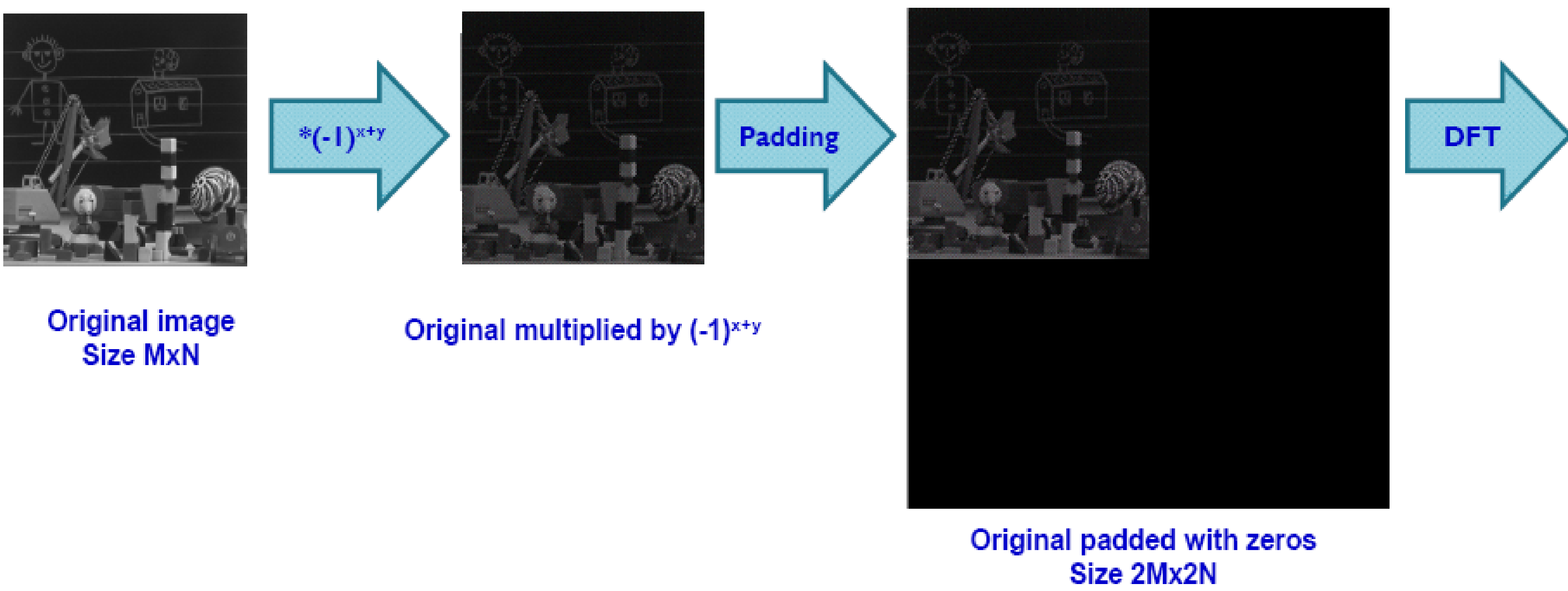
1) *Multiply $f(x,y)$ by $(-1)^{x+y}$ to center its transform*
2) *For a MxN input image $f(x,y)$, zero pad the image with M zeros in the vertical direction and N zeros in the horizontal direction to form the padded image $f_p(x,y)$*
3) *Compute the DFT, $F(u,v)$, of $f_p(x,y)$*
4) *Generate a real, symmetric filter function, $H(u,v)$, of size 2Mx2N that is centered at M and N*
5) *Perform filtering by computing the product $G(u,v) = H(u,v)F(u,v)$*
6) *Obtain the processed image by computing the IDFT of $G(u,v)$*

$$g_p(x,y) = \Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right](-1)^{x+y}$$

7) *Obtain the processed image by extracting the MxN region from the top-left quadrant of $g_p(x,y)$*

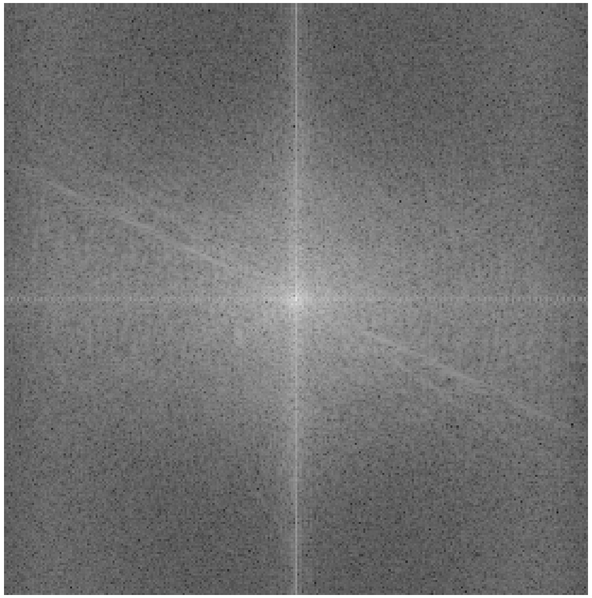
Fundamentals of Frequency Filtering

- **Steps for Filtering in the Frequency Domain - Example**

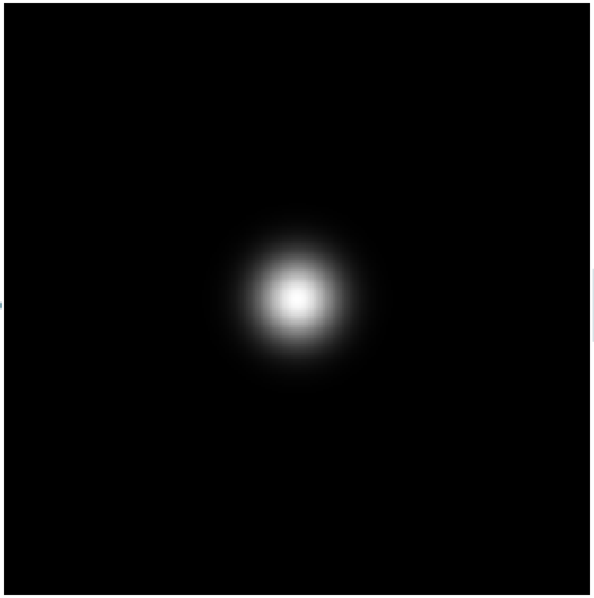


Fundamentals of Frequency Filtering

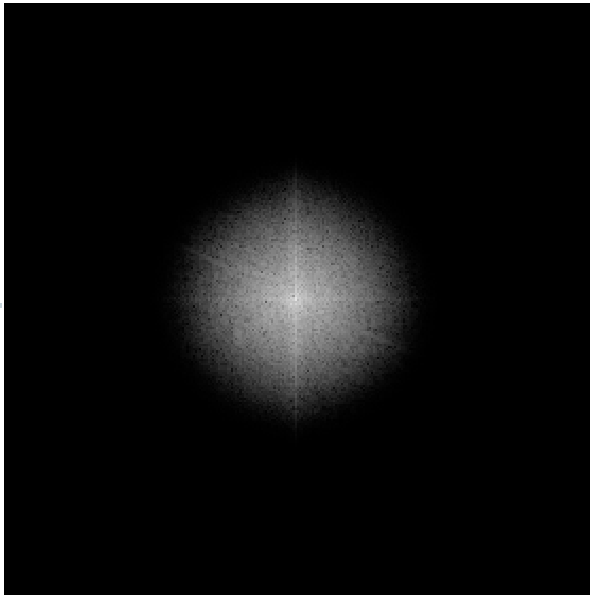
- **Steps for Filtering in the Frequency Domain - Example**



Magnitude of Spectrum
|F(u,v)|
Size 2Mx2N



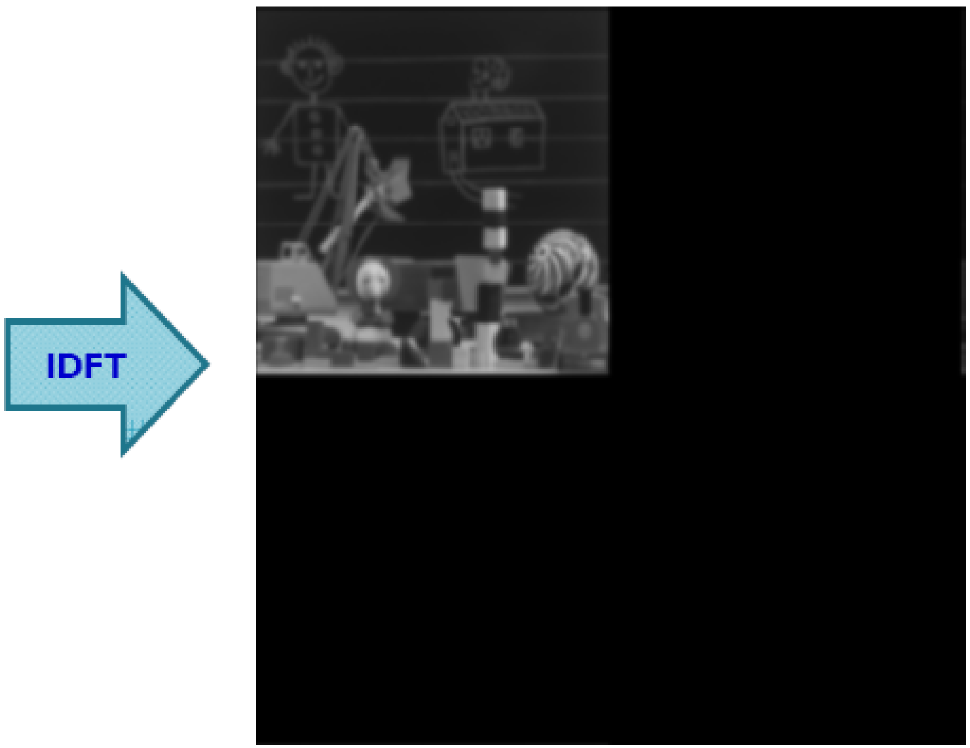
Specified Filter H(u,v)
Size 2Mx2N



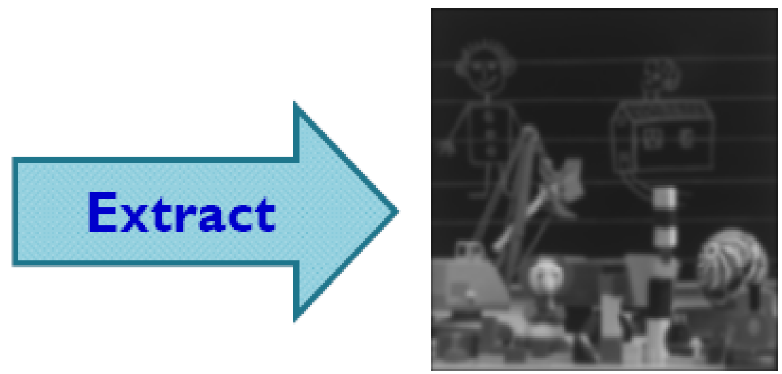
G(u,v) = |H(u,v) F(u,v)|
Size 2Mx2N

Fundamentals of Frequency Filtering

- **Steps for Filtering in the Frequency Domain - Example**



Filtered image $g_p(x,y)$
Size 2Mx2N



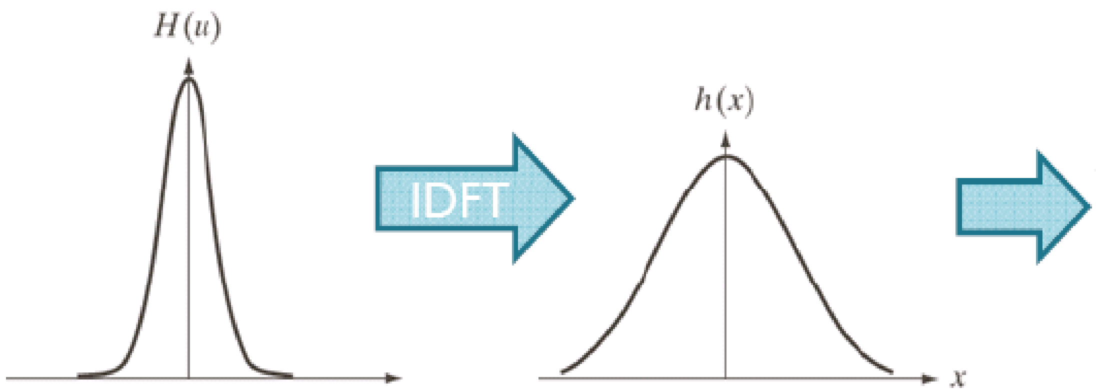
Filtered image g(x,y)
Size MxN

Fundamentals of Frequency Filtering

- **Correspondence Between Spatial and Frequency Filtering**
  - According to the convolution theorem, multiplication in frequency domain is equivalent to convolution in the spatial domain
  - As we saw earlier, the filter function is of the same size as the image.
  - However, when we discussed spatial filtering, we used smaller filter masks !! How can we explain this ?!
  - We usually use the IDFT of the filter function h(x,y) as a guidance in reconstructing small spatial filtering masks that would achieve the same task
    - Faster processing
    - The spatial mask coefficients are selected to capture the essence of full filter function

Fundamentals of Frequency Filtering

- **Correspondence Between Spatial and Frequency Filtering**
  - I-D Example

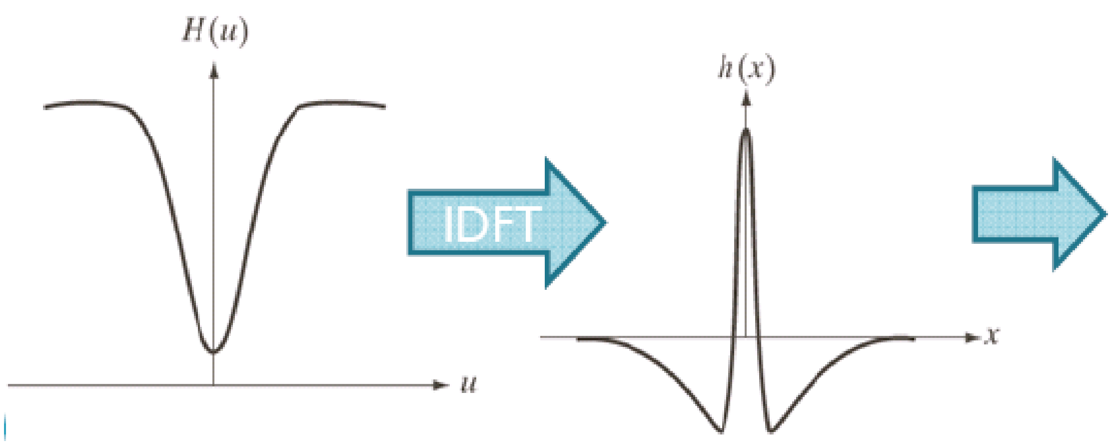


1/9*

1	1	1
1	1	1
1	1	1

1/16*

1	2	1
2	4	2
1	2	1



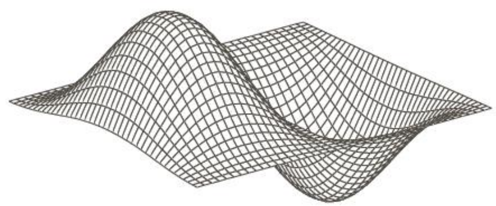
-1	-1	-1
-1	8	-1
-1	-1	-1

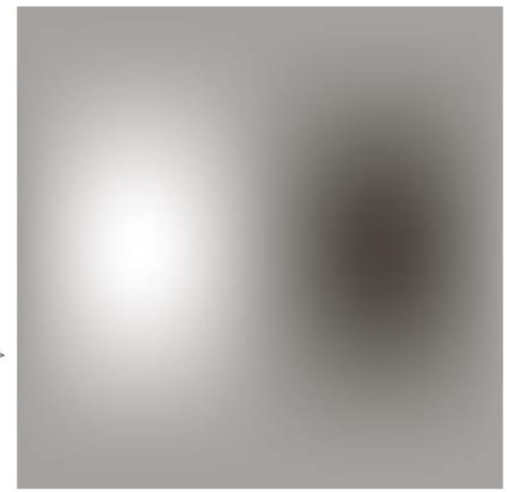
0	-1	0
-1	4	-1
0	-1	0

Fundamentals of Frequency Filtering

Image

−1	0	1
−2	0	2
−1	0	1





a b
c d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Fundamentals of Frequency Filtering

- **Correspondence Between Spatial and Frequency Filtering**

Original

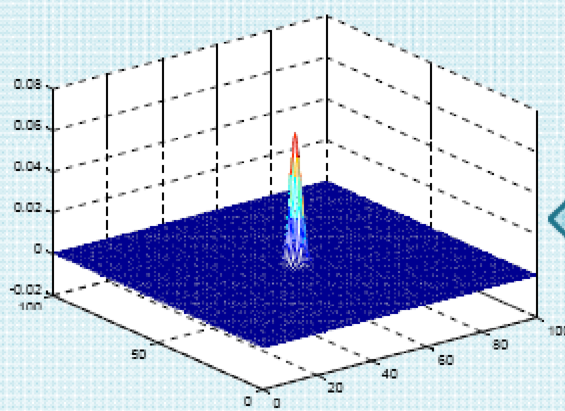
Filtering in the spatial domain

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

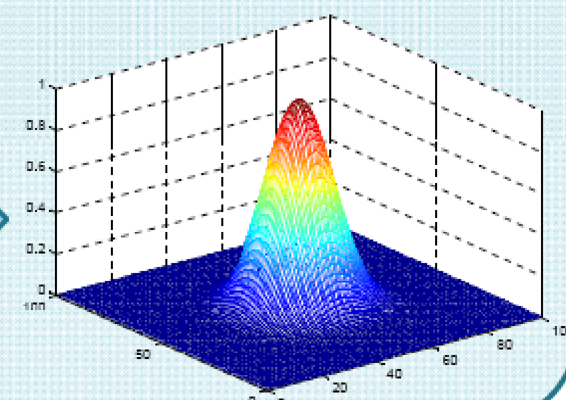
* 1/25

Filtering in the frequency domain using

H(u,v)



h(x,y)



Smoothing Using Frequency Filtering

- Smoothing or blurring is achieved in the frequency domain by using lowpass filters
- As the name indicates, lowpass filters preserve the low frequency components of the image while attenuating the high frequency components
- Common lowpass filters
  - Ideal
  - Butterworth
  - Gaussian
- In the following, we compare the performance of these types
- Keep in mind that we are talking about discrete filters that are centered in the middle of the spectrum

Smoothing Using Frequency Filtering

- **Ideal Lowpass Filter (ILPF)**
  - ILPF passes all frequencies within a circle of radius D_0 from the center of the spectrum (P/2,Q/2) and cuts all frequencies outside this circle
  - The ILPF is defined as

$$H(\mu,\upsilon)=\begin{cases} 1\,, & D(\mu,\upsilon)\ \leq\ D_0 \\ 0\,, & D(\mu,\upsilon)\ \leq\ D_0 \end{cases}$$

where

$$D(\mu,\upsilon)=\left[(\mu-P/2)^2+(\upsilon-Q/2)^2\right]^{1/2}$$

is the distance from each pixel to the center of the spectrum (P/2,Q/2)

  - The radius D_0 is called the <u>cutoff frequency</u>

Smoothing Using Frequency Filtering

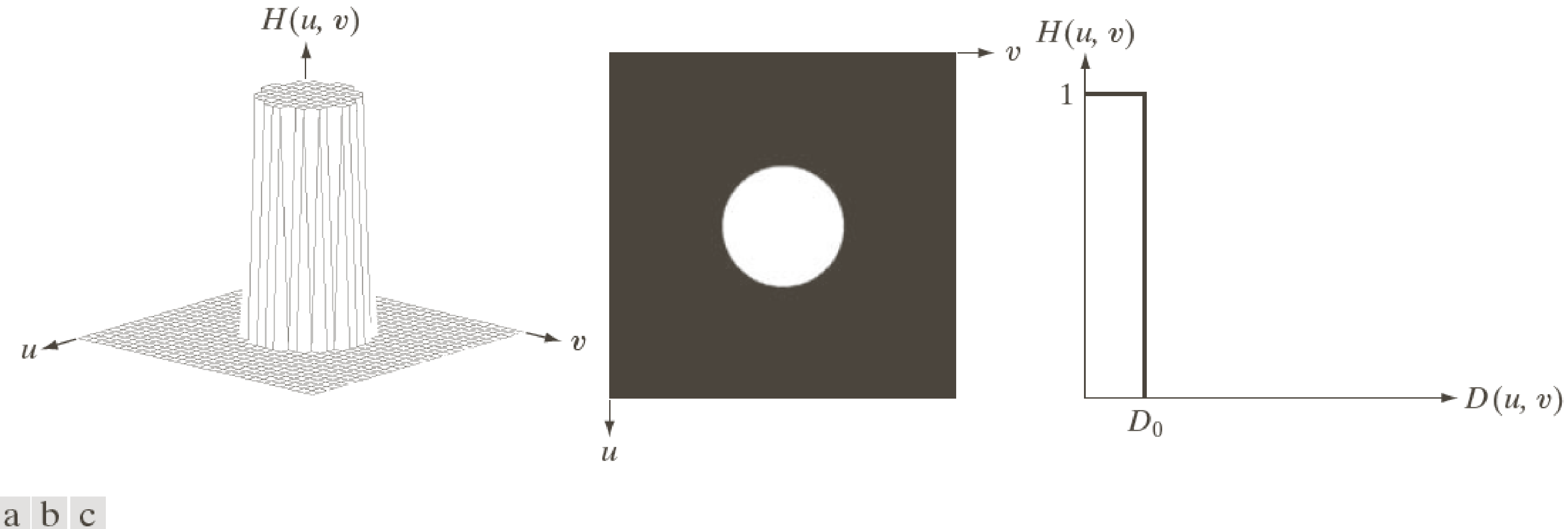


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Smoothing Using Frequency Filtering

- **Using Power Spectrum in Defining Filters**
  - One way to define the cutoff frequency is to compute circles that enclose specified amounts of total image power P_T which is defined as

$$PT = \sum_{u=0}^{P-1}\sum_{v=0}^{Q-1} P(u,v) = \sum_{u=0}^{P-1}\sum_{v=0}^{Q-1} \left|F(u,v)\right|^2$$

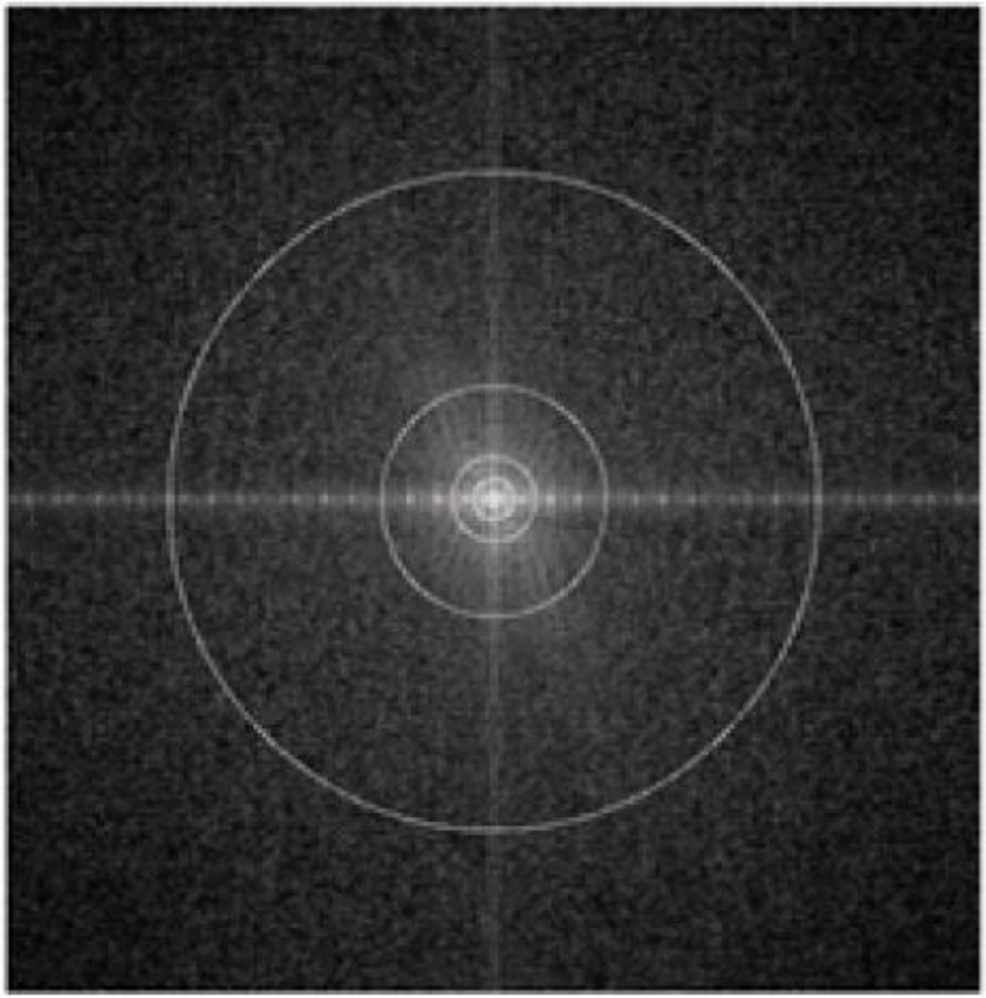
If the DFT of the filter is centered, then a circle with radius D_0 with origin at the center of frequency rectangle encloses α-percent of the power

$$\alpha = 100 \sum_{u}\sum_{v} P(u,v) / P_T$$

and summation is for all values of u and v that fall inside the circle

Smoothing Using Frequency Filtering

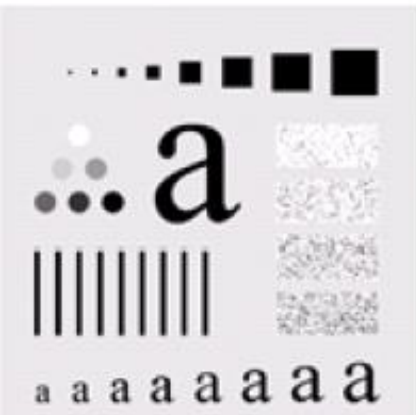
- **Using Power Spectrum in Defining Filters**

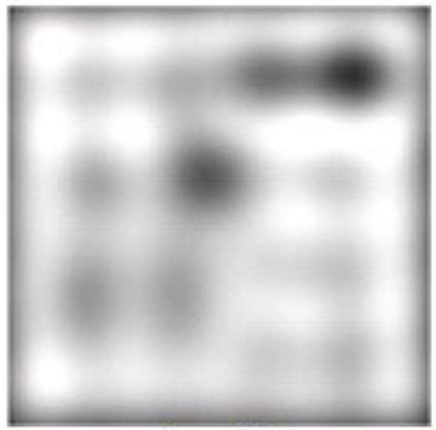


D_0	α
10	87.0
30	93.1
60	95.7
160	97.8
460	99.2

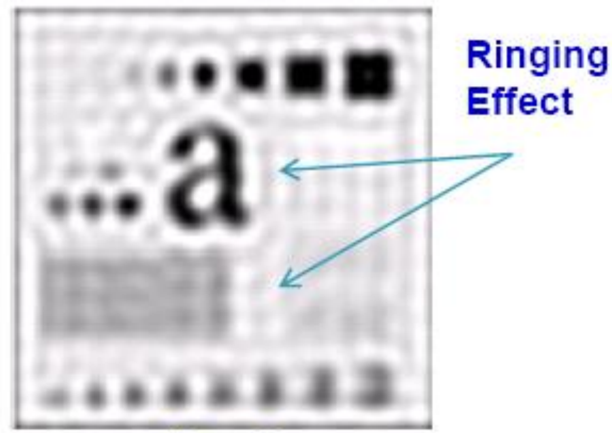
Smoothing Using Frequency Filtering

- **Ideal Lowpass Filter - example**
  - Let's smooth the image in the previous slide with ILPF with cutoff frequencies 10,30,60,160, and 460



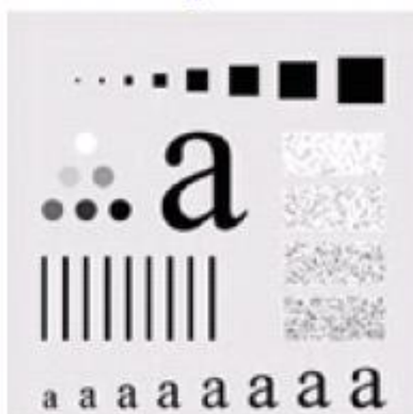


D_0 =10

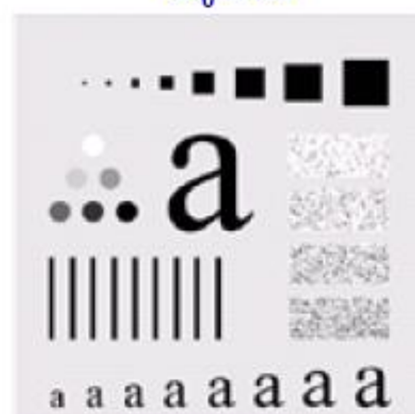


D_0 =30

D_0 =60



D_0 =160



D_0 =460

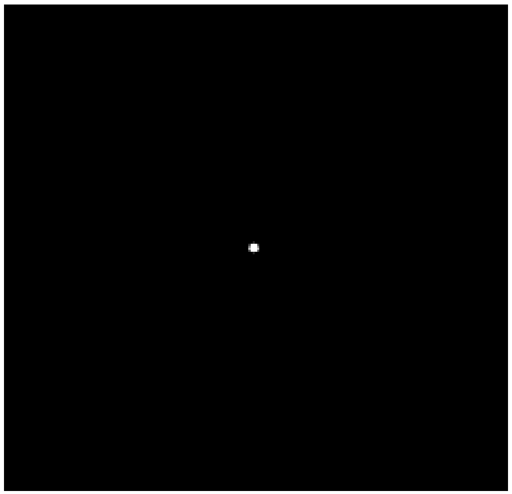
Smoothing Using Frequency Filtering

- **Ideal Lowpass Filter**
  - Why Ringing Effects ?
    - We saw earlier that the cross section of ILPF in the frequency domain is a pulse. It is expected that the IDFT of ILPF h(x,y) is a sinc function.
    - According to the convolution theorem, the multiplication performed in the frequency domain implies convolving the h(x,y) with f(x,y).
    - If we think of f(x,y) as set of impulses, each with a weight that represents pixel intensity, then convolution simply replaces a replica of h(x,y) at each impulse.
    - The main lobs of the sinc function are responsible for blurring while the side lobs are responsible for ringing
    - As the radius of the ILPF increase, its IDFT (the sinc function) approaches an impulse. In this case blurring and ringing is reduced.

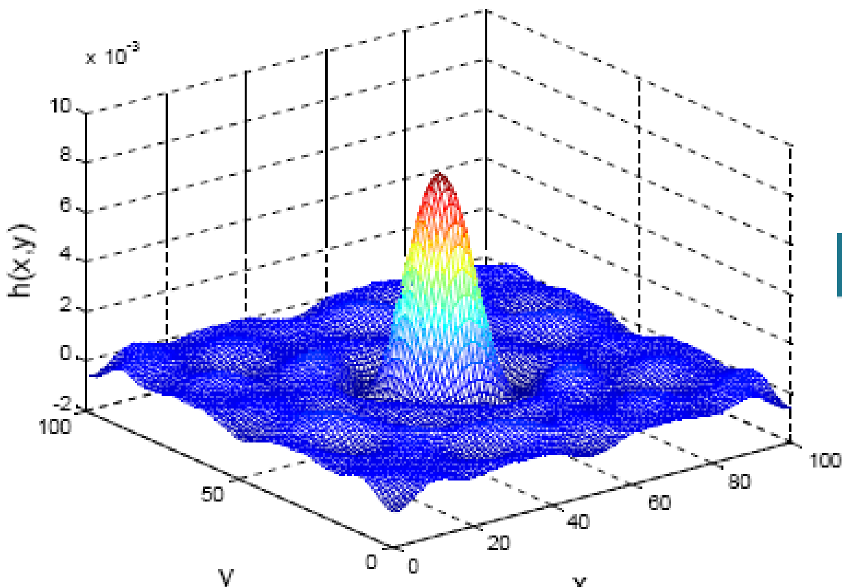
Smoothing Using Frequency Filtering

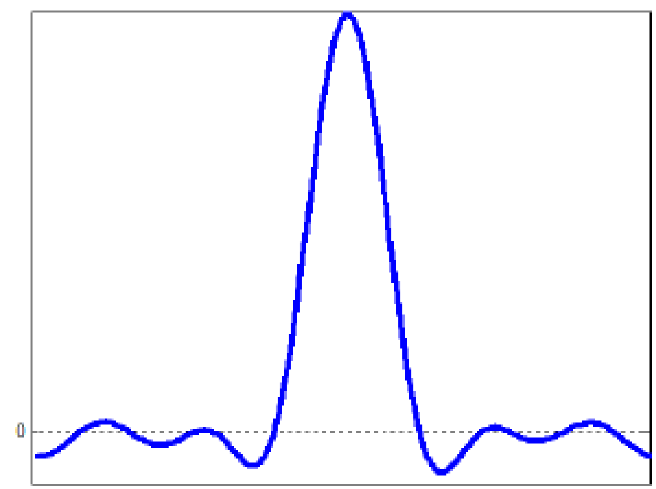
Ideal Lowpass Filter - Why Ringing Effects ?

ILPF
H(u,v)



ILPF
h(x,y)





ILPF Cross section

Smoothing Using Frequency Filtering

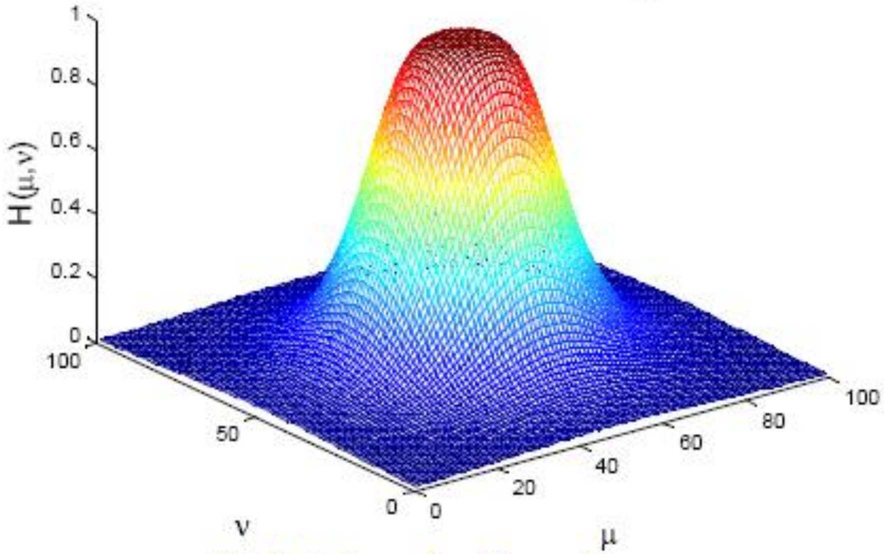
- **Butterworth Lowpass Filter (BLPF)**
  - The BLPF is defined as

$$H(\mu,\upsilon)=\frac{1}{1+\left[D(\mu,\upsilon)/D_0\right]^{2n}}$$

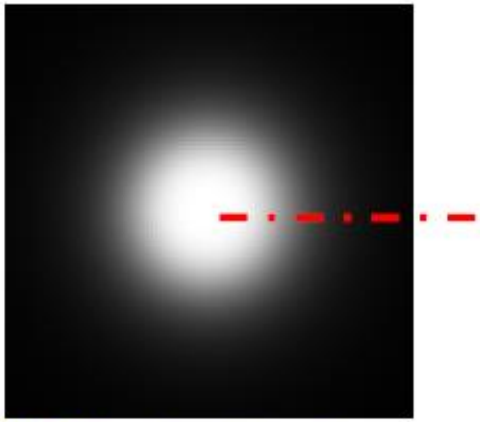
  - n is called the order of the filter. As n increases, the steepness of BLPF increases and approaches that of ILPF
  - Unlike ILPF, the BLPF has no sharp discontinuity that gives a clear cutoff between filtered and passed frequencies
  - The cutoff frequency is usually defined as the locus of points for which H(u,v) is down to a certain fraction of its maximum value; usually 50% (-3dB).

Smoothing Using Frequency Filtering

- **Butterworth Lowpass Filter (BLPF)**

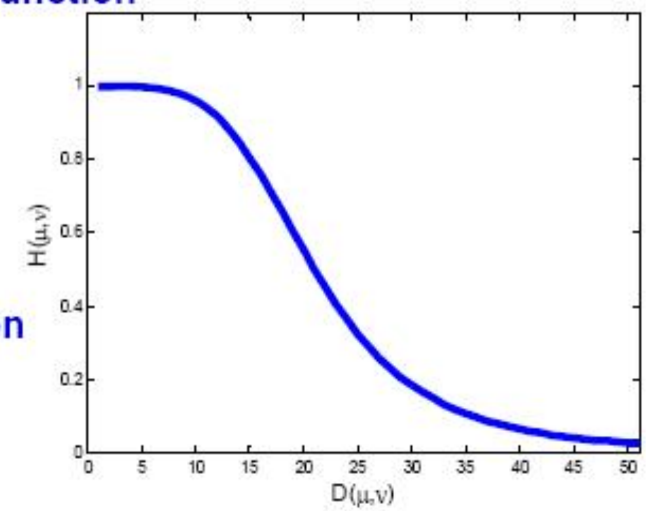


BLPF Transfer Function



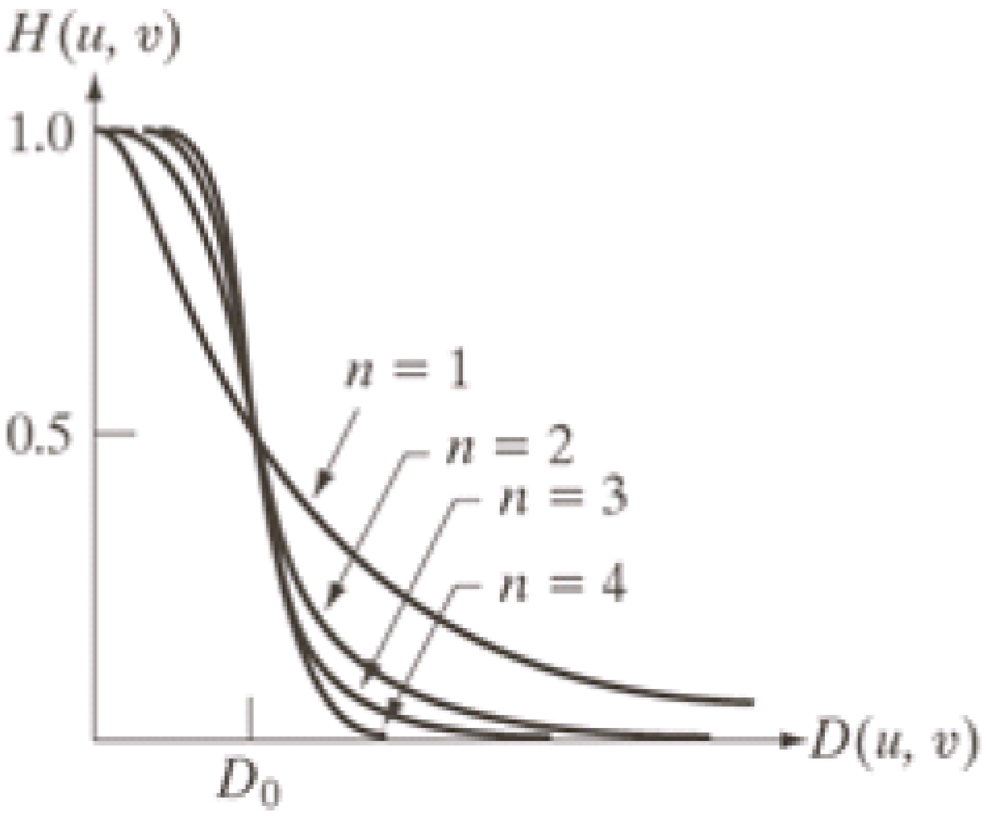
BLPF Displayed as image

Filter Radial Cross Section



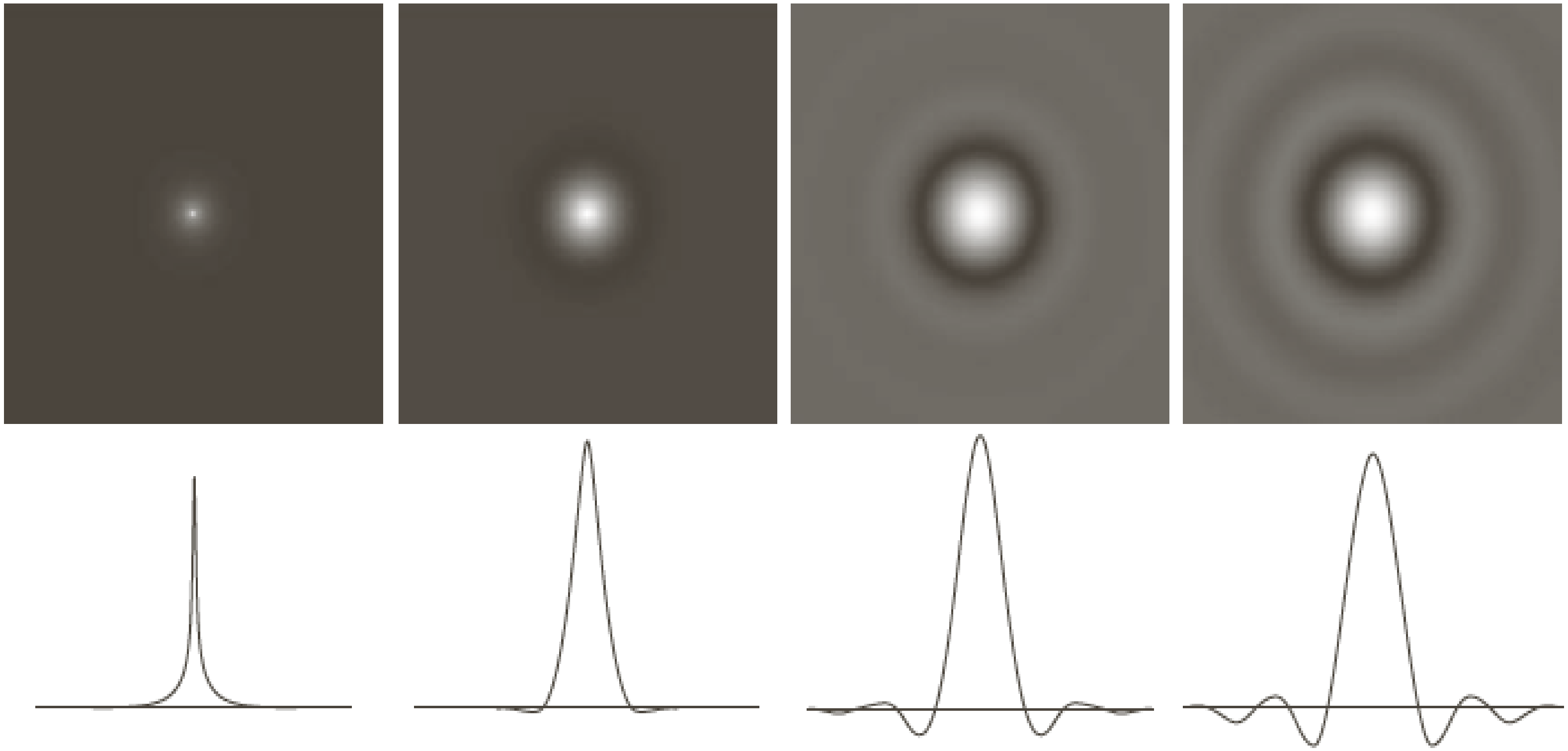
Smoothing Using Frequency Filtering

- **Butterworth Lowpass Filter (BLPF)**
  - **Effect of Changing BLPF order – Frequency Domain**



As n increases, the BLPF approaches ILPF

Smoothing Using Frequency Filtering



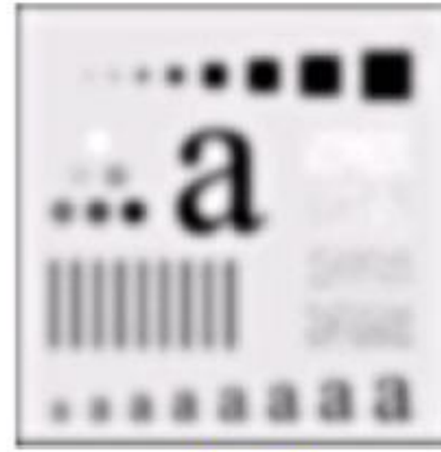
a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

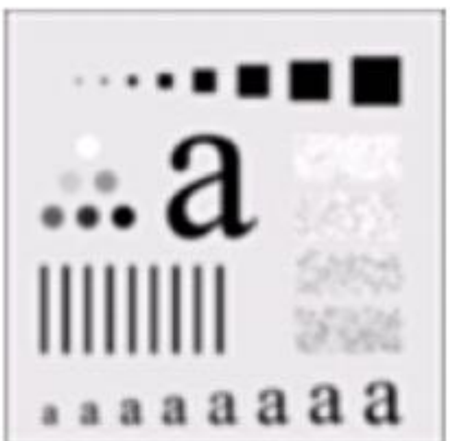
Smoothing Using Frequency Filtering

- **Butterworth Lowpass Filter (BLPF) - Example**

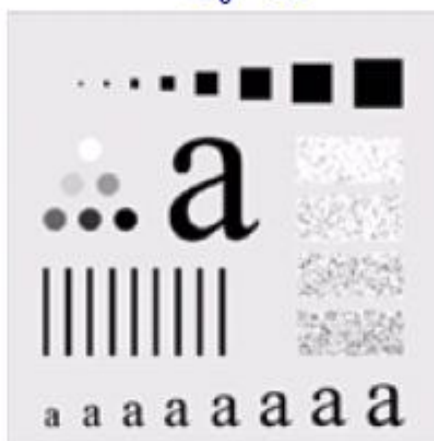
D_0 =10



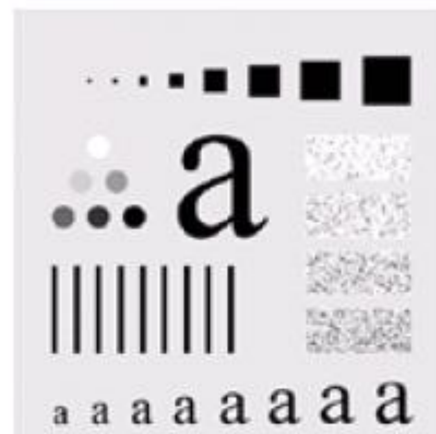
D_0 =30



D_0 =60



D_0 =160



D_0 =460

BLPF order is 2

Smoothing Using Frequency Filtering

- **Gaussian Lowpass Filter (GLPF)**
  - The GLPF is defined as

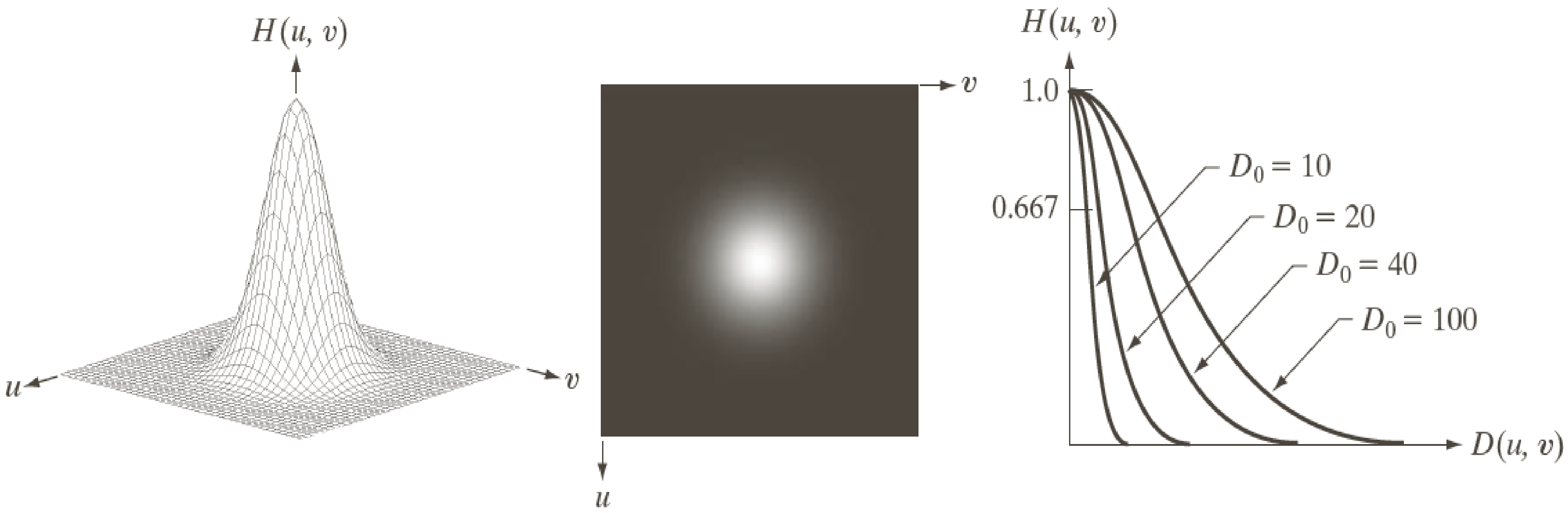
$$H(\mu,\upsilon) = e^{-\frac{D^2(\mu,\upsilon)}{2D_0^2}}$$

  - Unlike ILPF, the GLPF has no sharp discontinuity that gives a clear cutoff between filtered and passed frequencies

  - The cutoff frequency is usually defined as the locus of points for which H(u,v) is down to a certain fraction of its maximum value; usually 50% (-3dB).

  - **Note: GPLF has no ringing at all since its IDFT is also a gaussian**

Smoothing Using Frequency Filtering



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

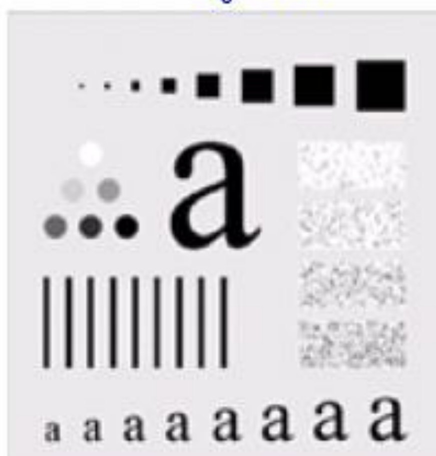
Smoothing Using Frequency Filtering

- **Gaussian Lowpass Filter (GLPF) - Example**

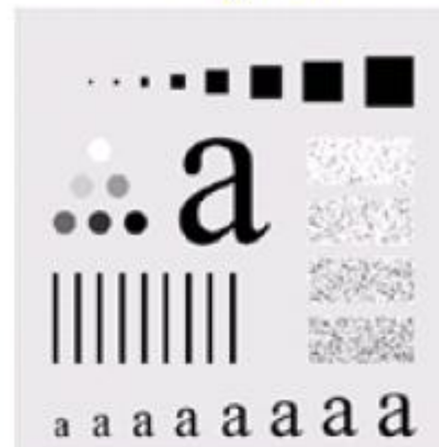
$D_0 = 10$

$D_0 = 30$

$D_0 = 60$

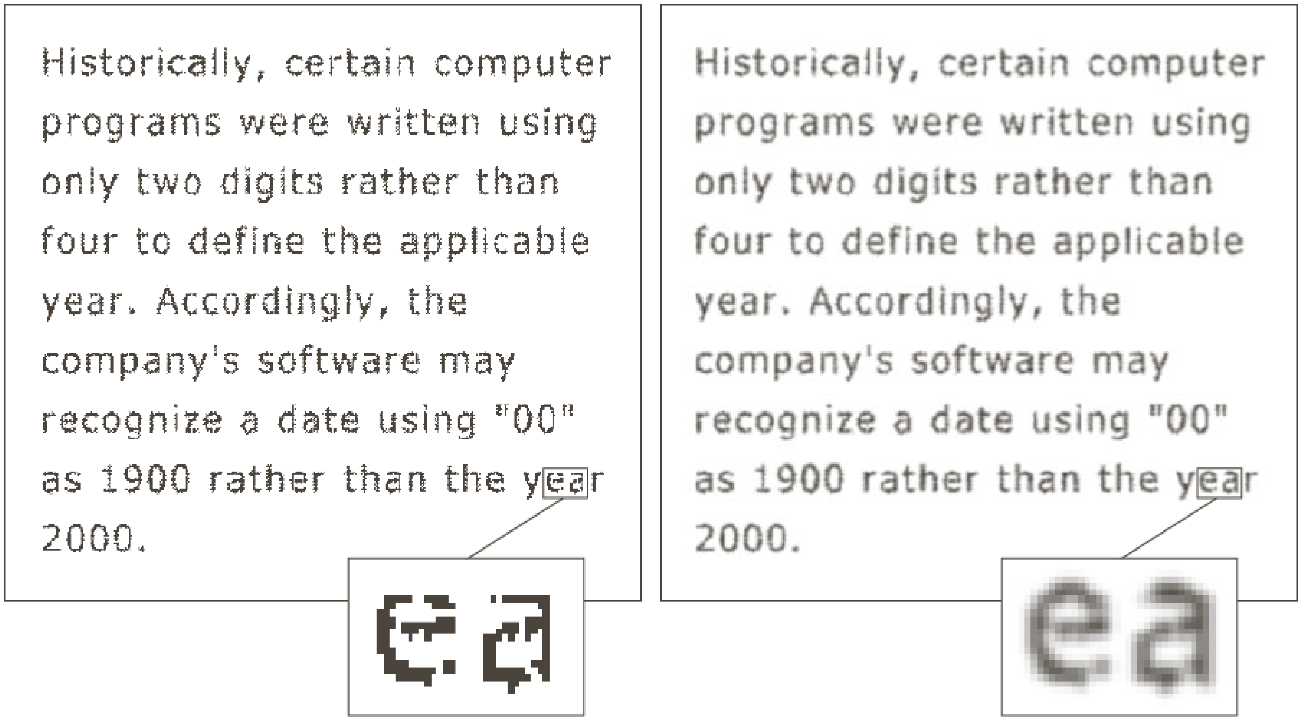


$D_0 = 160$



$D_0 = 460$

Smoothing Using Frequency Filtering



a b

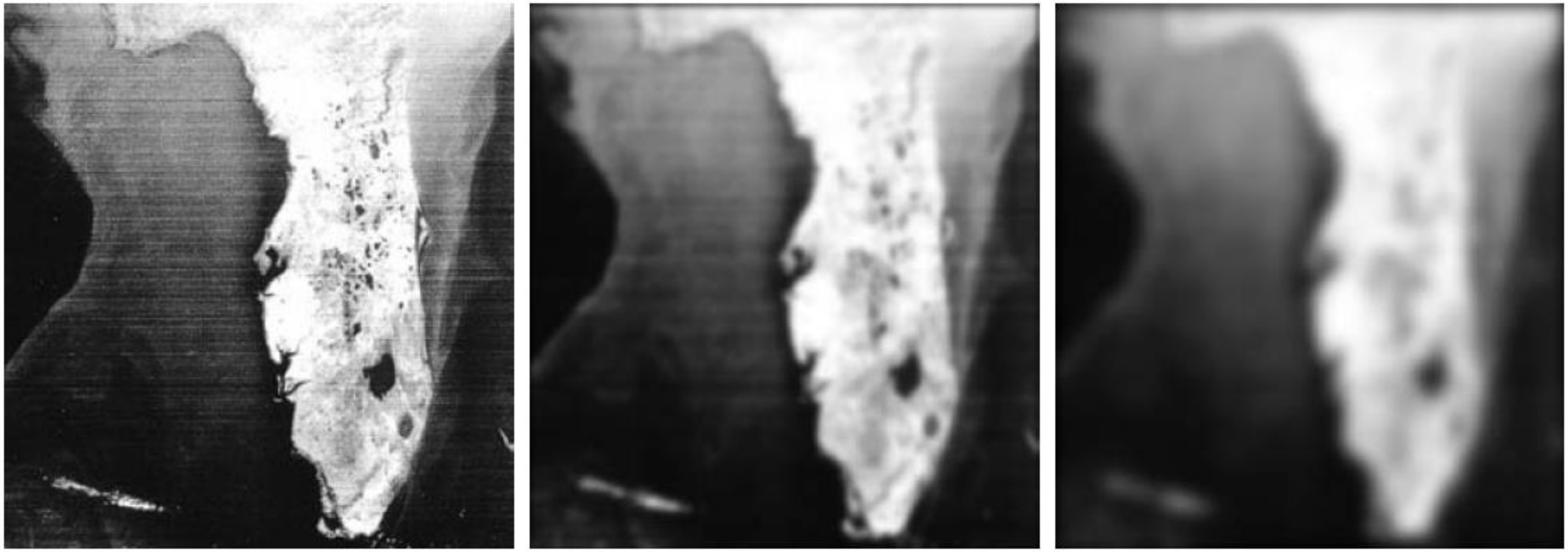
FIGURE 4.49
(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Smoothing Using Frequency Filtering

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Smoothing Using Frequency Filtering



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Sharpening Using Frequency Filtering

- Enhancing sharp details and edges is performed by using highpass filters
- Highpass filters preserves the high frequency components of the image (which correspond to edges, abrupt changes , or noise) while attenuating the low frequency components
- A highpass filter is obtained from a low pass filter using

$$H_{HP}(\mu,v) = 1 - H_{LP}(\mu,v)$$

- Common High filters
  - Ideal
  - Butterworth
  - Gaussian

Sharpening Using Frequency Filtering

- **Ideal Highpass Filter (IHPF)**
  - ILPF passes all frequencies within a circle of radius D_0 from the center of the spectrum (P/2,Q/2) and cuts all frequencies outside this circle
  - The ILPF is defined as

$$H(\mu,\upsilon)=\begin{cases} 0\ , & D(\mu,\upsilon)\ \leq\ D_0 \\ 1\ , & D(\mu,\upsilon)\ \leq\ D_0 \end{cases}$$

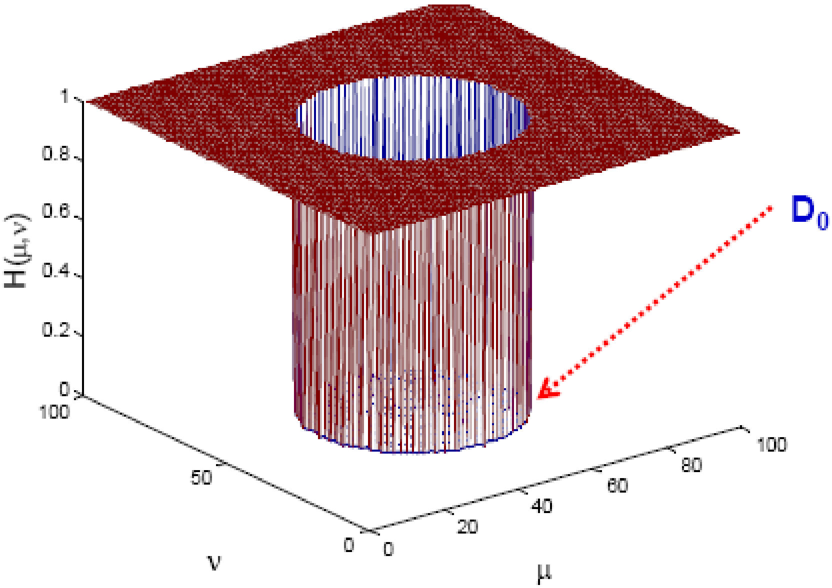
where

$$D(\mu,\upsilon)=\left[(\mu-P/2)^2+(\upsilon-Q/2)^2\right]^{1/2}$$

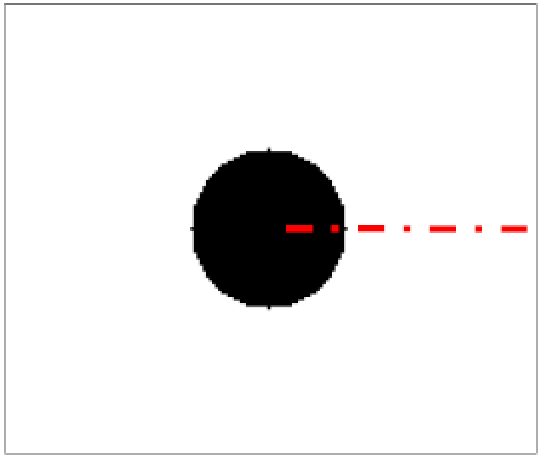
is the distance from each pixel to the center of the spectrum (P/2,Q/2)

  - The radius D_0 is called the <u>cutoff frequency</u>

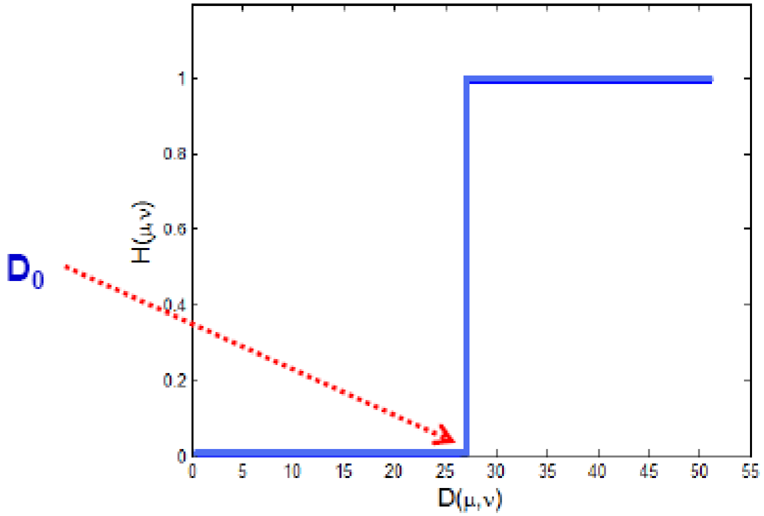
Sharpening Using Frequency Filtering



IHPF Transfer Function



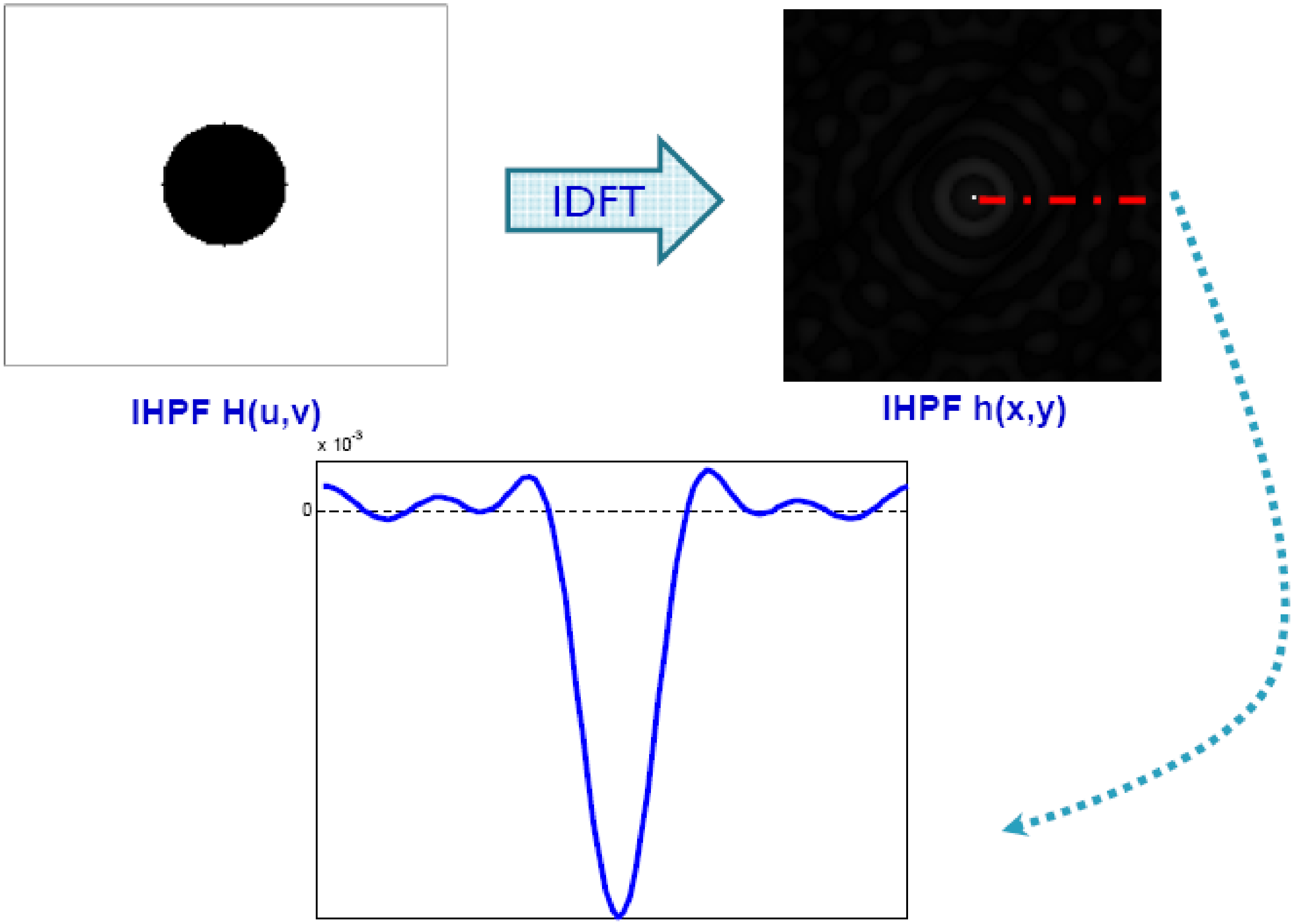
IHPF Displayed as image



Filter Radial Cross Section

Sharpening Using Frequency Filtering

- **Ideal Highpass Filter**
  - Ringing effects is unavoidable in IHPF as it is derived from the ILPF

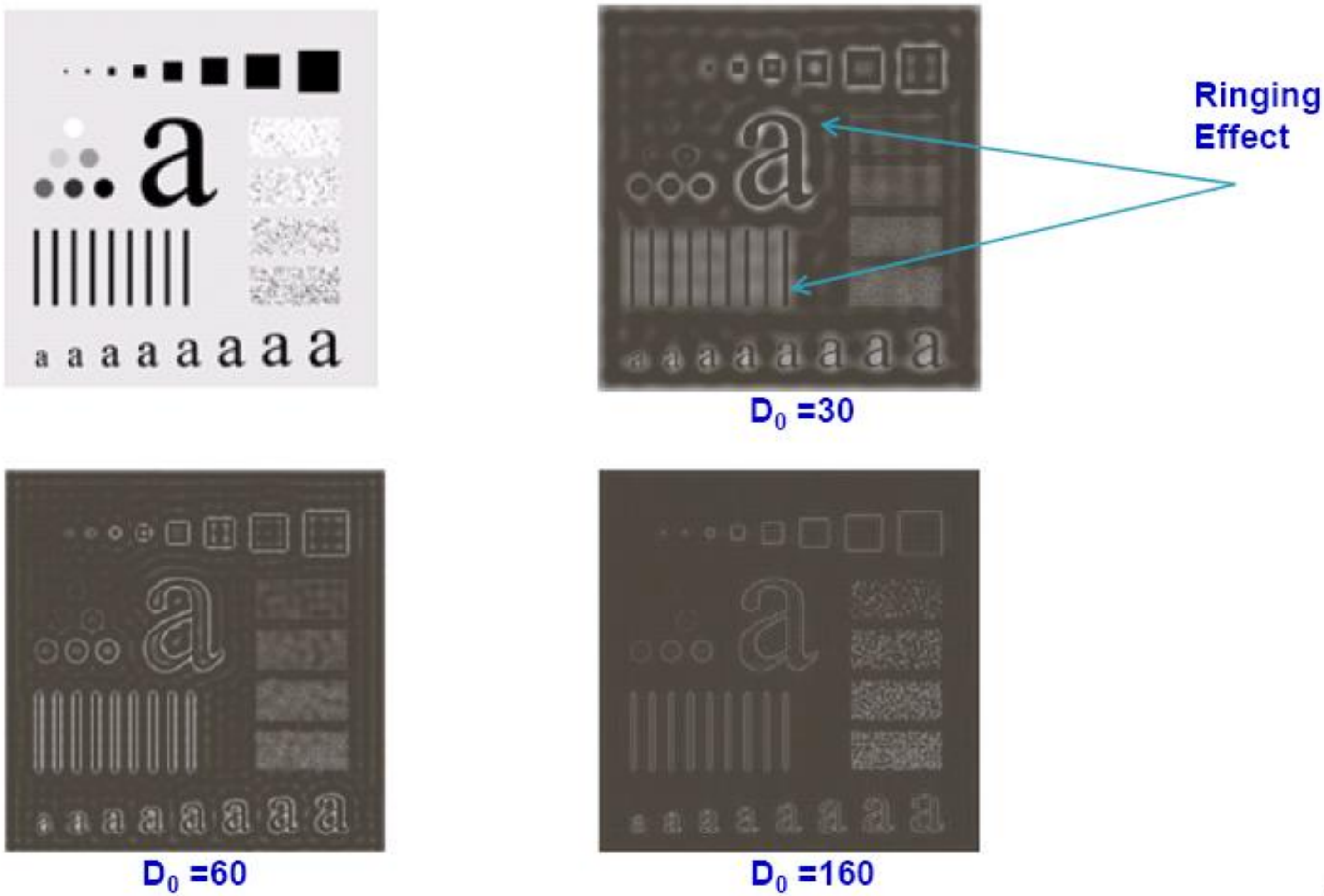


IHPF H(u,v)

IHPF h(x,y)

Sharpening Using Frequency Filtering

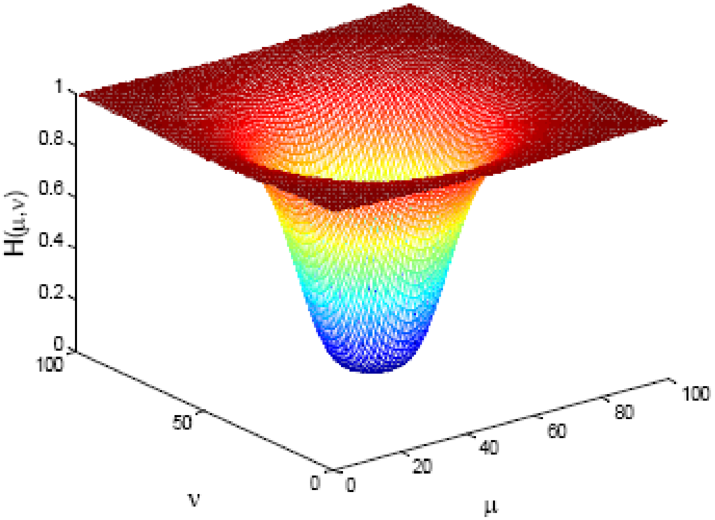
- **Ideal Highpass Filter - example**
  - Result of highpass filtering using D0 = 30, 60, and 160



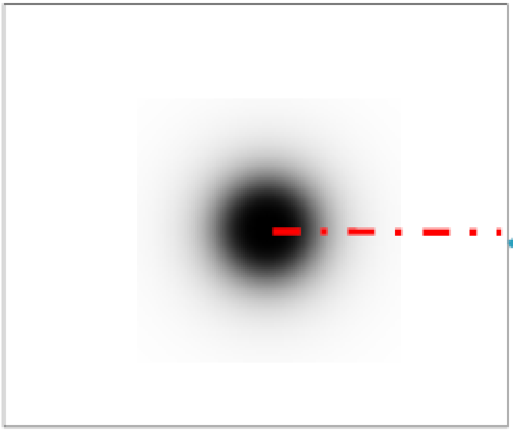
Sharpening Using Frequency Filtering

- **Butterworth Highpass Filter (BHPF)**
  - The BHPF is defined as

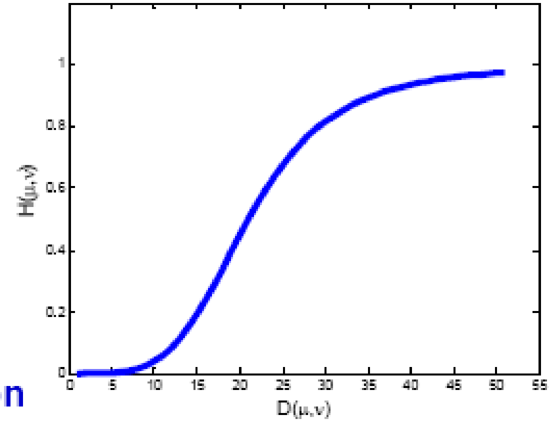
$$H(\mu,\upsilon)=\frac{1}{1+\left[D0\,/\,D(\mu,\upsilon)\right]^{2n}}$$



BHPF Transfer Function



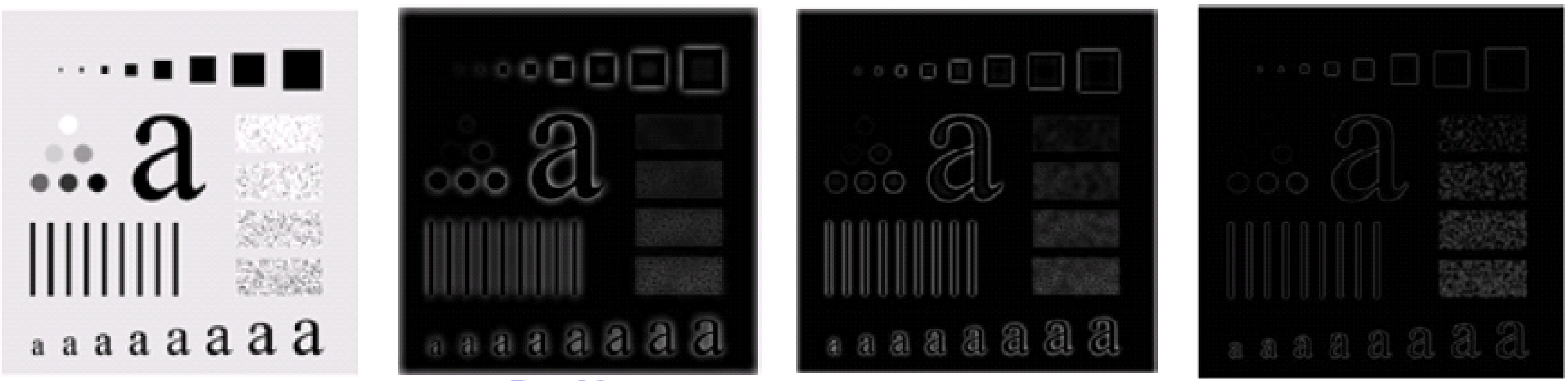
BHPF Displayed as image



Filter Radial Cross Section

Sharpening Using Frequency Filtering

- **Butterworth Highpass Filter - example**
  - Result of highpass filtering using D0 = 30, 60, and 160 and n = 2



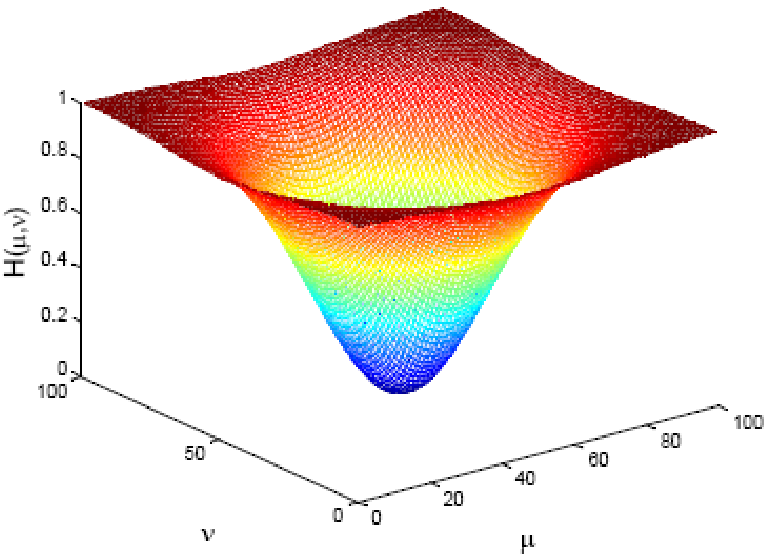
D_0 =30 D_0 =60 D_0 =160

  - Note that ringing effects decrease as we increase D_0
  - Similar to BLPF, the ringing in BHPF increases as we increase n

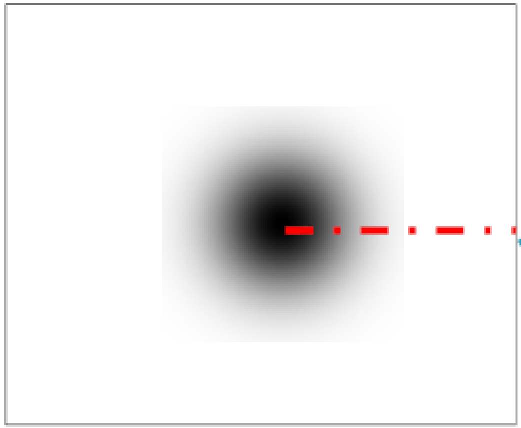
Sharpening Using Frequency Filtering

- **Gaussian Highpass Filter (GHPF)**
  - The GHPF is defined as

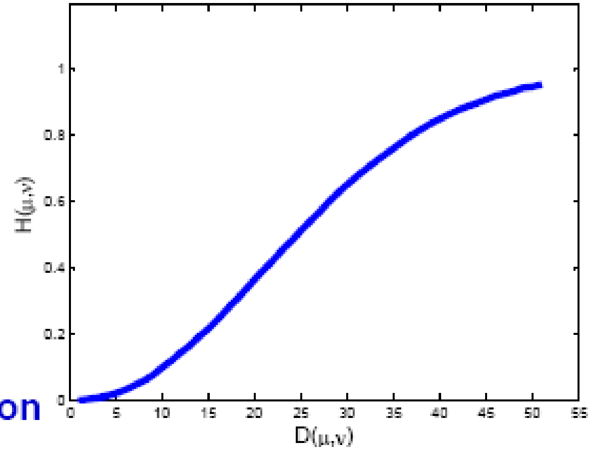
$$H(\mu,\upsilon)=1-e^{-\frac{D^2(\mu,\upsilon)}{2D_0^2}}$$



GHPF Transfer Function



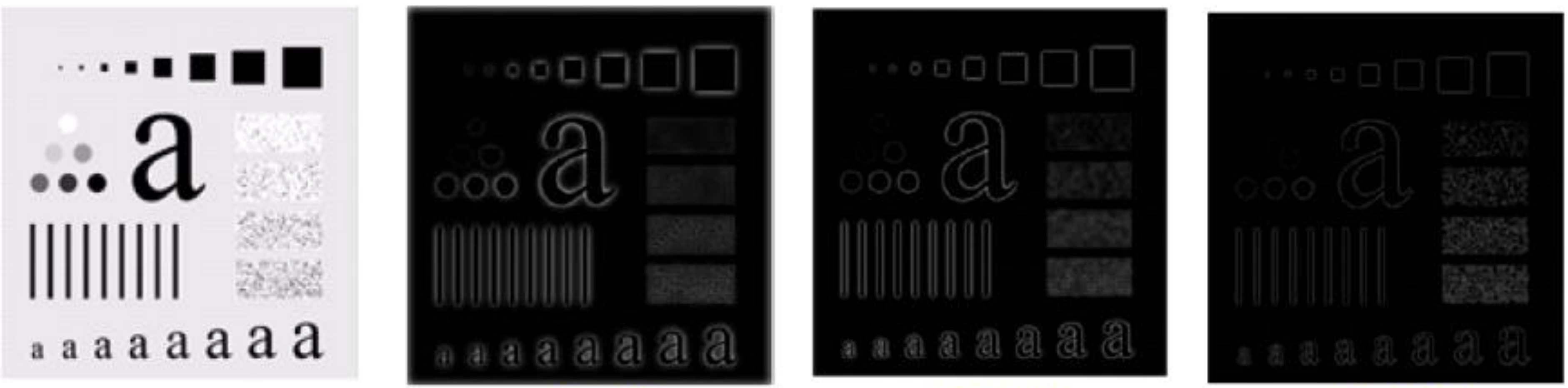
GHPF Displayed as image



Filter Radial Cross Section

Sharpening Using Frequency Filtering

- **Gaussian Highpass Filter - example**
  - Result of highpass filtering using D0 = 30, 60, and 160



D_0 =30 D_0 =60 D_0 =160

  - Note that GHPF has no ringing effects

Sharpening Using Frequency Filtering

- **Applications – Example**

Original — Filtered using BHPF n =4 — Threshold Image

- Note how the highpass filtered image has lost gray tones because the DC component was removed
- Thresholding was applied to point out ridges in the fingerprint

Sharpening Using Frequency Filtering

- **The Laplacian in Frequency Domain**
  - It can be shown that the Laplacian can be implemented in frequency domain as a filter H(u,v) as

$$H(\mu,\upsilon) = -4\pi^2(\mu^2 + \upsilon^2)$$

  - And with respect to the center of the frequency rectangle

$$H(\mu,\upsilon) = -4\pi^2\left((\mu - P/2)^2 + (\upsilon - Q/2)^2\right) = -4\pi^2 D(\mu,\upsilon)$$

  - Thus, the Laplacian of an image f(x,y) is

$$\nabla^2 f(x,y) = \Im^{-1}\left[H(\mu,\upsilon)F(\mu,\upsilon)\right]$$

Sharpening Using Frequency Filtering

- **The Laplacian in Frequency Domain**
  - Enhancement using Laplacian is achieved by

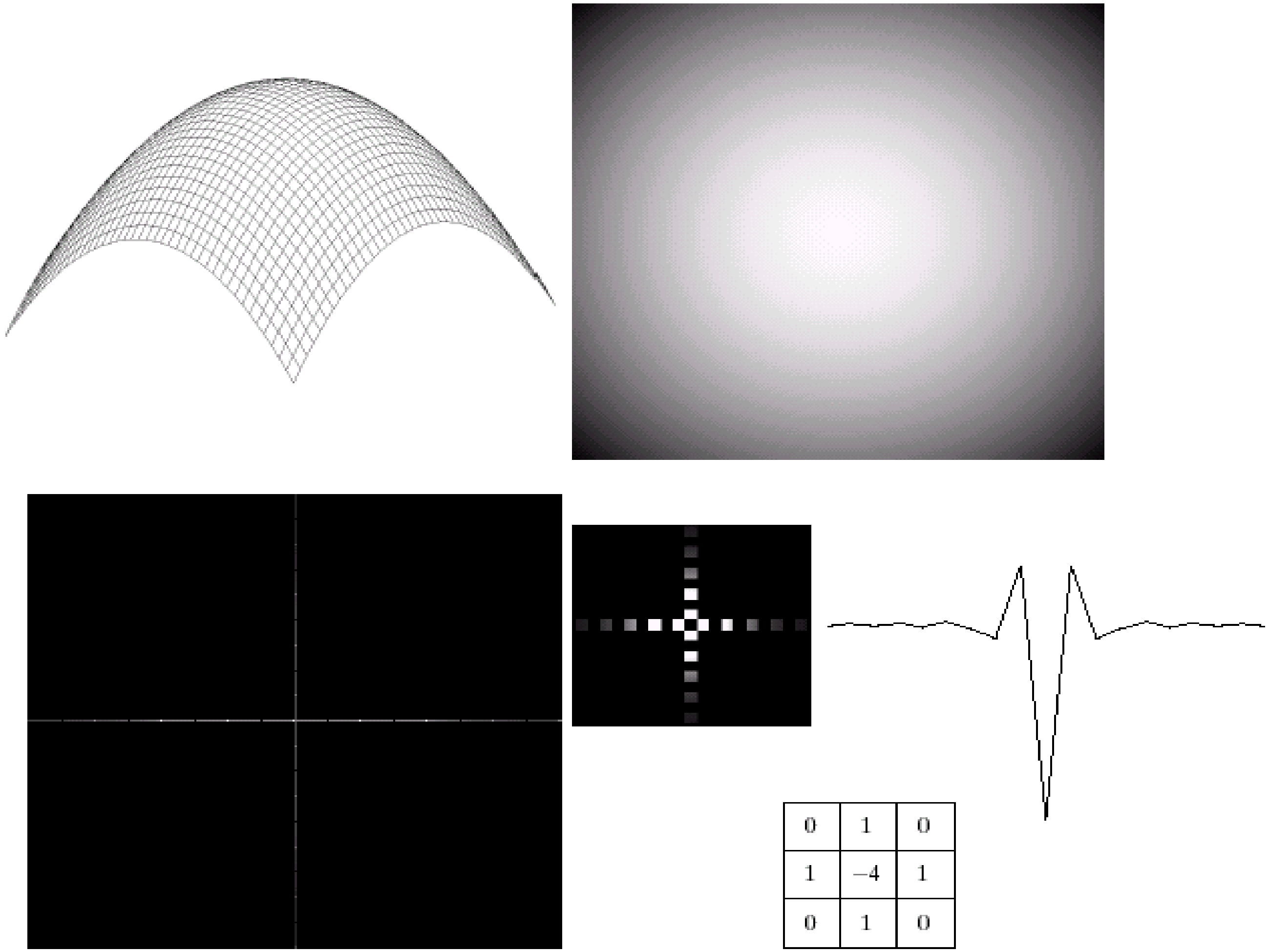
$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

  - Or, in frequency domain

$$\begin{aligned} g(x,y) &= \Im^{-1}\left[F(\mu,\upsilon) - H(\mu,\upsilon)F(\mu,\upsilon)\right] \\ &= \Im^{-1}\left[(1 - H(\mu,\upsilon))F(\mu,\upsilon)\right] \\ &= \Im^{-1}\left[(1 + 4\pi^2 D(\mu,\upsilon))F(\mu,\upsilon)\right] \end{aligned}$$

  - Although this formulation is elegant, it is hard to find the scaling factors in the frequency domain. So, we usually find the IDFT of the Laplacian then we carry out enhancement in spatial domain by normalizing the Laplacian to ~[-1,1]

Sharpening Using Frequency Filtering



0	1	0
1	–4	1
0	1	0

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

Sharpening Using Frequency Filtering

- **The Laplacian in Frequency Domain – Example**

Original

Enhanced Image Using Laplacian

Selective Filtering

- The lowpass and highpass filters discussed so far operate over the entire range of the frequency rectangle

- In several applications, the interest is usually about specific frequency bands or smaller regions of the frequency rectangle

- Filters in the first category are called Bandreject and Bandpass filters

- Filters in the second category are called notch filters

Selective Filtering

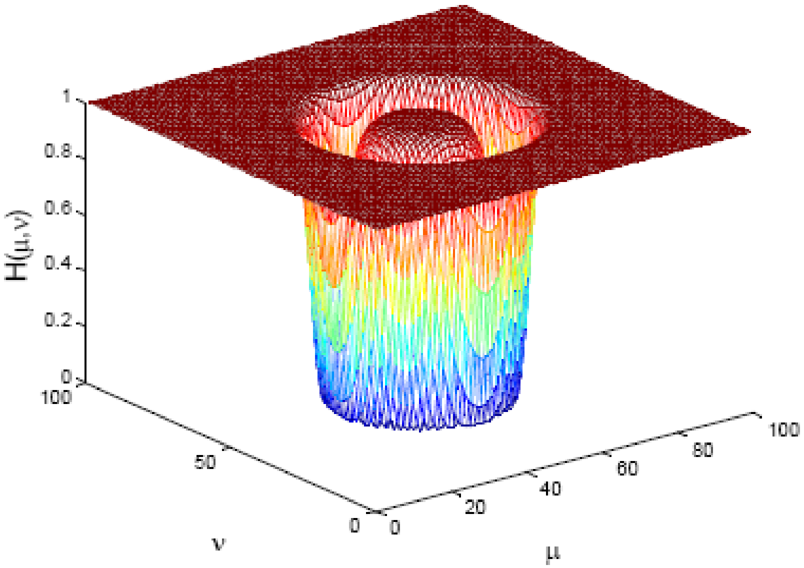
- **Bandreject Filters**
  - They are used to reject a certain band of frequencies
  - They can be easily reconstructed using the filter types we discussed earlier

Ideal	$H(\mu,\upsilon)=\begin{cases} 0\ ,\ D_0-\dfrac{W}{2}\leq D(\mu,\upsilon)\leq D_0+\dfrac{W}{2} \\ 1\ ,\ otherwise \end{cases}$
Butterworth	$H(\mu,\upsilon)=\dfrac{1}{1+\left[\dfrac{DW}{D^2(\mu,\upsilon)-D_0^2}\right]^{2n}}$
Gaussian	$H(\mu,\upsilon)=1-e^{-\left[\frac{D^2(\mu,\upsilon)-D_0^2}{DW}\right]}$

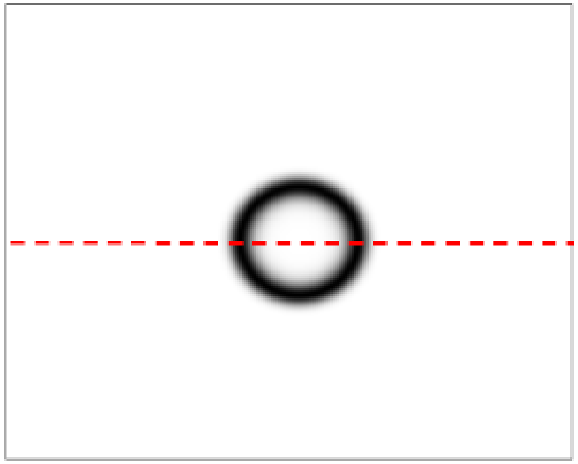
W is the width of the reject band and $\mathbf{D_0}$ is the radial center of the band

Selective Filtering

- **Bandreject Filters**

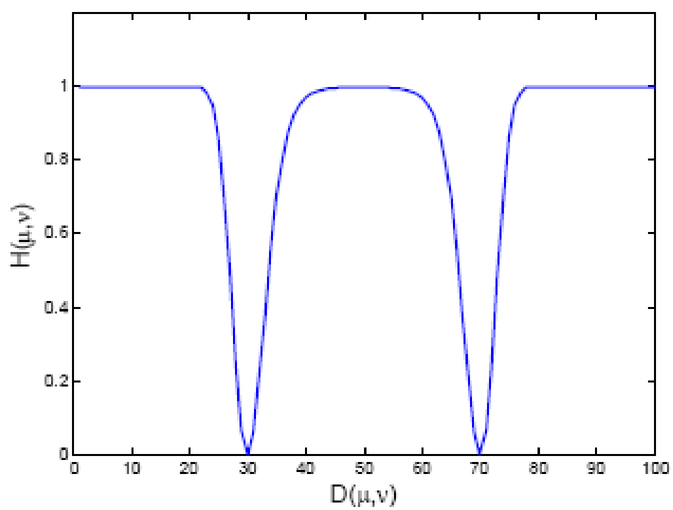


Gaussian BRF
D0 = 20 W = 8



GBRF Displayed as image

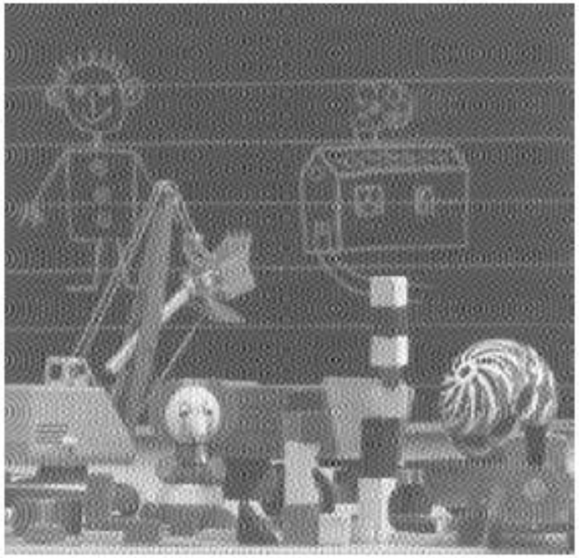
Filter Radial Cross Section



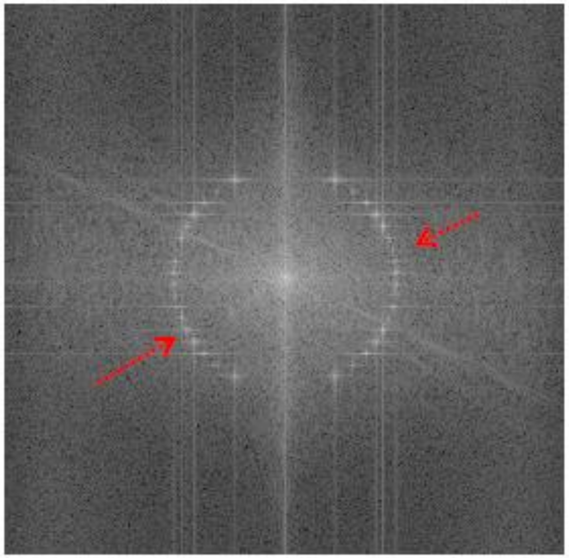
Selective Filtering

- **Bandreject Filters - Example**

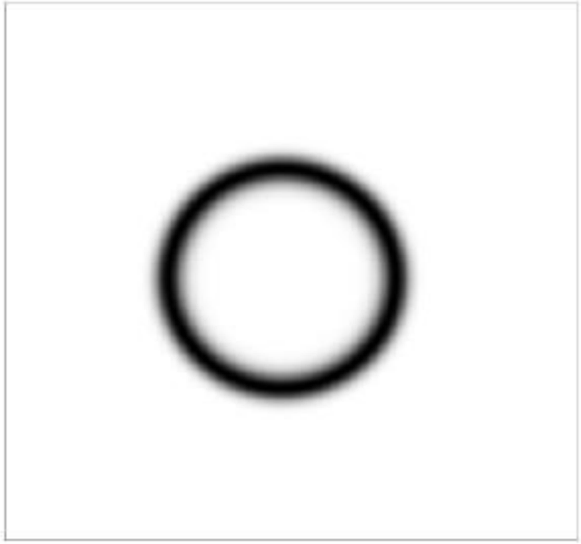
Corrupted Image



Frequency Spectrum of Image



Gaussian Band-Reject Filter

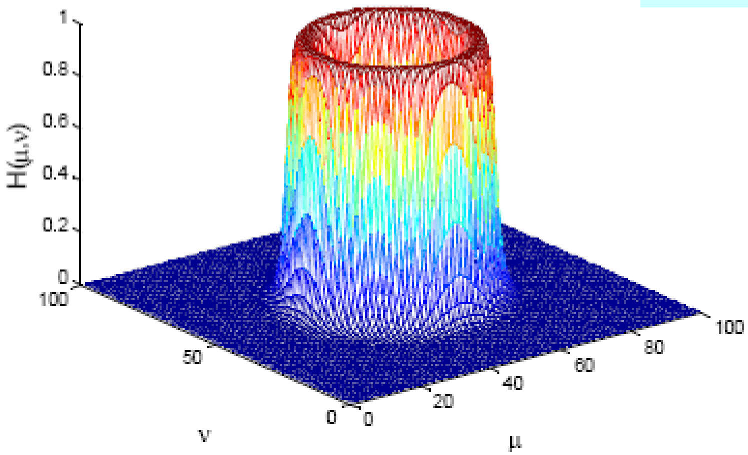


Filtered Image

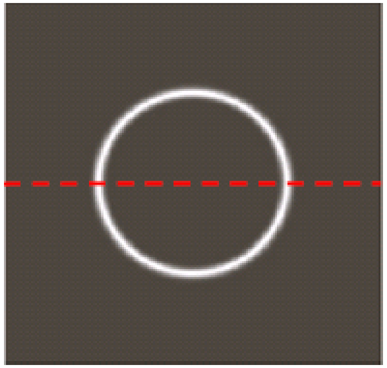
Selective Filtering

- **Bandpass Filters**
  - They are used to pass a certain band of frequencies
  - They can be easily reconstructed from bandreject filters using

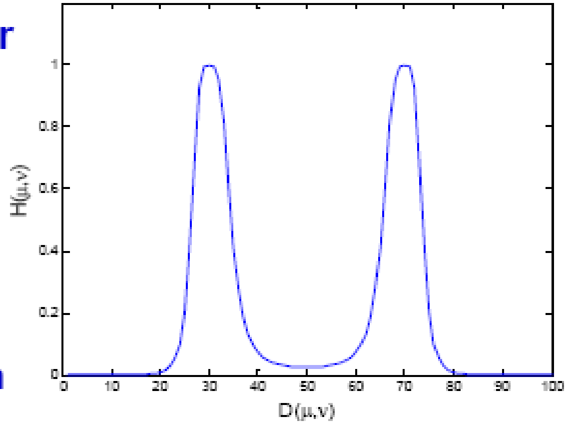
$$H_{BP}(\mu,v) = 1 - H_{BR}(\mu,v)$$



Butterworth Bandpass Filter



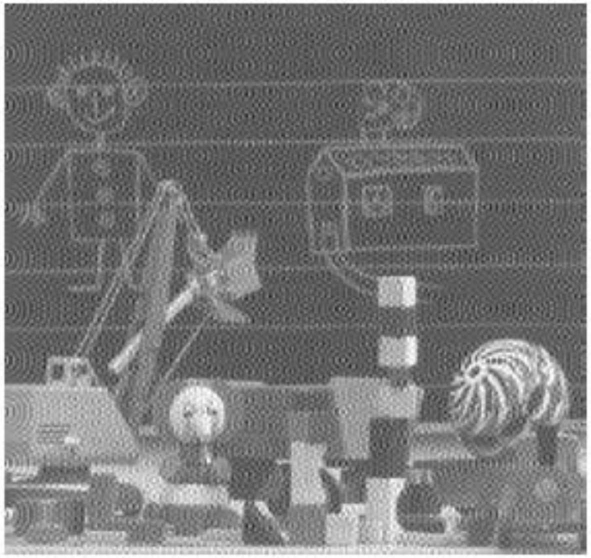
BBPF Displayed as image



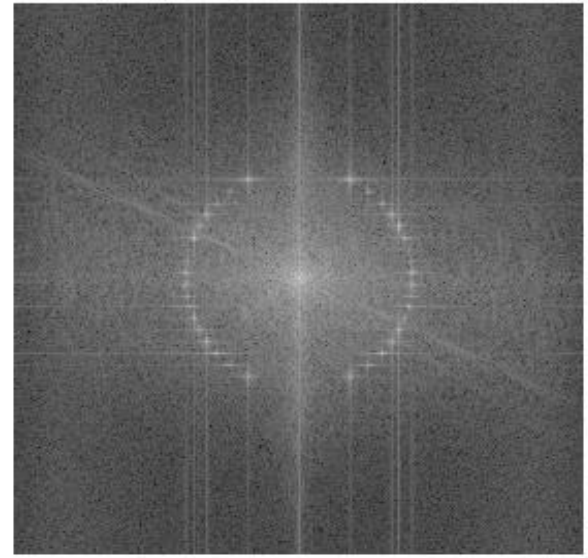
Filter Radial Cross Section

- **Bandpass Filters - Example**

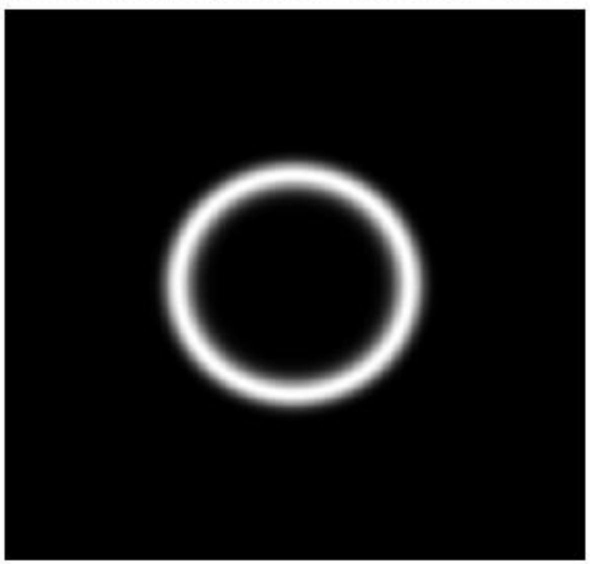
Corrupted Image



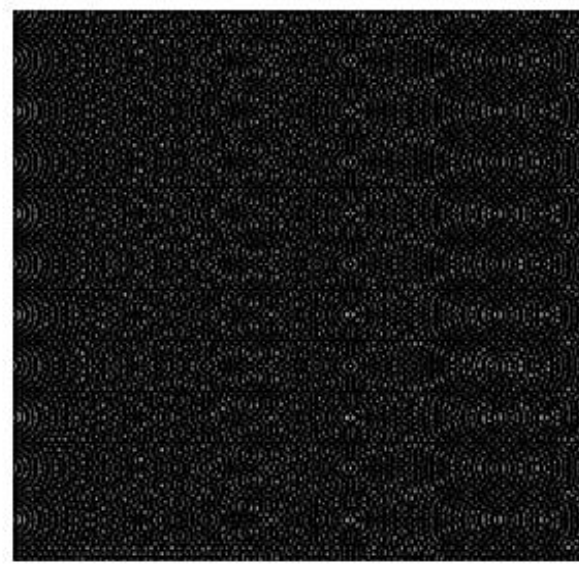
Frequency Spectrum of Image



Gaussian Bandpass Filter

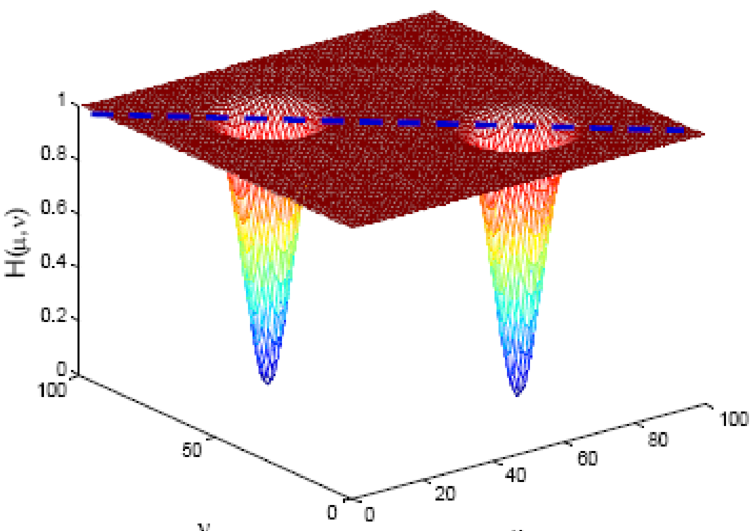


Extracted Degradation

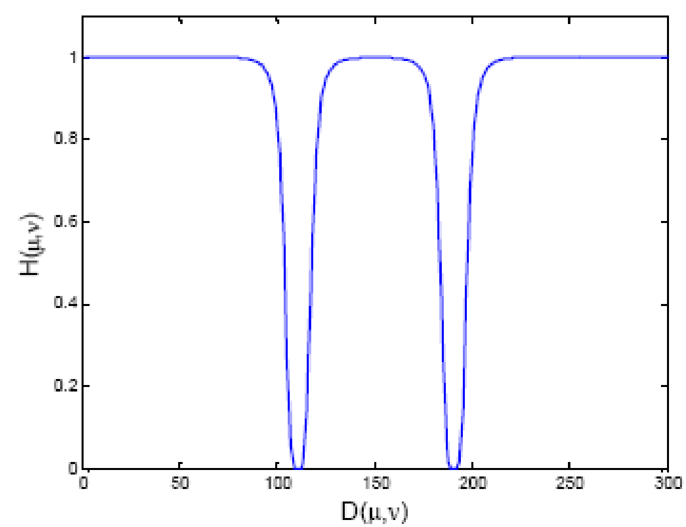


Selective Filtering

- **Notch Filters**
  - They are the most useful of selective filters
  - They can be used to pass (notch-pass) or reject (notch-reject) frequencies in a predefined neighborhood about the center of the frequency rectangle
  - Notch filters are required to be symmetric around the origin. This implies that a notch centered at (u_0,v_0) must have a corresponding notch at $(-u_0,-v_0)$



Gaussian Notch-Reject Filter



Selective Filtering

- **Notch Filters**
  - Notch filters are reconstructed as the product of two highpass filters whose centers are translated to the center of the notches

$$H_{NR}(\mu,\upsilon)=\prod_{k-1}^{Q}H_k(\mu,\upsilon)H_{-k}(\mu,\upsilon)$$

$H_k(u,v)$ and $H_{-k}(u,v)$ are the highpass filters centered at (u_k,v_k) and $(-u_k,-v_k)$

Q is the number of notch pairs

  - The centers of the highpass filters are specified with respect to the center (M/2,N/2)

$$D_k(\mu,\upsilon)=\left[(\mu-M/2-\mu_k)^2+(\upsilon-N/2-\upsilon_k)^2\right]^{1/2}$$

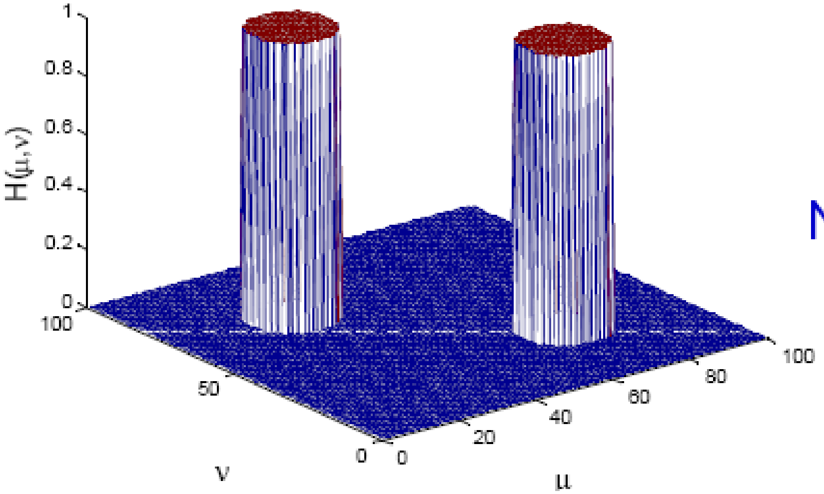
$$D_{-k}(\mu,\upsilon)=\left[(\mu-M/2+\mu_k)^2+(\upsilon-N/2+\upsilon_k)^2\right]^{1/2}$$

Selective Filtering

- **Notch Filters - Example**
  - Notch filter with three notch pairs that is constructed using BHPF

$$H_{NR}(\mu,\upsilon)=\prod_{k-1}^{3}\left[\frac{1}{1+[D_{0k}/D_k(\mu,\upsilon)]^{2n}}\right]\left[\frac{1}{1+[D_{0k}/D_{-k}(\mu,\upsilon)]^{2n}}\right]$$

- **Notch-pass Filters**
  - They can be constructed using $H_{NP}(\mu,\upsilon)=1-H_{NR}(\mu,\upsilon)$



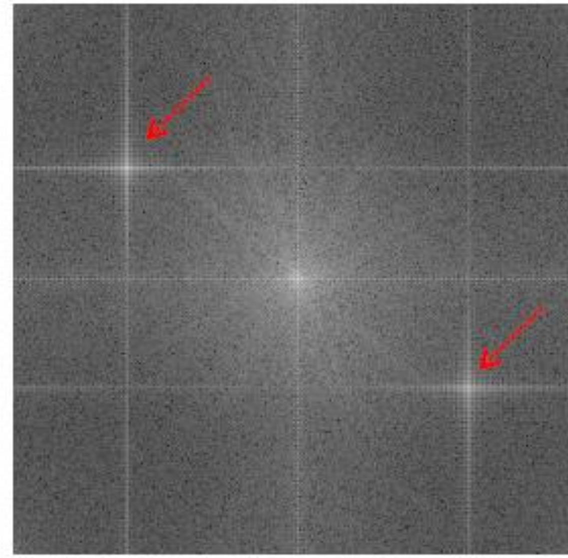
Ideal
Notch-pass Filter

Selective Filtering

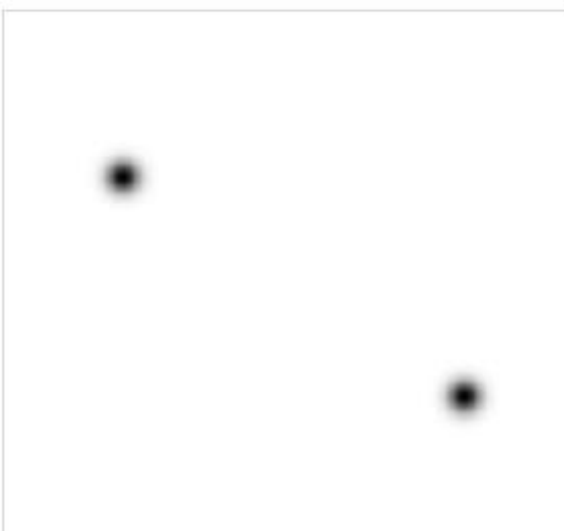
- **Notch-reject Filters - Example**

Corrupted Image

Frequency Spectrum of Image



Butterworth Notch-reject Filter



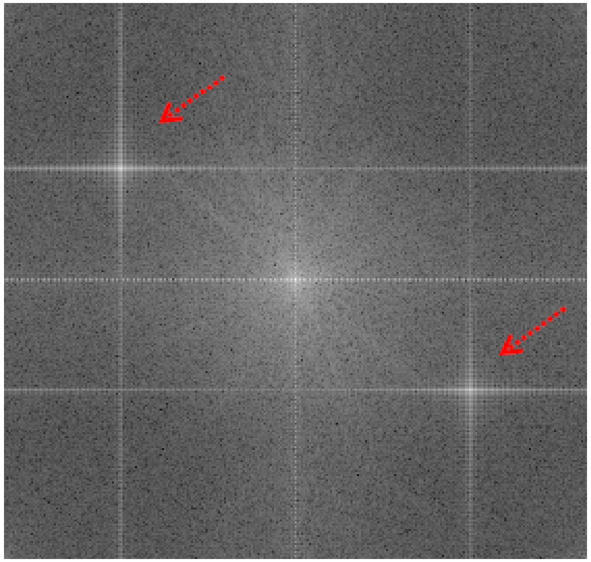
Filtered Image

Selective Filtering

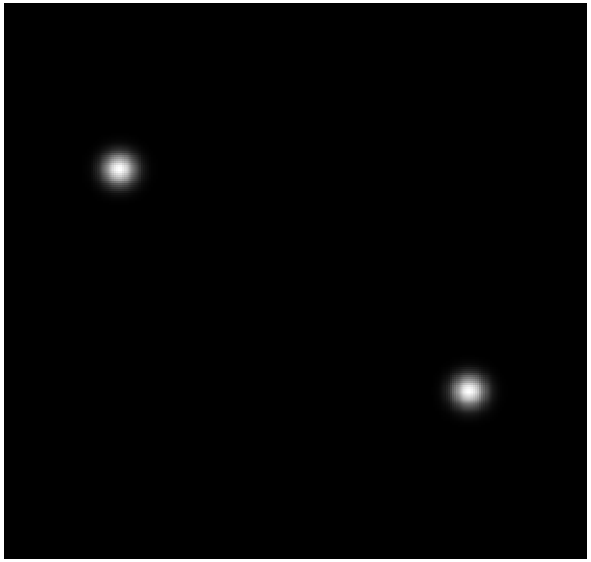
- **Notch-pass Filters - Example**

Corrupted Image

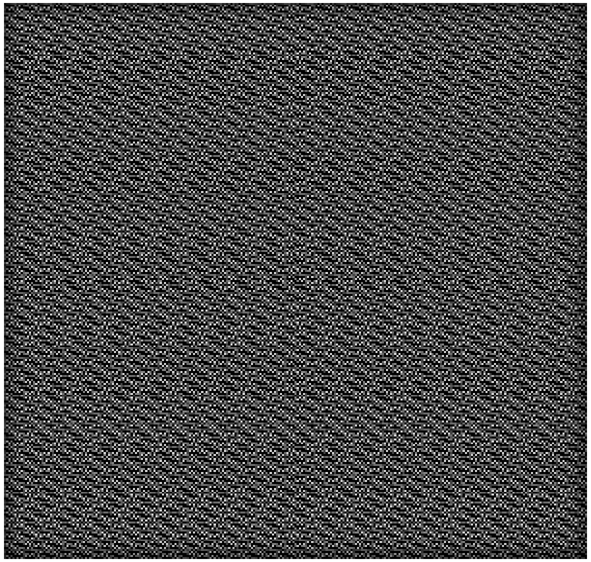
Frequency Spectrum of Image



Butterworth Bandpass Filter



Extracted Degradation



Homomorphic Filtering

- The image formation model using illumination and reflectance is given by

$$f(x,y) = i(x,y)r(x,y)$$

- This equation can't be used to operate on the illumination or reflectance functions separately in the frequency domain since the product of the two functions in the spatial domain is not equivalent to multiplying their transforms
- However, if we define

$$\begin{aligned} z(x,y) &= ln\left[f(x,y)\right] = ln\left[i(x,y)r(x,y)\right] \\ &= ln\left[i(x,y)\right] + ln\left[r(x,y)\right] \end{aligned}$$

- Then

$$\begin{aligned} \Im\{z(x,y)\} &= \Im\{ln[f(x,y)]\} \\ &= \Im\{ln[i(x,y)]\} + \Im\{ln[r(x,y)]\} \\ Z(\mu,\upsilon) &= F_i(\mu,\upsilon) + F_r(\mu,\upsilon) \end{aligned}$$

Homomorphic Filtering

- Now, we filter Z(u,v) by a filter H(u,v)

$$
\begin{aligned}
S(\mu,\upsilon) &= Z(\mu,\upsilon)H(\mu,\upsilon) \\
&= H(\mu,\upsilon)F_i(\mu,\upsilon) + H(\mu,\upsilon)F_r(\mu,\upsilon)
\end{aligned}
$$

- The filtered image in the spatial domain is

$$
\begin{aligned}
s(x,y) &= \Im^{-1}\{S(\mu,\upsilon)\} \\
&= \Im^{-1}\{H(\mu,\upsilon)F_i(\mu,\upsilon)\} + \Im^{-1}\{H(\mu,\upsilon)F_r(\mu,\upsilon)\}
\end{aligned}
$$

- If we define

$$
\begin{aligned}
i'(x,y) &= \Im^{-1}\{H(\mu,\upsilon)F_i(\mu,\upsilon)\} \\
r'(x,y) &= \Im^{-1}\{H(\mu,\upsilon)F_r(\mu,\upsilon)\}
\end{aligned}
$$

- then

$$
s(x,y) = i'(x,y) + r'(x,y)
$$

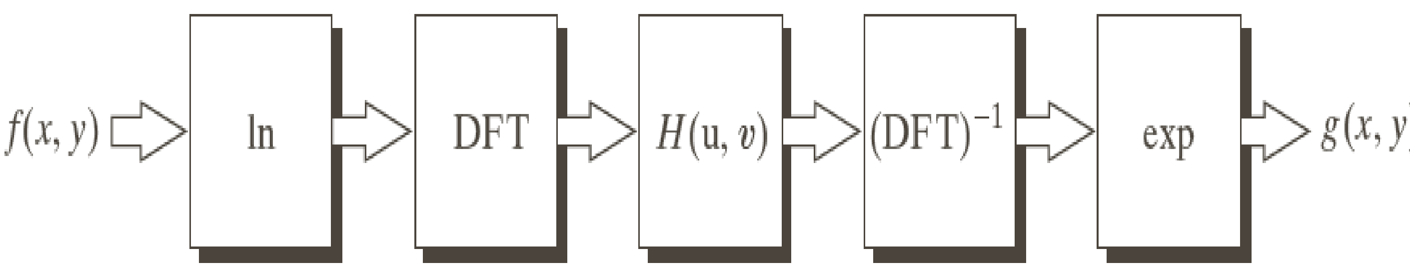
Homomorphic Filtering

- The output image is computed by exponentiation

$$
\begin{aligned}
g(x,y) &= e^{s(x,y)} \\
&= e^{i'(x,y) + r'(x,y)} \\
&= e^{i'(x,y)}\, e^{r'(x,y)} \\
&= i_0(x,y) r_0(x,y)
\end{aligned}
$$

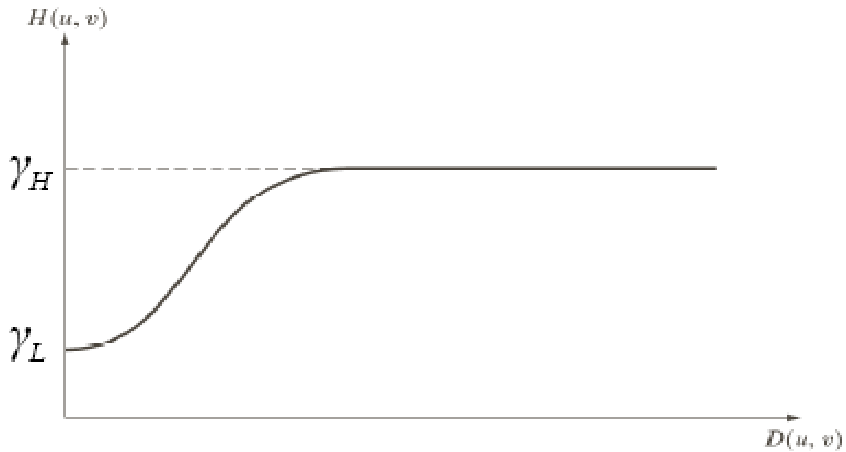
- This approach is called homomorphic filtering which is summarized in the figure below

FIGURE 4.60 Summary of steps in homomorphic filtering.



Homomorphic Filtering

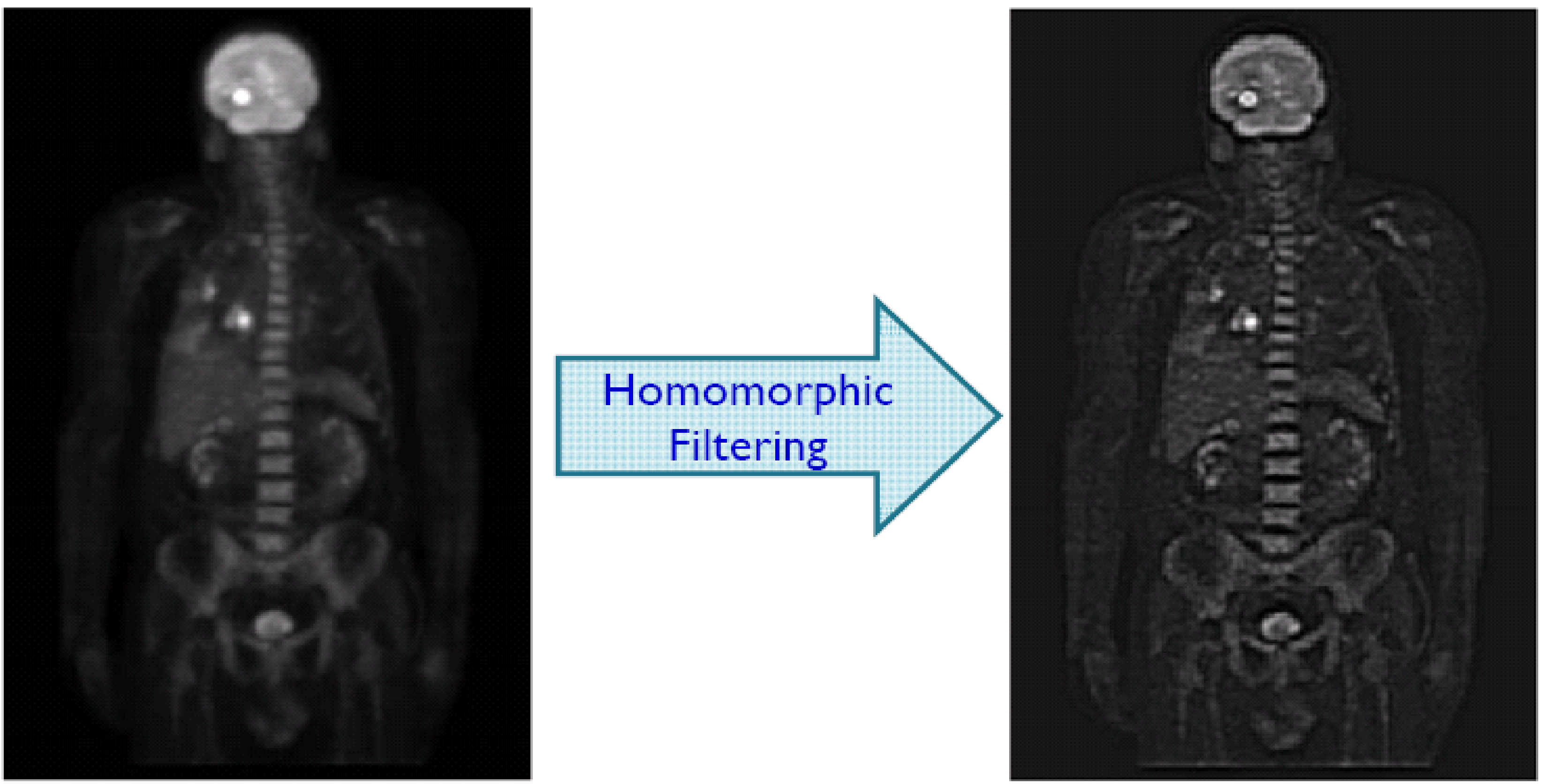
- Homomorphic filtering is of great importance if we want to operate on the illumination and reflectance components separately
- The illumination component is characterized with slow variations while the reflectance tends to vary greatly; especially at the borders of dissimilar objects
- We can design a single filter that affects the low and high frequencies in different controllable ways



$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{c[D^2(u,v)/D_0^2]}\right]$$

Homomorphic Filtering

- **Example**
  - A PET image that is blurred and many of its low-intenisty features are obscured by the high intensity of the hot spots
  - Use the filter in the previous slide with $\gamma_H = 2$, $\gamma_L = 0.25$, c=1, and D0 = 80



Homomorphic Filtering

a b

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)