Morphological Image Processing

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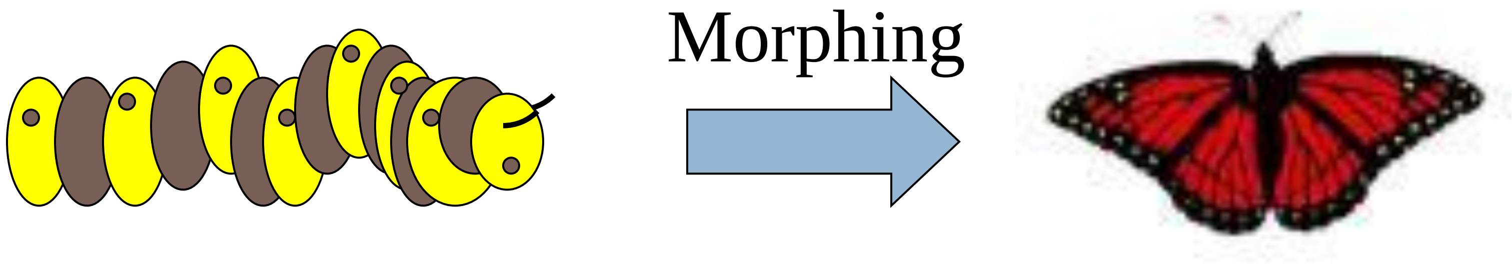
Outline

- Introduction
- Erosion and Dilation
- Opening and Closing
- Boundary Extraction
- Hole Filling
- Hit-or-Miss Transformation

Introduction

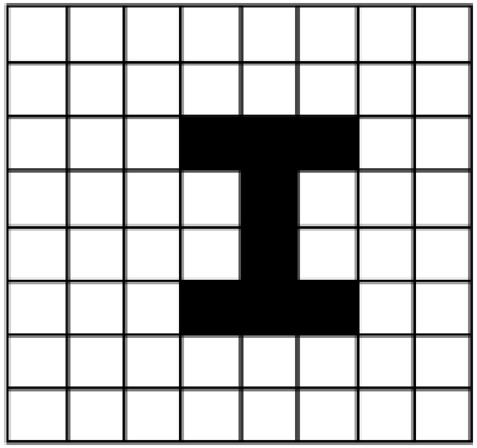
- Morphological operations come from the word "morphing" in Biology which means "changing a shape".



- In the context of image processing , it refers to a set of mathematical tools that can be used to
  - Extract useful description and representation of regions in images (boundaries, skeletons, convex hull)
  - Remove imperfections introduced during segmentation (thinning, regions filling)

Introduction

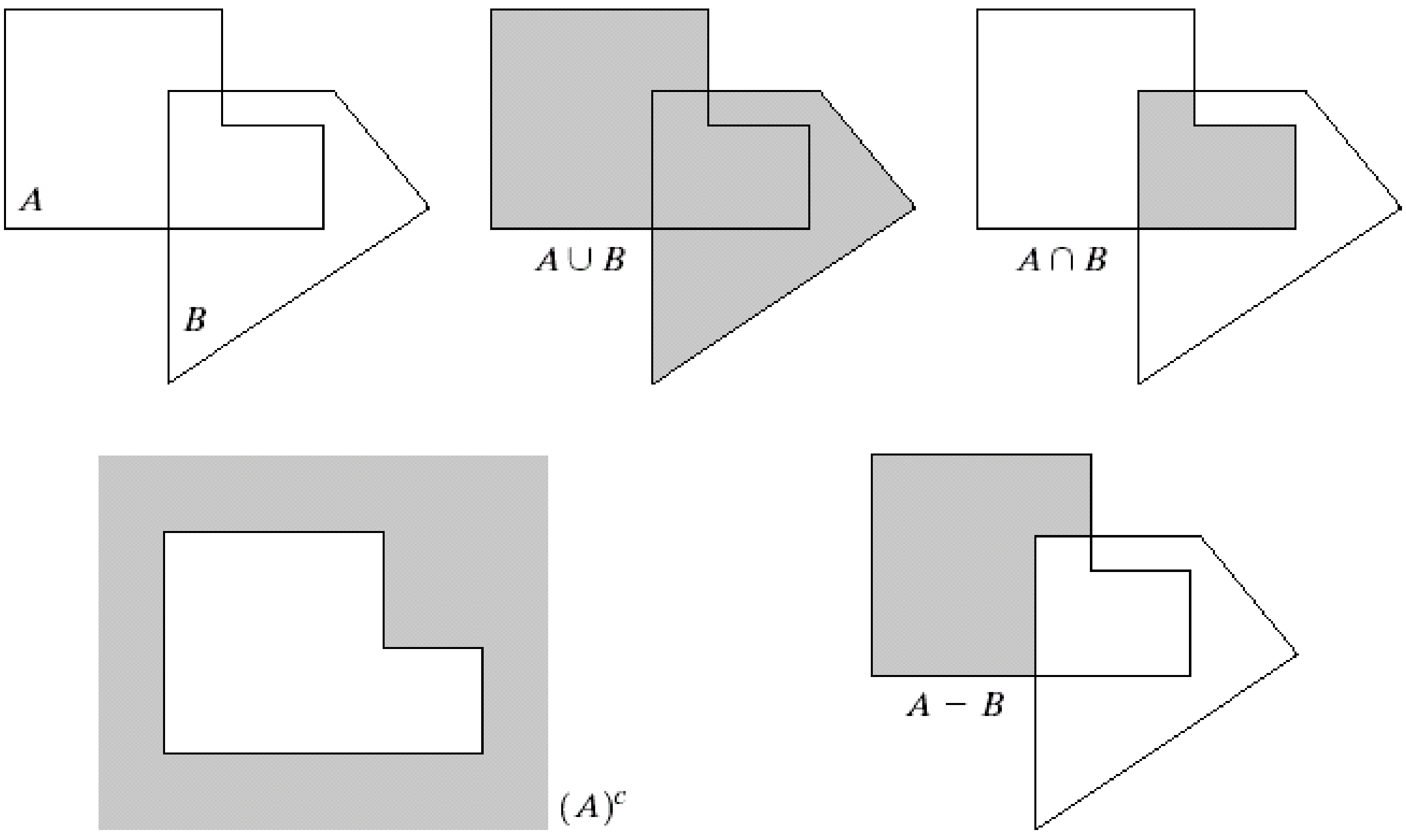
- A binary image can be considered as a set by considering "black" pixels (with image value "1") as elements in the set and "white" pixels (with value "0") as outside the set, or vice versa.



- Morphological filters are essentially set operations

Basic Set Operations

- Let x, y, z, … represent locations of 2D pixels, e.g. x = (x1, x2), S denote the complete set of all pixels in an image, let A, B, … represent subsets of S
  - Each set may represent one object. Each pixel (x,y) has its status: belong to a set or not belong to a set.

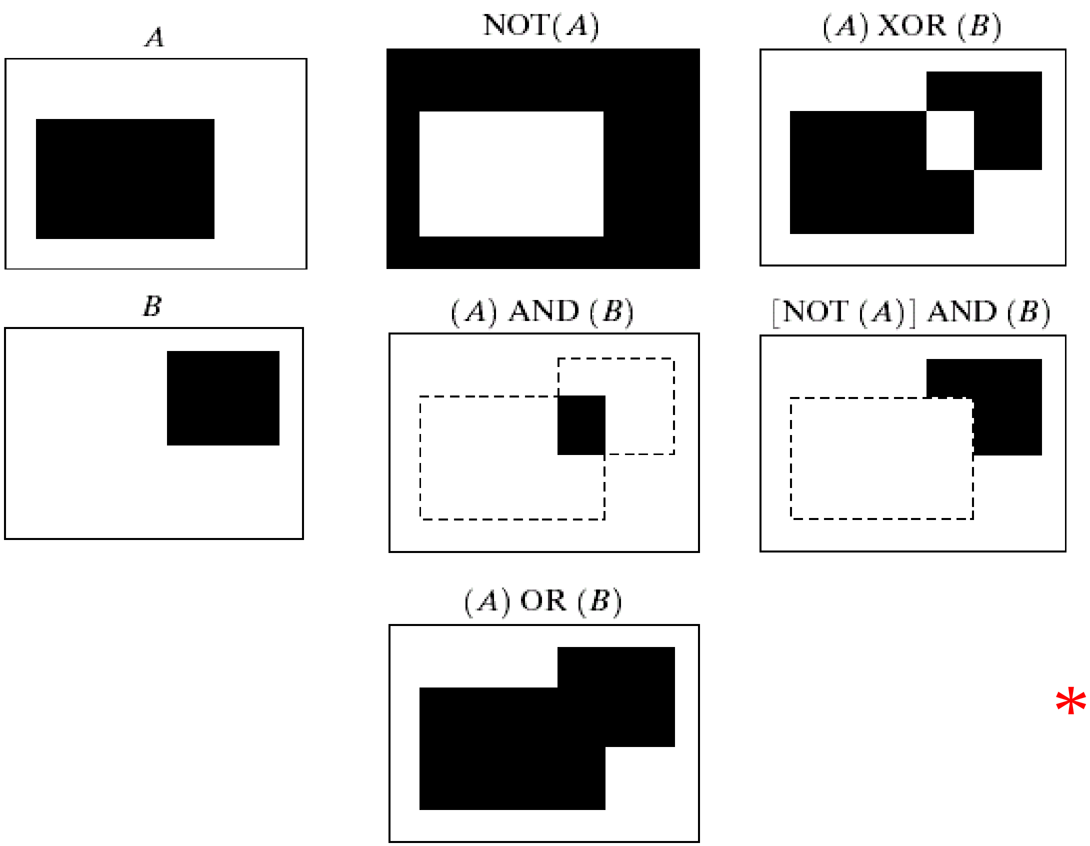


a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Basic Set Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

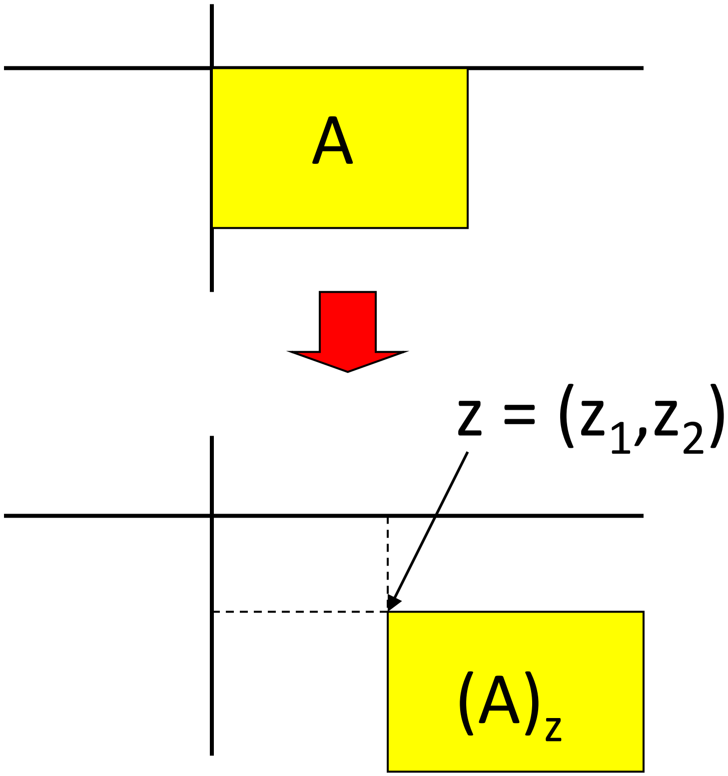


*For binary images only

Basic Set Operations

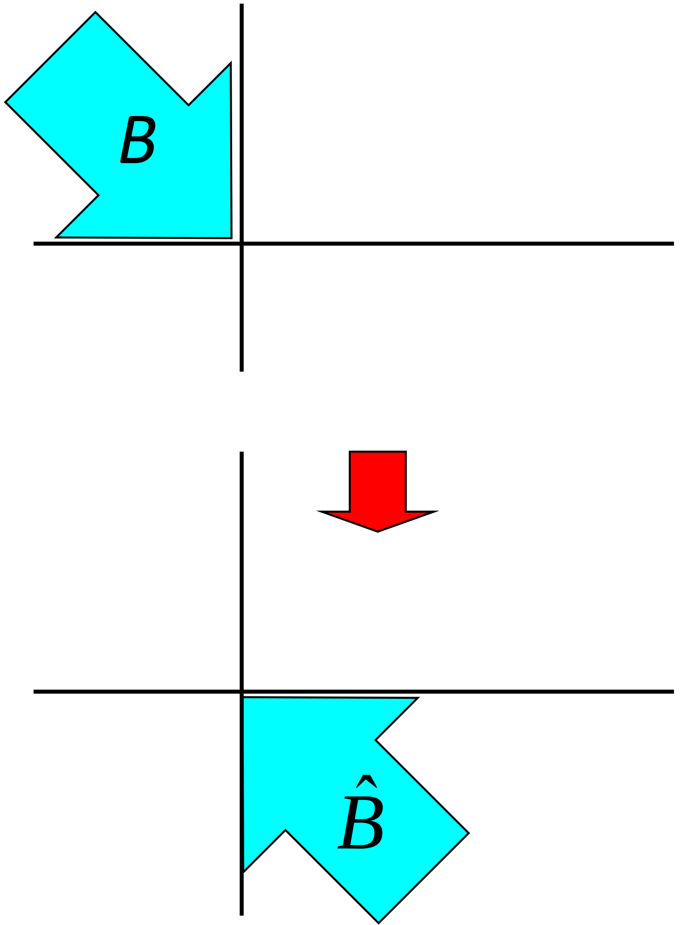
Translation

$$(A)_z = \{c \mid c = a + z, \quad \text{for } a \in A\}$$



Reflection

$$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$$



Structuring Elements

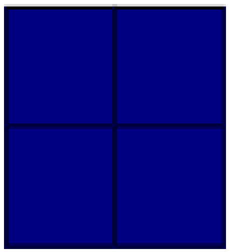
- All morphological operations are based on using structuring elements (similar to filter masks)

1	1	1
1	**1**	1
1	1	1

0	1	0
1	**1**	1
0	1	0

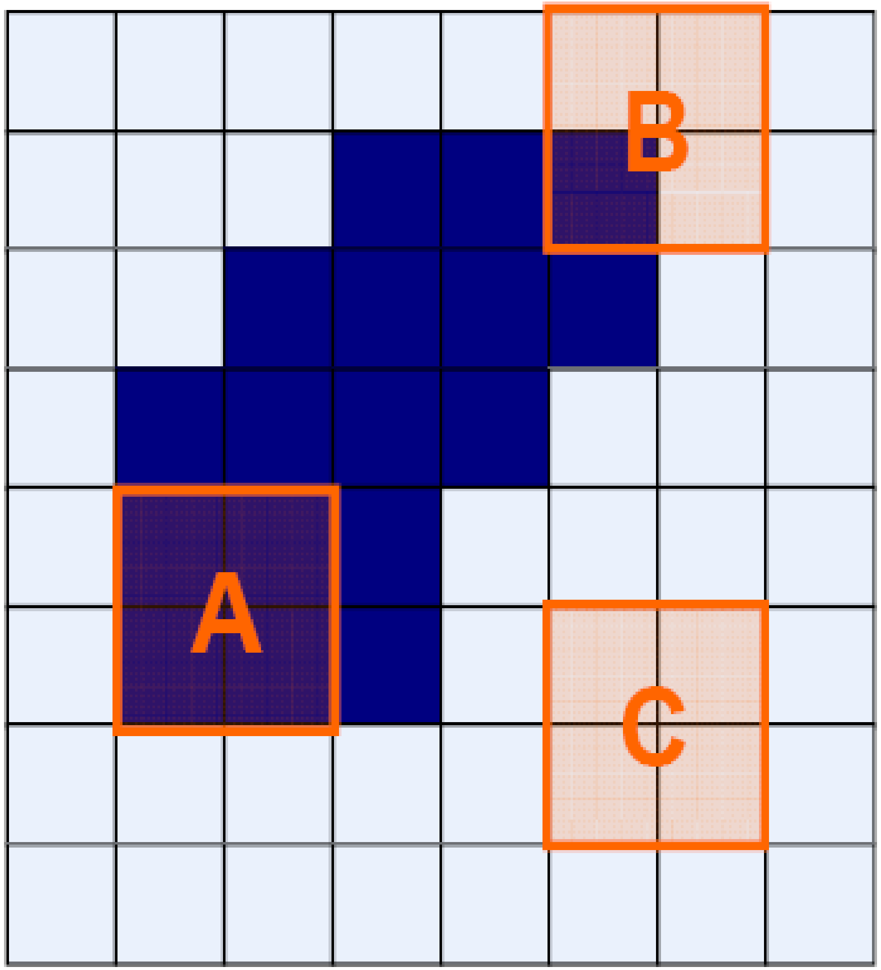
- Morphological operations are based moving the structuring element over the image pixels to check for a HIT or FIT
  - FIT - all *ON pixels* in the structuring element cover *ON pixels* in the image
  - HIT - any *ON* pixel in the structuring element covers an *ON* pixel in the image

Structuring Elements



Structuring Element

- A – Fit
- B – Hit
- C – Miss



Structuring Elements

- Structuring elements can be of any size and make any shape
- Rectangular shapes are usually used with the origin being at the center pixel

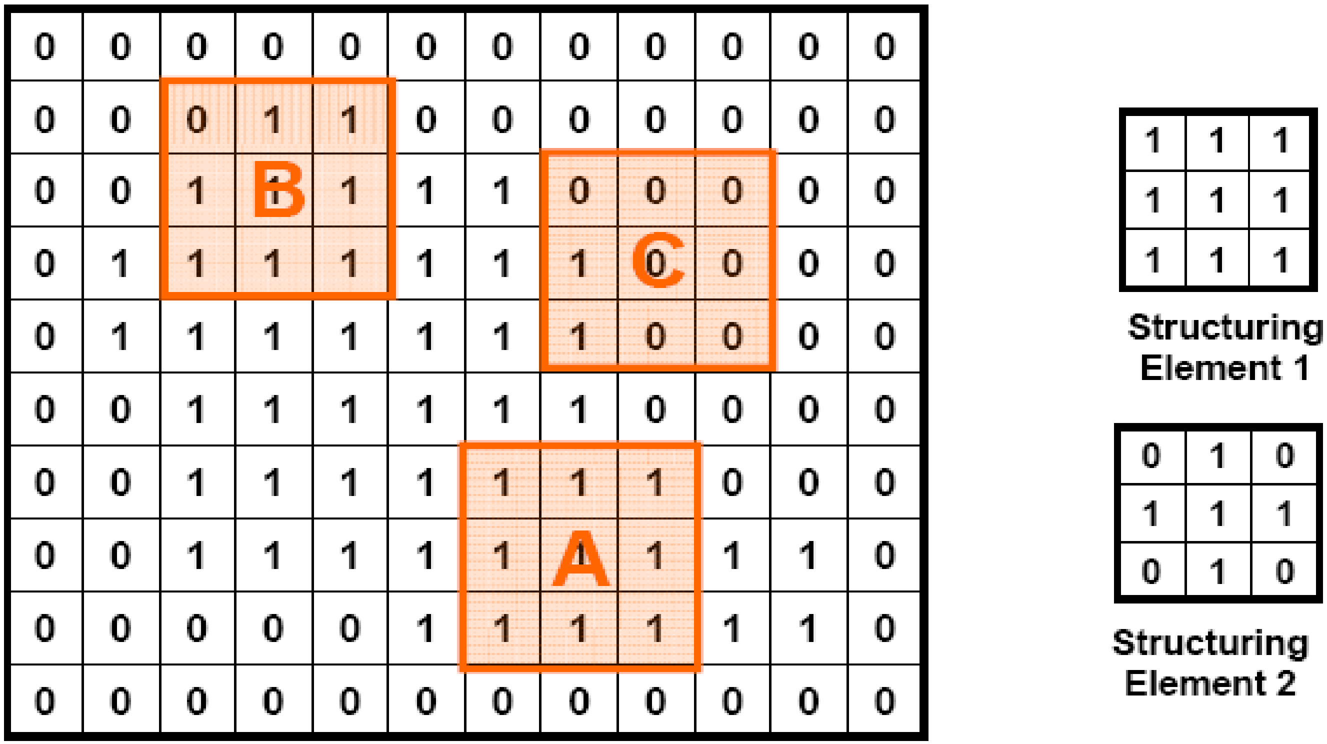
1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed

Structuring Elements



Location	Structuring Element 1	Structuring Element 2
A	FIT	FIT
B	HIT	FIT
C	HIT	HIT

Dilation

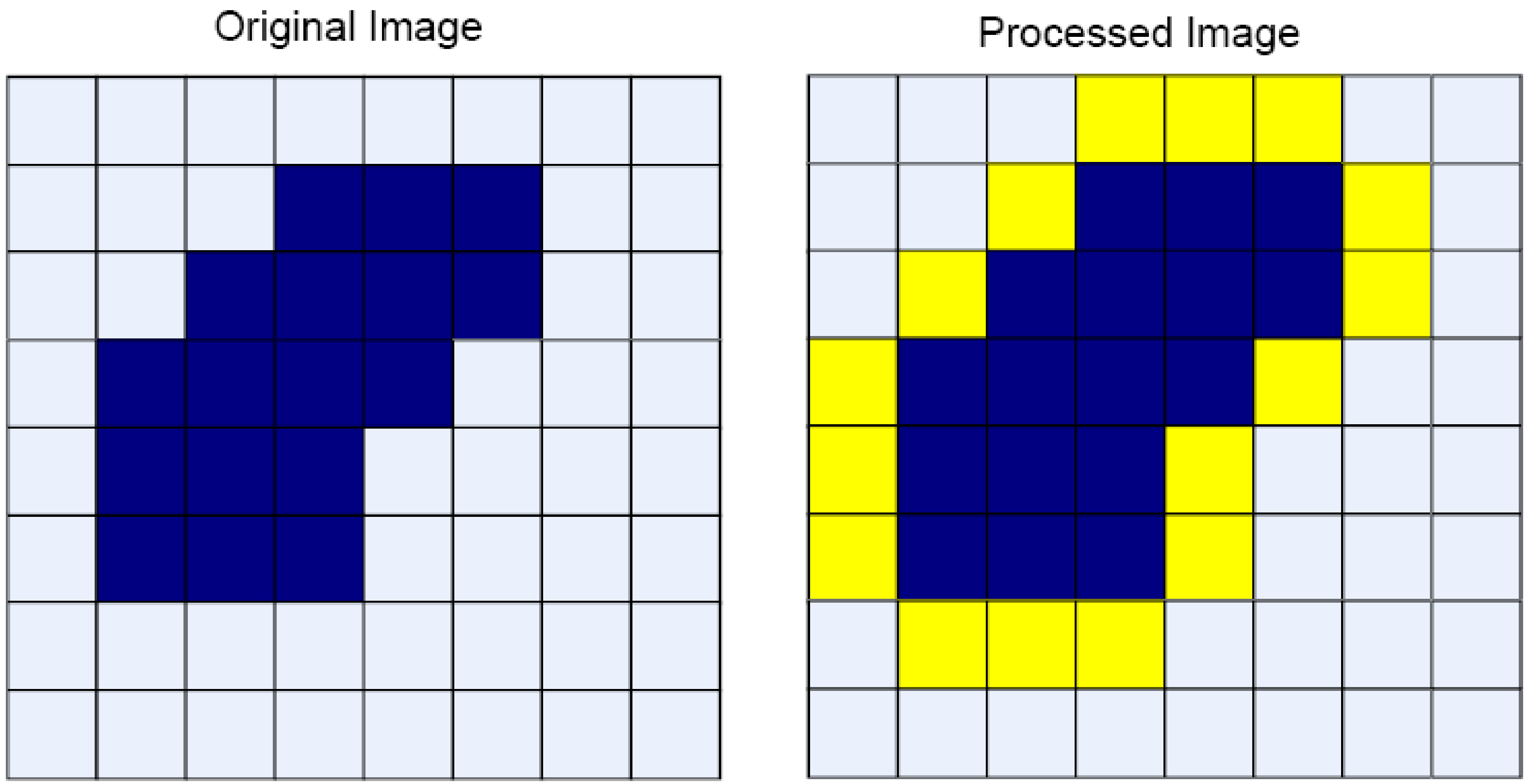
- Dilation of image A by structuring element B is given by $A \oplus B$ *such that*

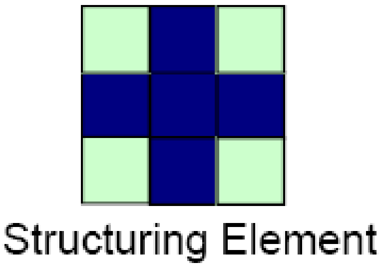
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

- The structuring element B is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

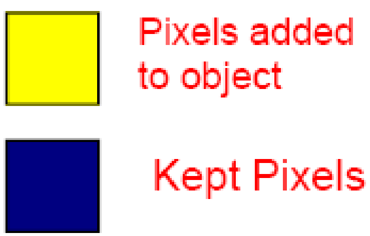
$$g(x,y) = \begin{cases} 1 & \text{, if } B \text{ hits } A \\ 0 & \text{, otherwise} \end{cases}$$

Dilation





- * Keep pixels at which the structuring element hits at least one ON pixel
- * Dilation enlarges objects



Dilation

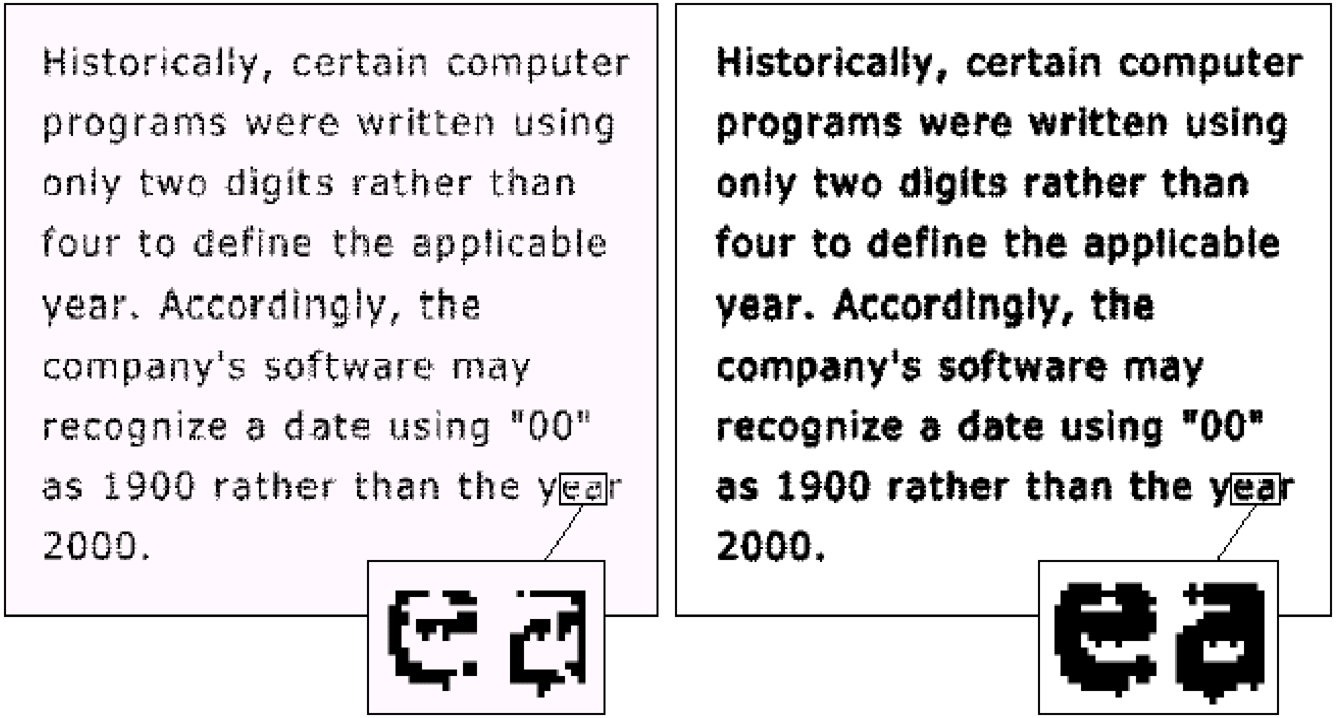
Original image

Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Watch out: In these examples a 1 refers to a black pixel!

Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

What Is Dilation For?

- Dilation can repair breaks

- Dilation can repair intrusions

Erosion

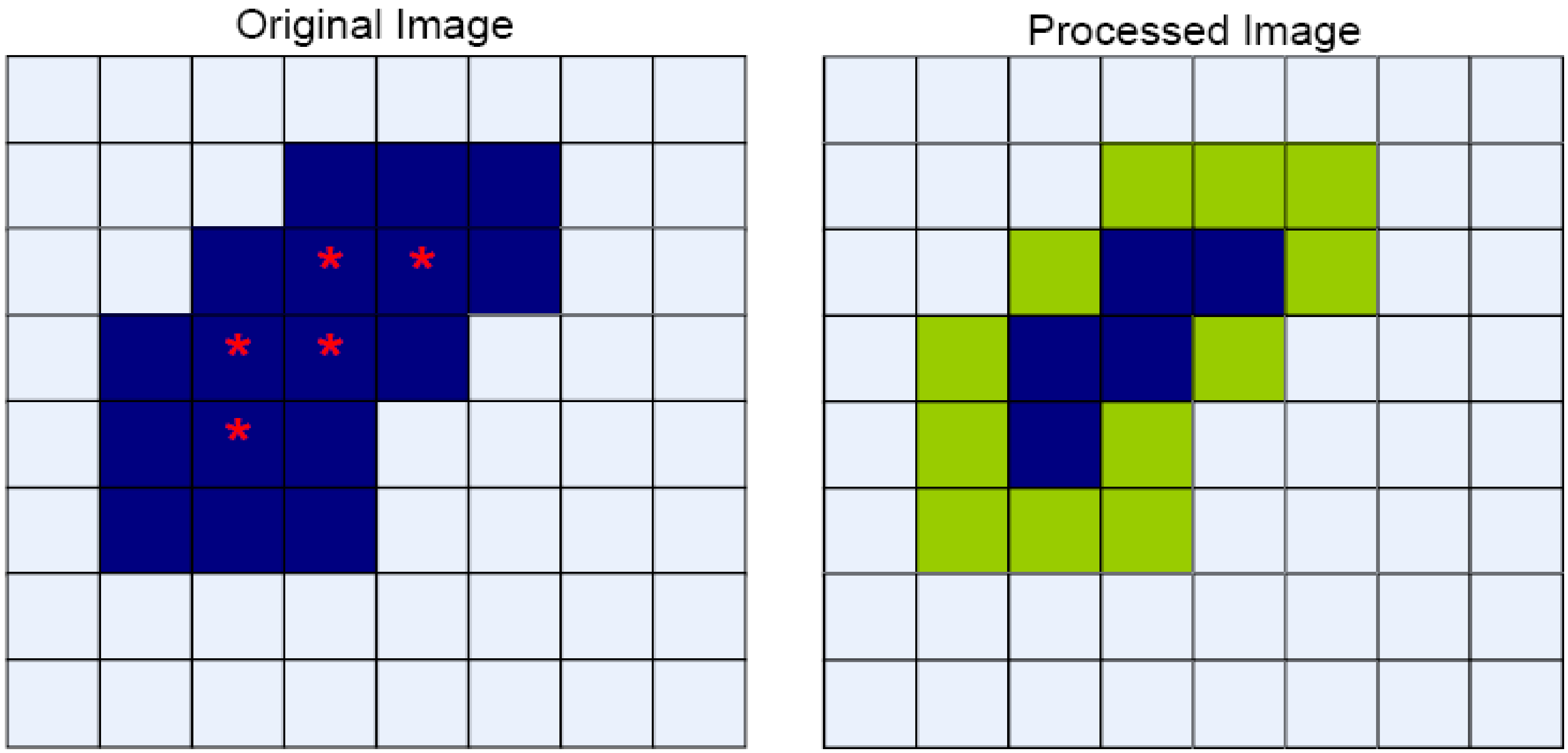
- Erosion of image A by structuring element B is given by $A \ominus B$ *such that*

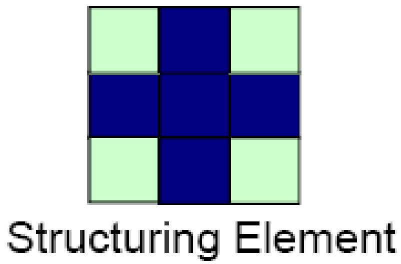
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- The structuring element B is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

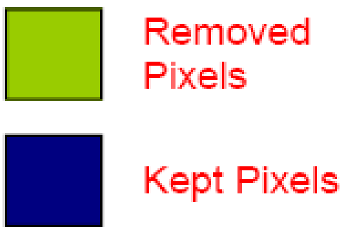
$$g(x, y) = \begin{cases} 1 & \text{, if } B \text{ fits } A \\ 0 & \text{, otherwise} \end{cases}$$

Erosion



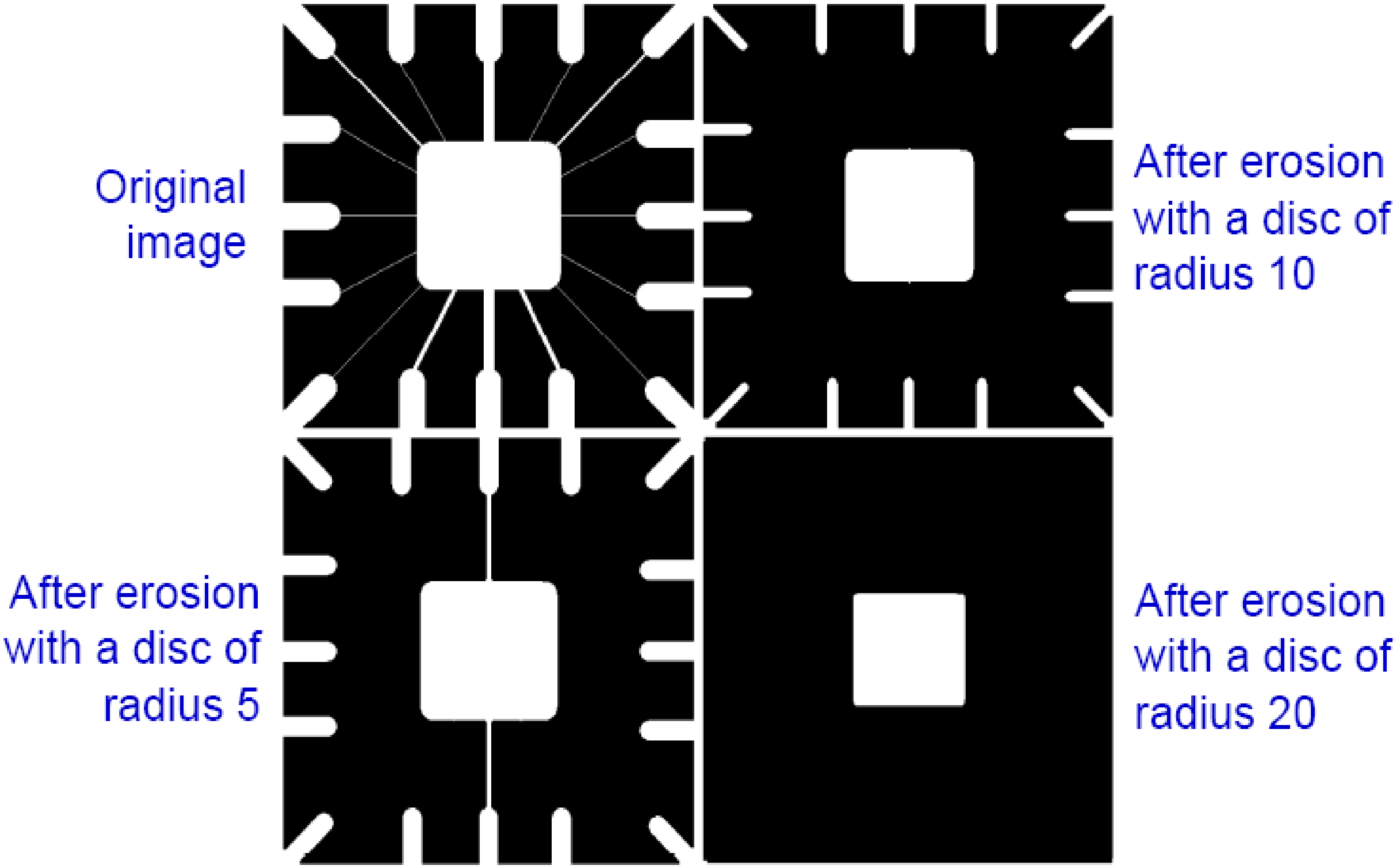


- * Keep pixels at which the structuring element scores a FIT
- * Erosion shrinks objects



Erosion

Using Erosion to remove image component



Erosion

Original image

Erosion by 3*3 square structuring element

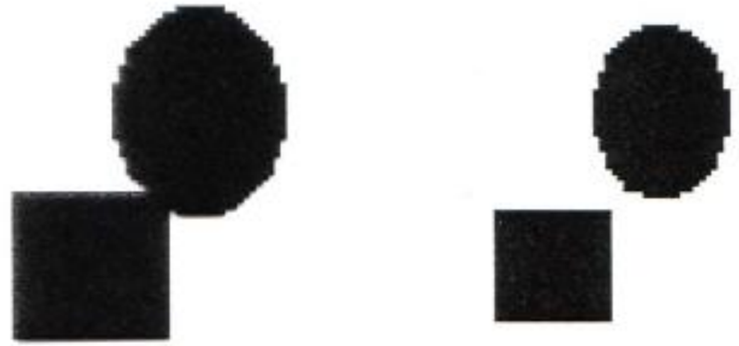
Erosion by 5*5 square structuring element

- **Watch out:** In these examples a 1 refers to a black pixel!

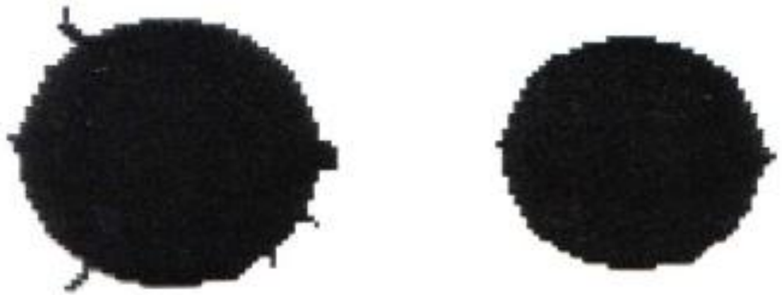
Erosion

What Is Erosion For?

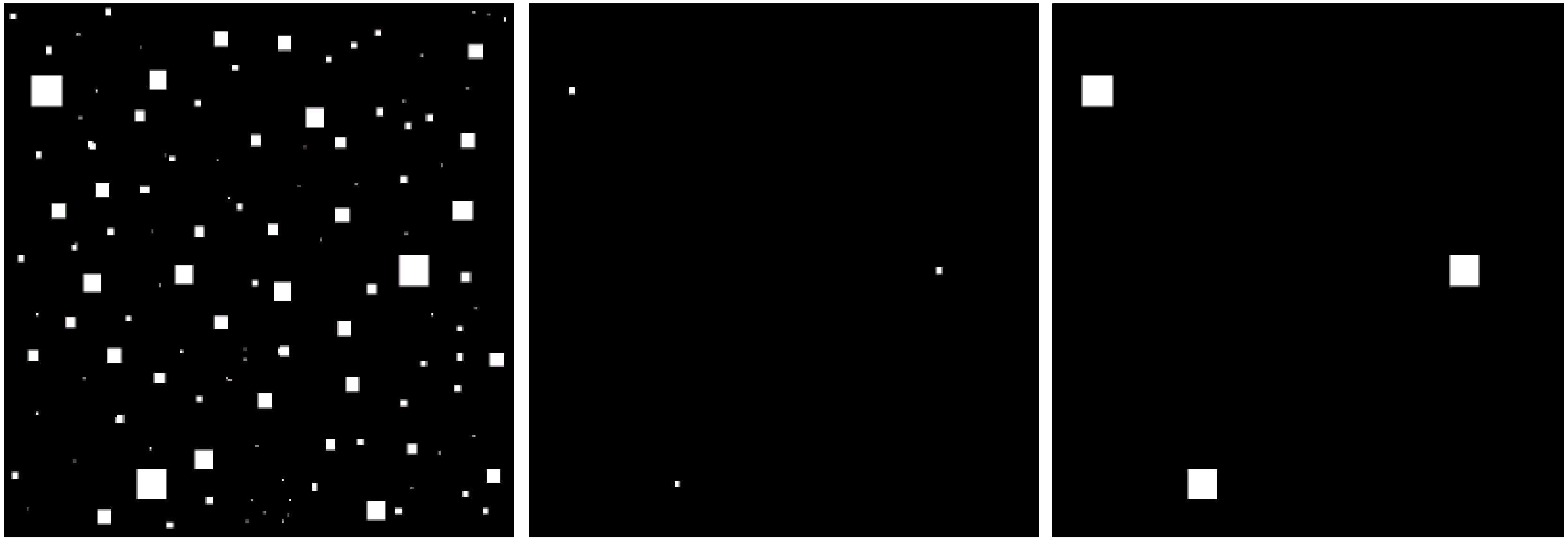
- Erosion can split apart joined objects



- Erosion can strip away extrusions



Application of Dilation and Erosion



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Erosion thins objects in a binary image

- Image details smaller than SE are removed

Dilation grows object in a binary image

Opening

- The opening operation on image A by structuring element B is defined as

$$A \circ B = (A \ominus B) \oplus B$$

in other words, it is erosion followed by dilation

- Generally, opening is used to
  - Smooth the contour of an object
  - Break narrow paths between large objects
  - Eliminate thin protrusions

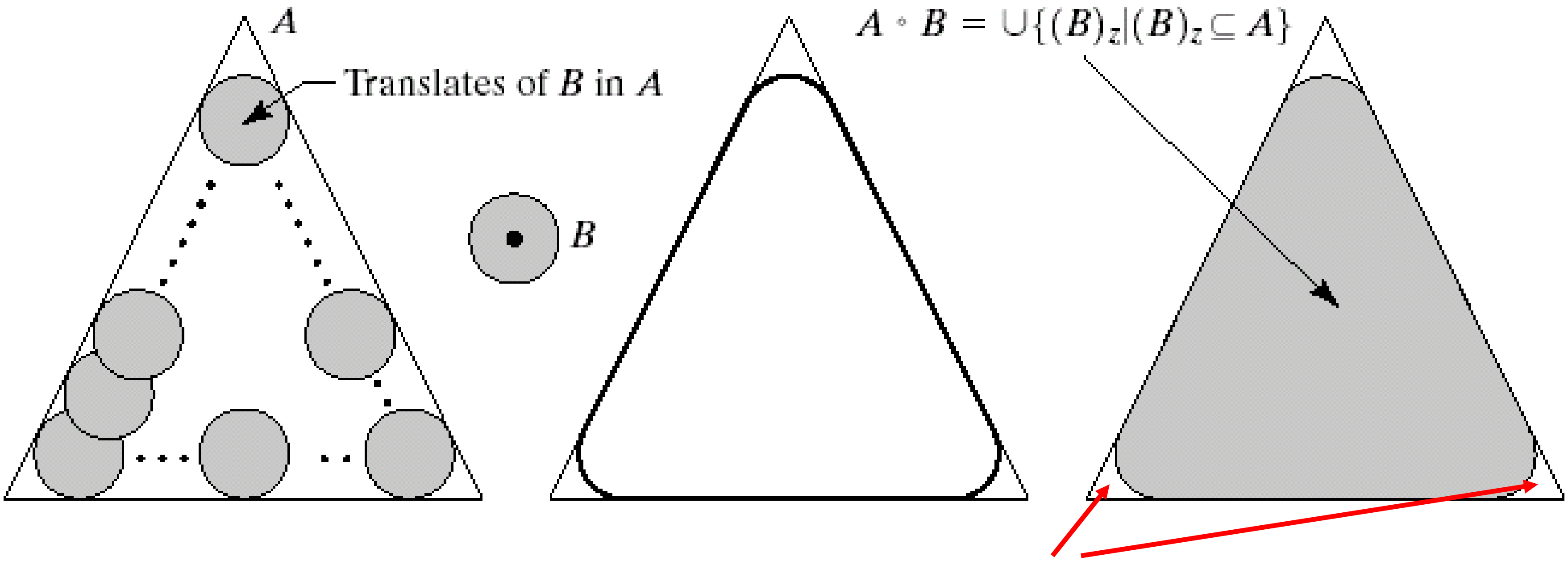
Opening

$$A \circ B = (A \ominus B) \oplus B$$

or

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

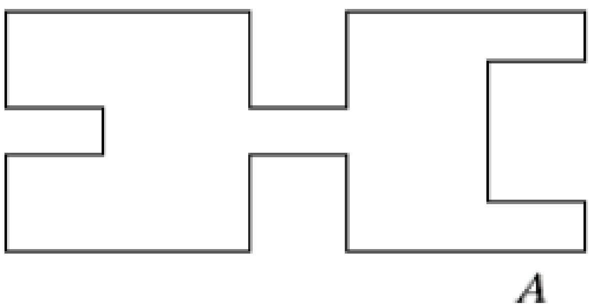
= Combination of all parts of A that can completely contain B



Opening eliminates narrow and small details and corners.

Opening

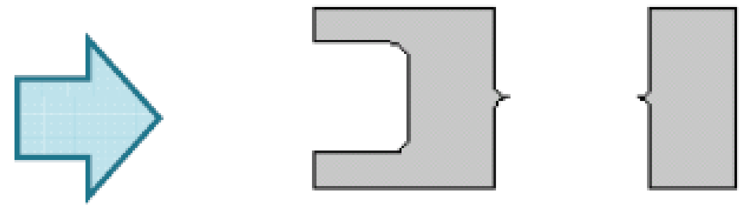
- **Example**



Original shape

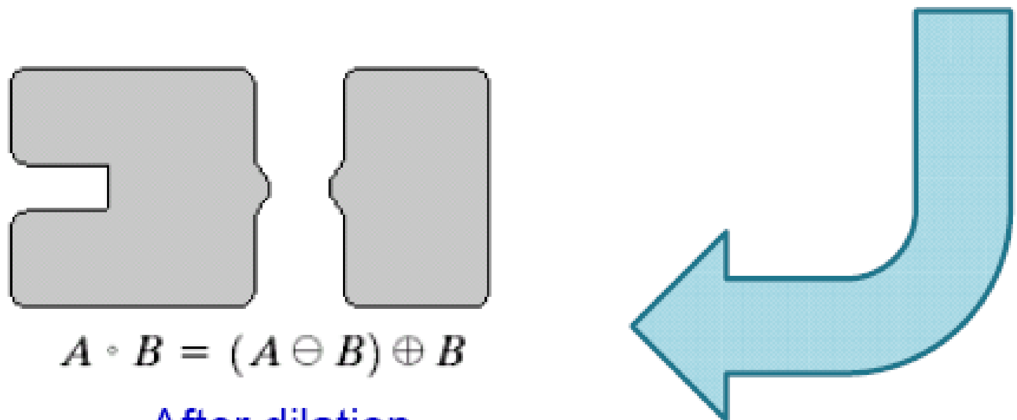
0	0	1	0	0
0	1	1	1	0
1	1	**1**	1	1
0	1	1	1	0
0	0	1	0	0

Structuring Element



$A \ominus B$

After erosion



$A \circ B = (A \ominus B) \oplus B$

After dilation
(opening)

Closing

- The closing operation on image A by structuring element B is defined as

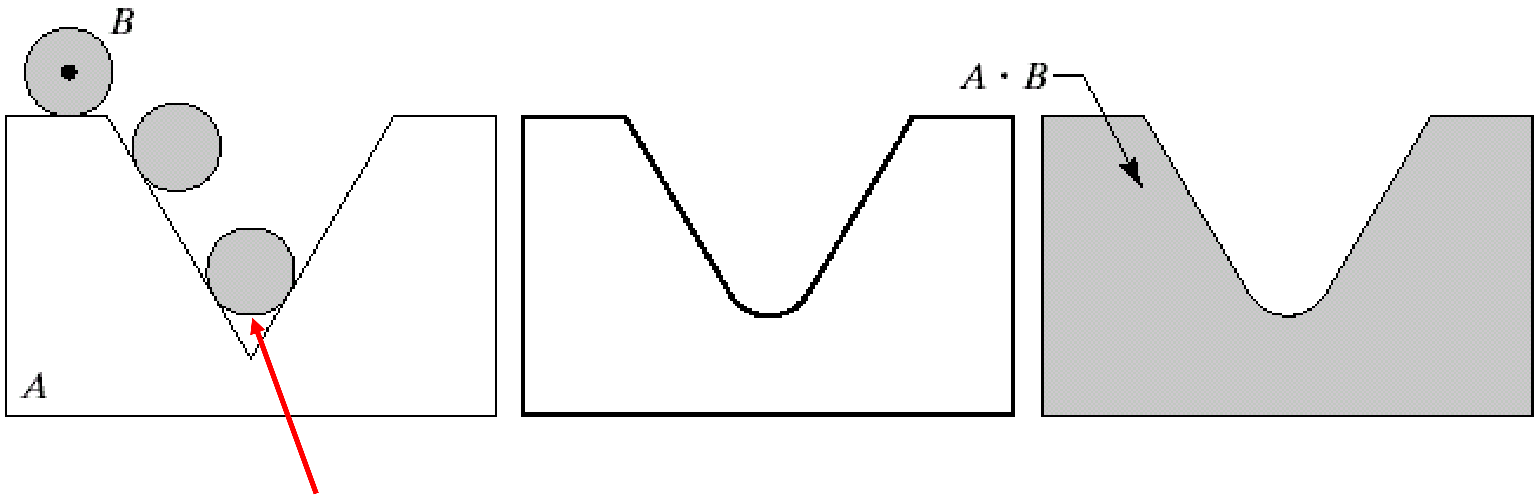
$$A \bullet B = (A \oplus B) \ominus B$$

in other words, it is dilation followed by erosion

- Generally, closing is used to
  - Smooth the contour of an object
  - Fuse narrow paths between large objects
  - Eliminate thin small holes and fill gaps in the contour

Closing

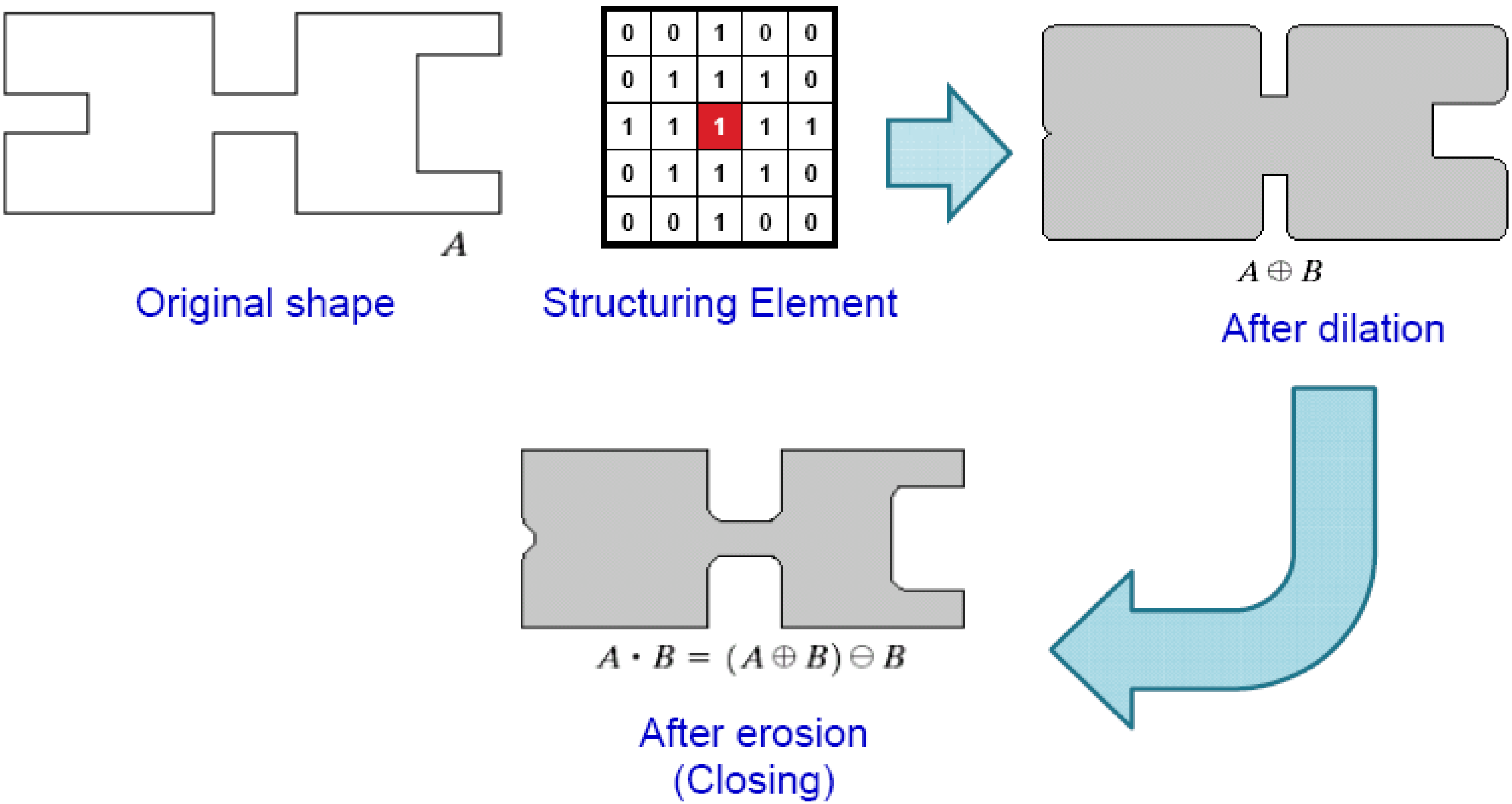
$$A \bullet B = (A \oplus B) \ominus B$$



Closing fills narrow gaps and notches

Closing

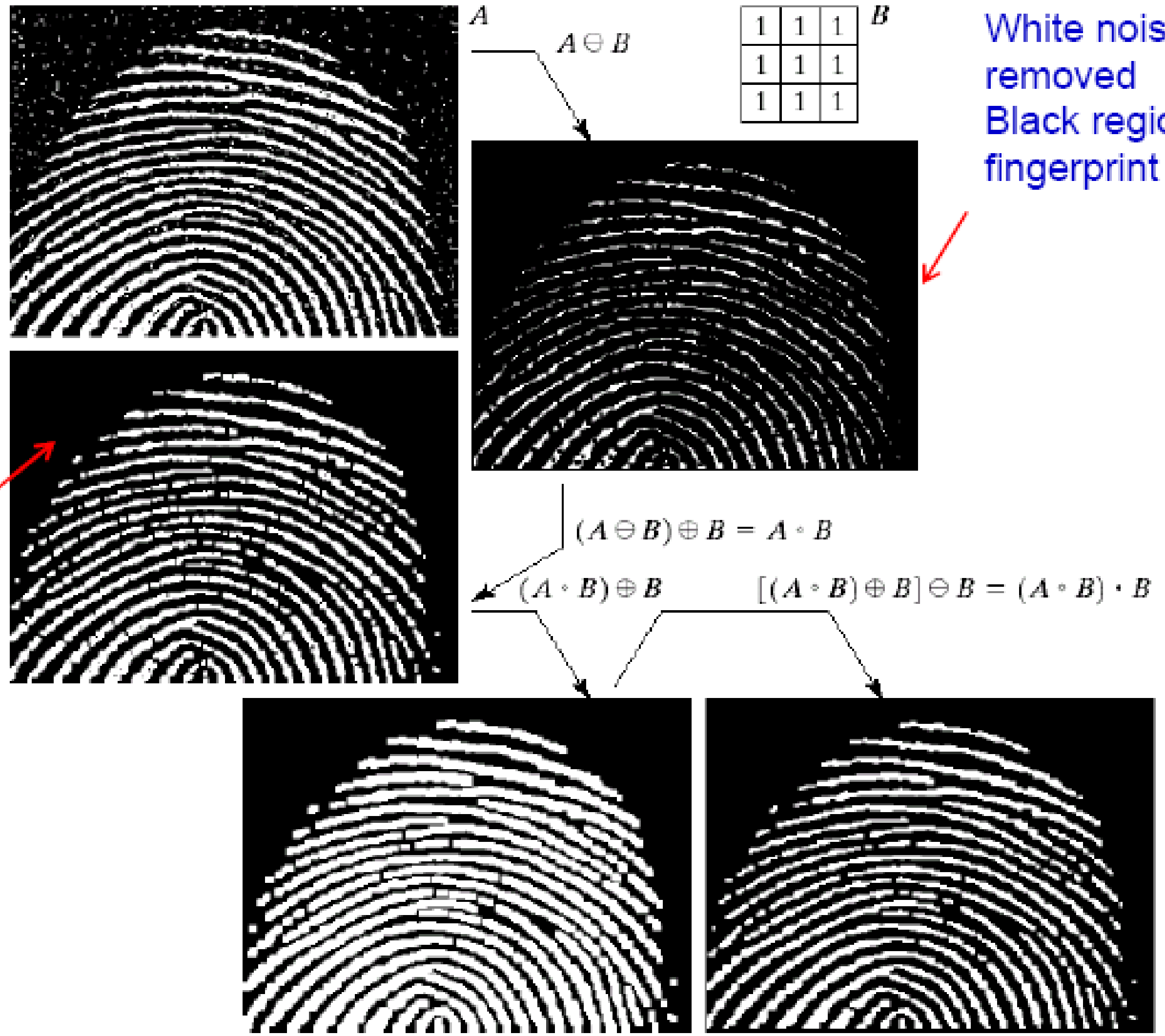
- **Example**



Example

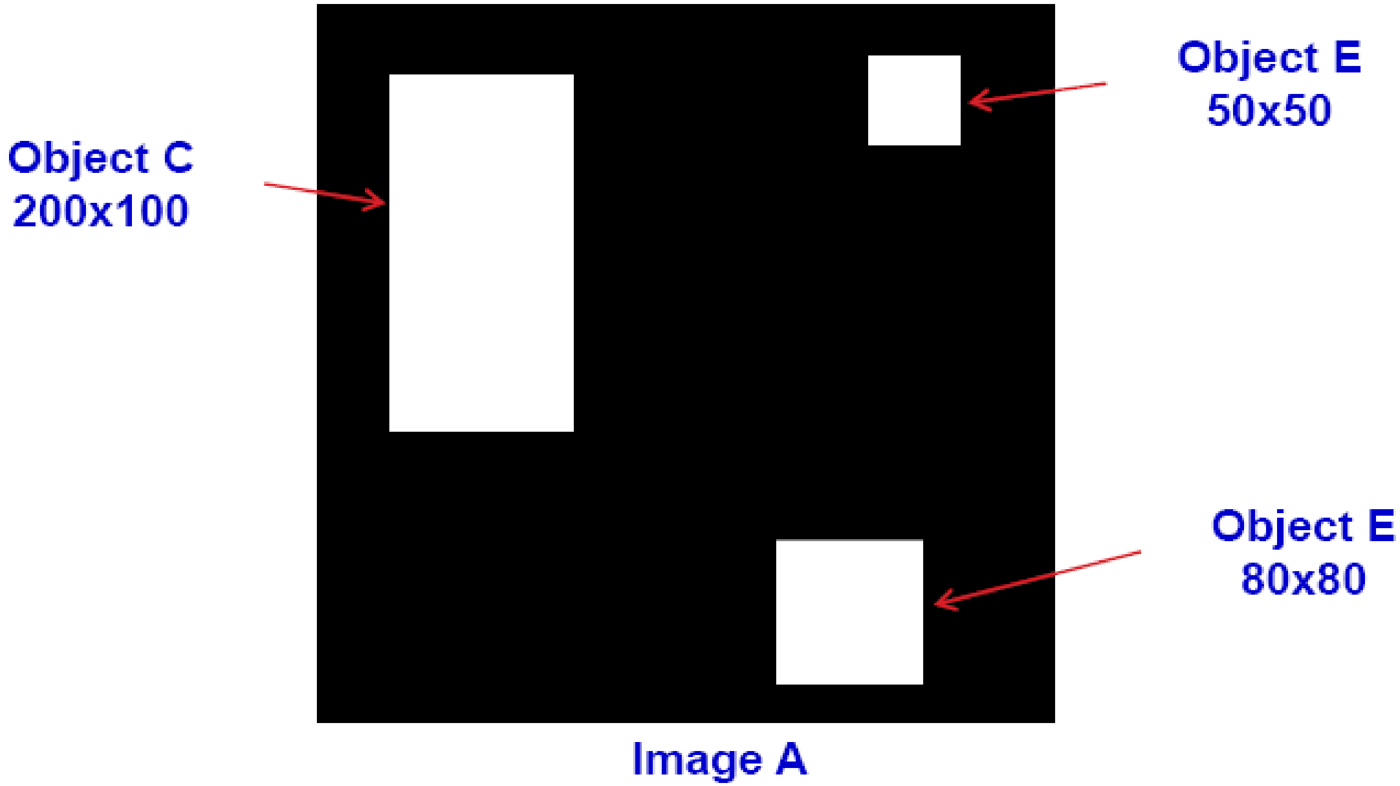
White noise regions removed
Black regions within the fingerprint have enlarged

Black regions within the fingerprint shrunken
But gaps are introduced between ridges



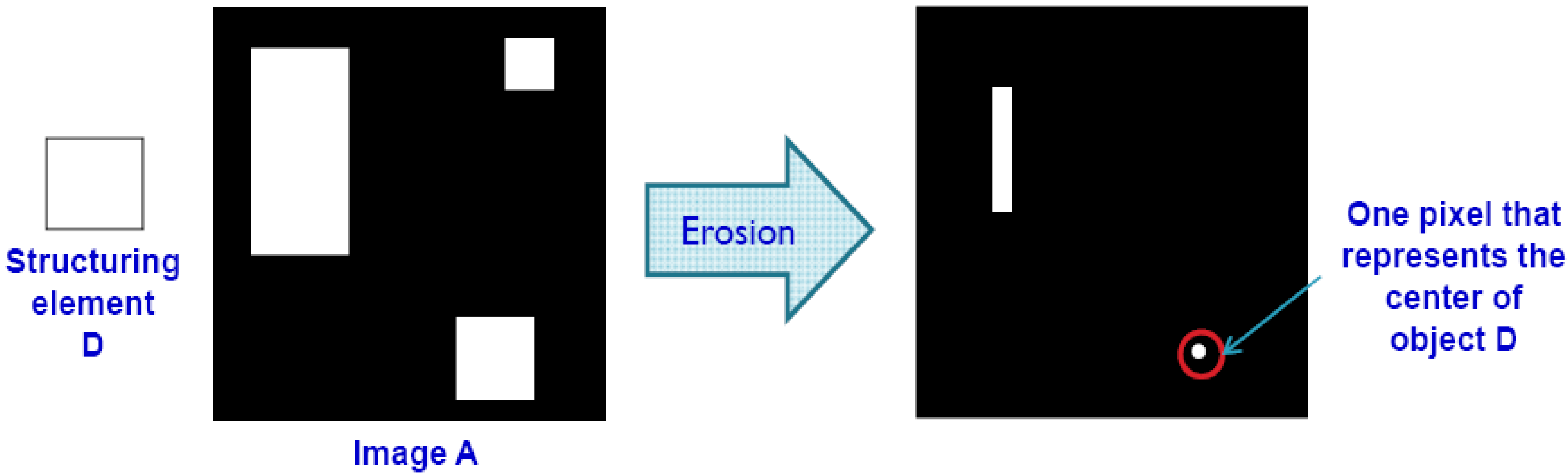
Hit-or-Miss Transformation

- A basic tool for shape detection
- Suppose we have an image A that consists of three objects C, D, and E and we want to detect the presence of shape D; assuming we know its shape



Hit-or-Miss Transformation

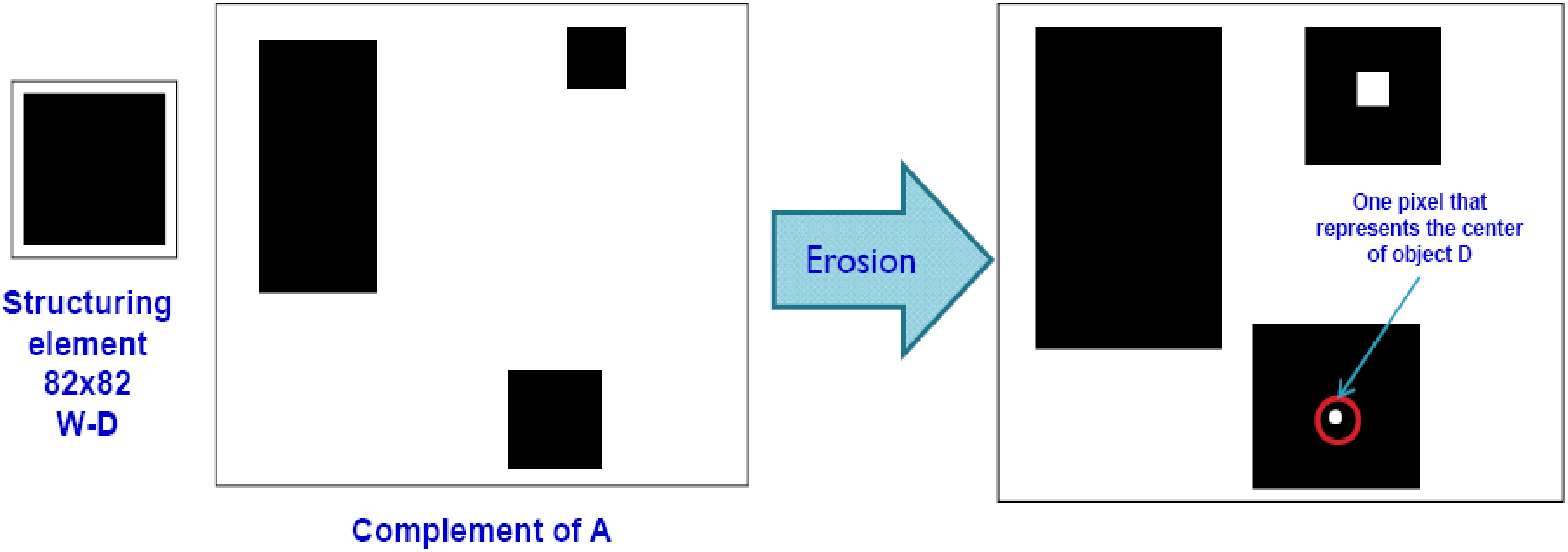
- We can start by eroding A with a structuring element that with the same shape as D



- However, the result may contain parts of larger objects

Hit-or-Miss Transformation

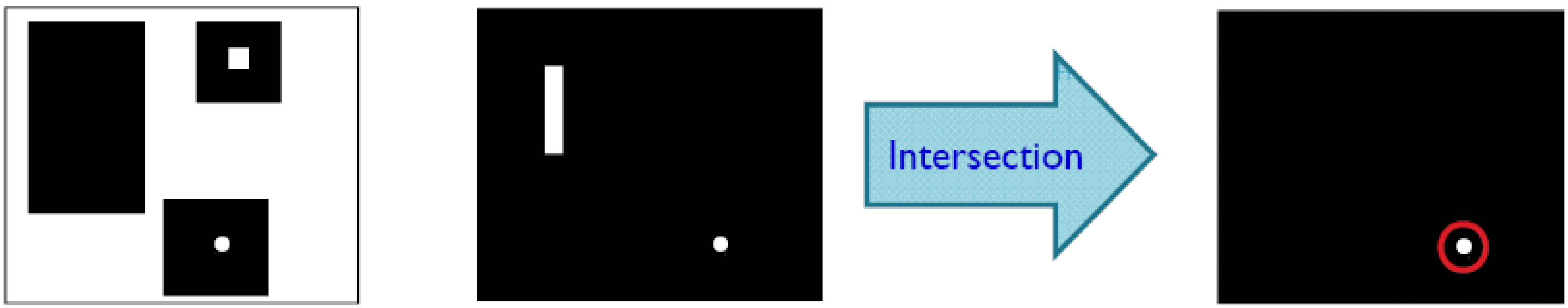
- Consider the complement of A and a new structuring element that is one pixel thicker than the object



- Note how the center of object D was detected again !

Hit-or-Miss Transformation

- Consider the intersection of the erosion results from the previous two slides



- Object is successfully located !
- Formally, the hit-or-miss transformation on image A to detect some object X, is performed by

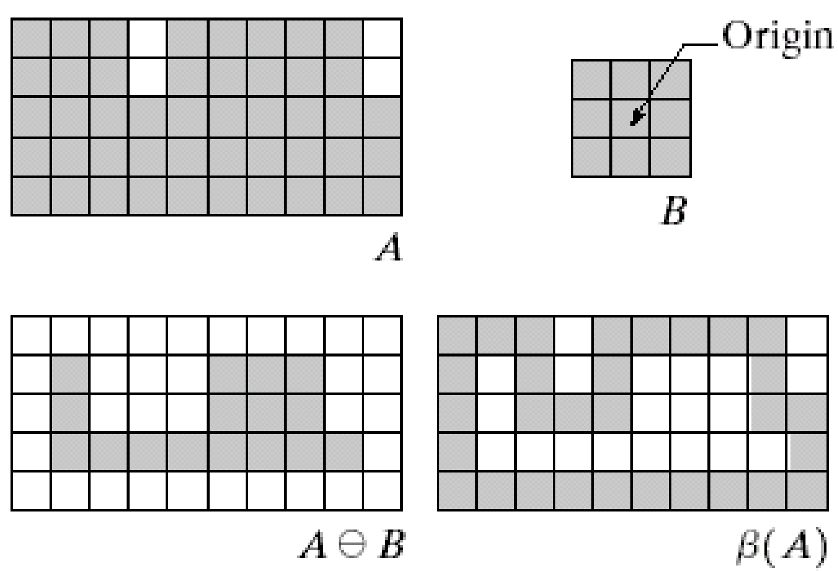
$$A \circledast X = (A \ominus X) \cap [Ac \ominus (W - X)]$$

where W is a the object X thickened by one pixel

Boundary Extraction

- The boundary of set A denoted by $\beta(A)$ can be obtained by first eroding A by a suitable structuring element B then perform the set difference between A and its erosion

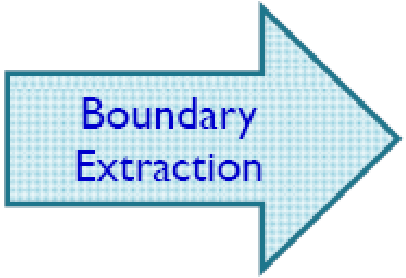
$$\beta(A) = A - (A \ominus B)$$



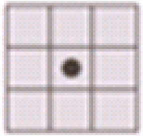
Boundary Extraction

- **Example**

Original binary image



Boundary detected based on morphological operators



B

Structuring element

Hole Filling

- A hole may be defined as a background region surrounded by a connected border of foreground pixels
- The algorithm presented here assumes that we know one pixel for each hole in the image
- **Algorithm**
  - Form an array X_0 of the same size as A and initialize it with zeros except locations that correspond to pixels inside the regions to be filled
  - Apply the following equation iteratively on array X until $X_k = X_{k-1}$ to form the filled holes

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

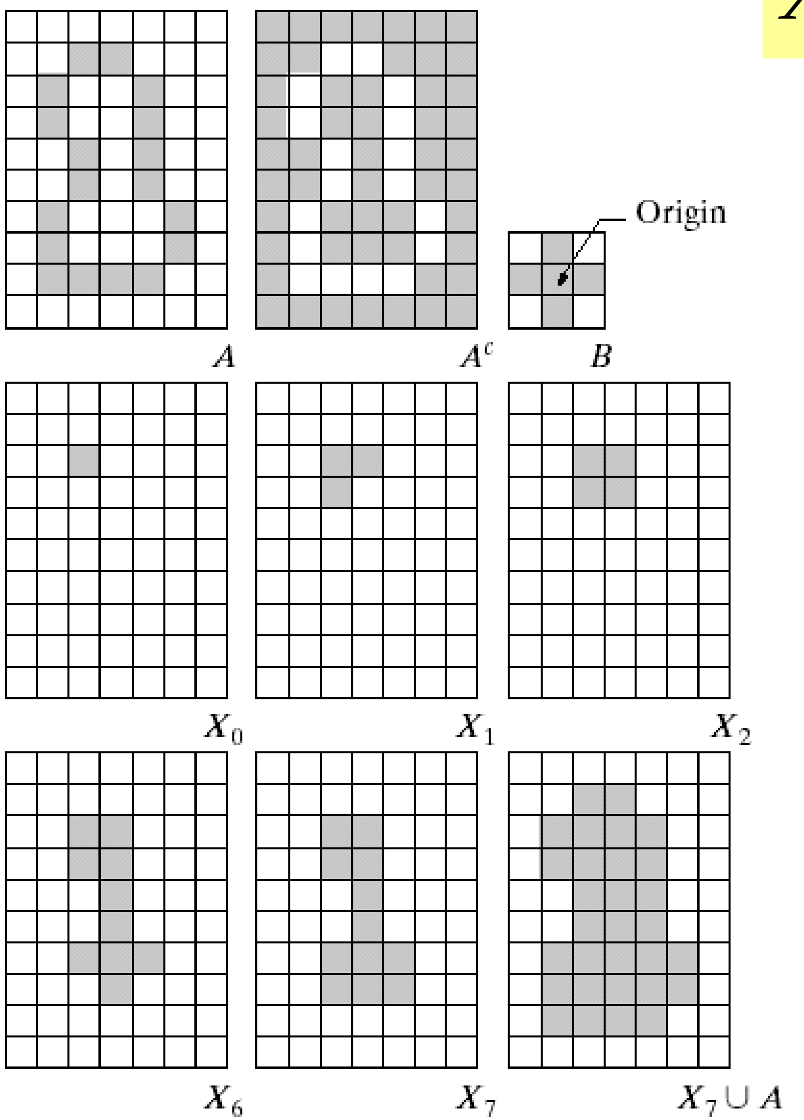
  - The original image with filled holes is found by

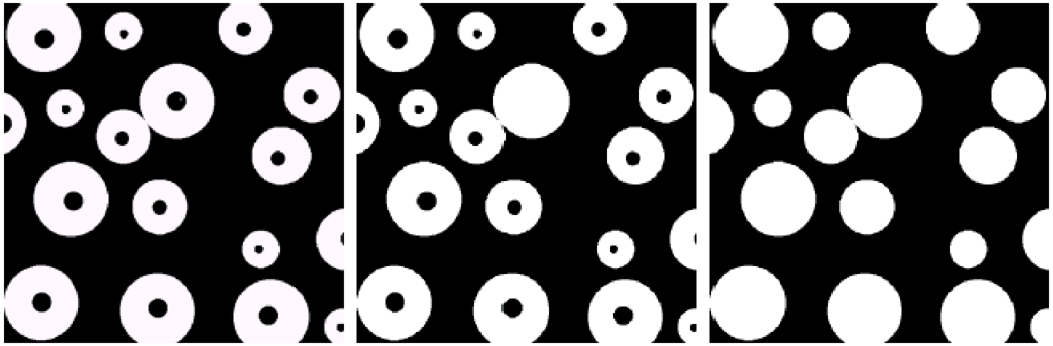
$$A_{filled} = X_k \cup A$$

Hole Filling

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c$$

where X_0 = seed pixel p





Original image

Results of region filling

Connected Components

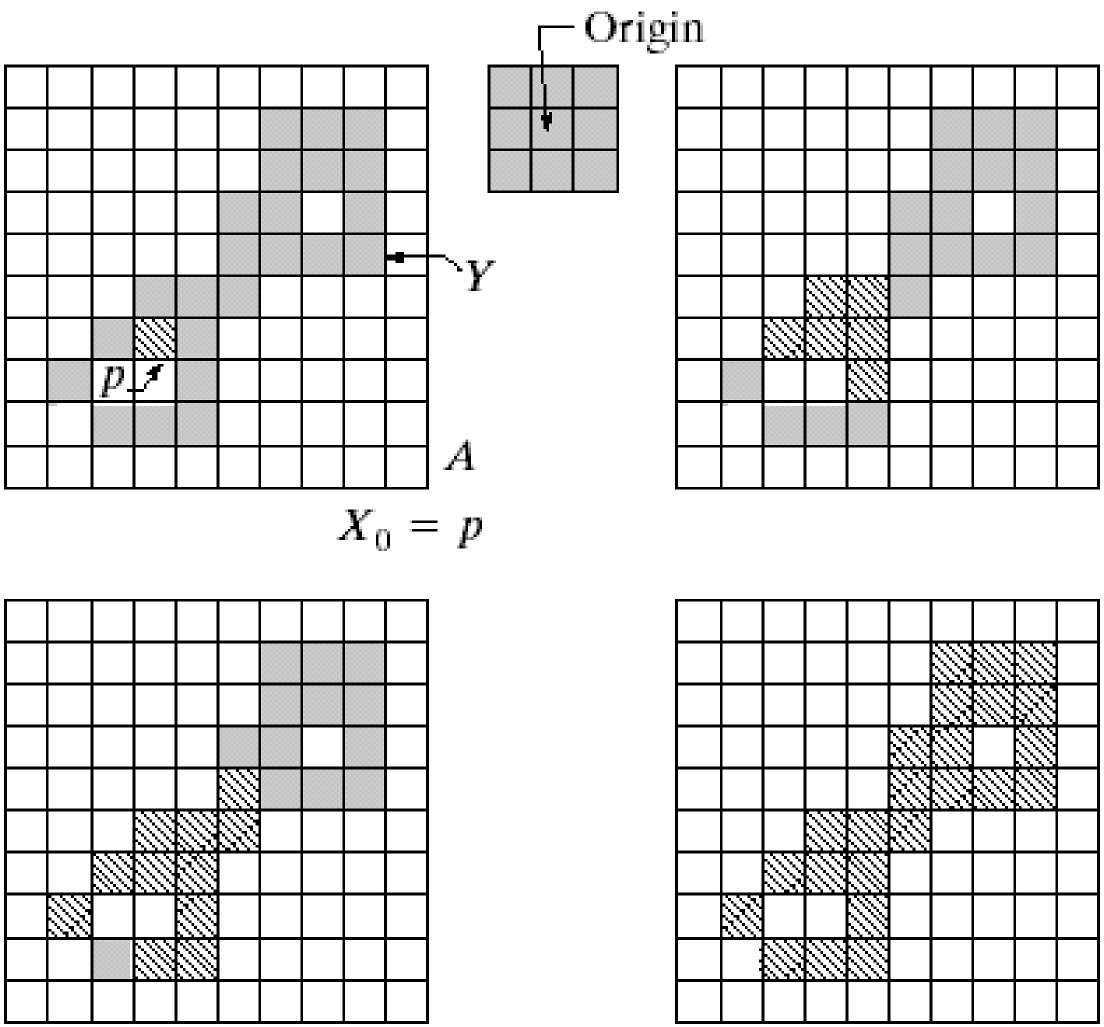
- Let Y represent a connected component contained in a set A
  - Assume that a point p of Y is known
  - Then the following yields all the element of

$$X_k = (X_{k-1} \oplus B) \bigcap A, \quad k = 1,2,3,\ldots$$

    - where X_0=p, B is the suitable structuring element
    - The algorithm terminates if X_k=X_{k-1}, and let Y=X_k

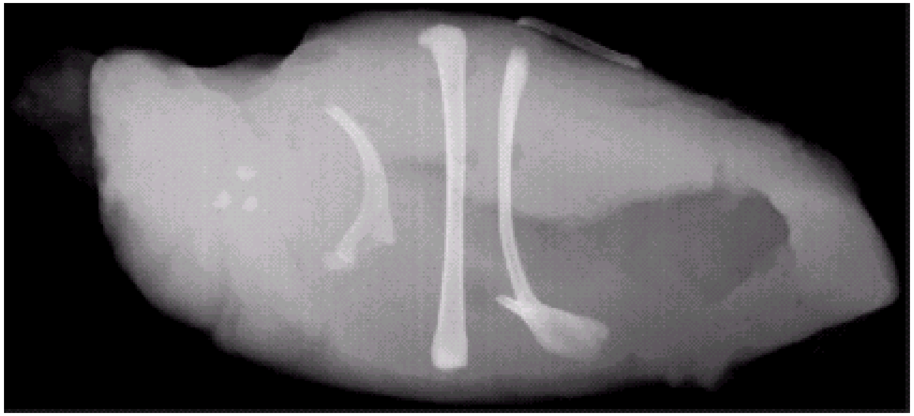
Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A$$ where X_0 = seed pixel p



Connected Components

X-ray image of bones



Thresholded image

Connected components

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- A set A is said to be convex
  - if the straight line segment joining any two points in A lies entirely within A
- The convex hull H(=C(S)) of an arbitrary set S is the smallest convex set containing S
- The set difference H-S is called the convex deficiency of S
- Morphological algorithm for obtaining the convex hull C(A) of a set A

➡ Let B^i , i=1,2,3,4, representing the four structuring elements

➡ $X_k^i = (X_{k-1}^i \circledast B^i) \cup A, \quad i = 1,2,3,4 \; and \; k = 1,2,3,...$

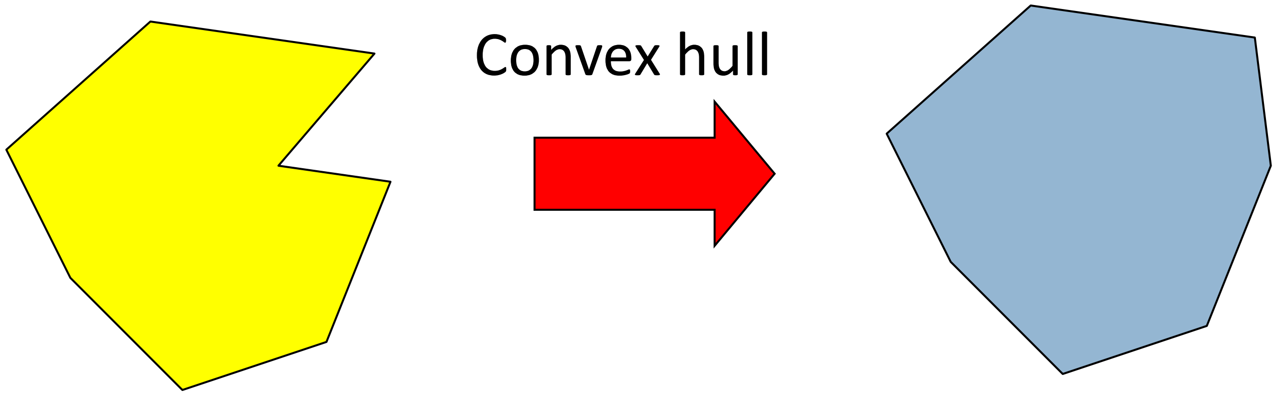
with $X_0^i = A$

➡ When the results converge to D^i , the convex hull of A is given by

$$C(A) = \bigcup_{i=1}^{4} D^i$$

Convex Hull

Convex hull has no concave part.



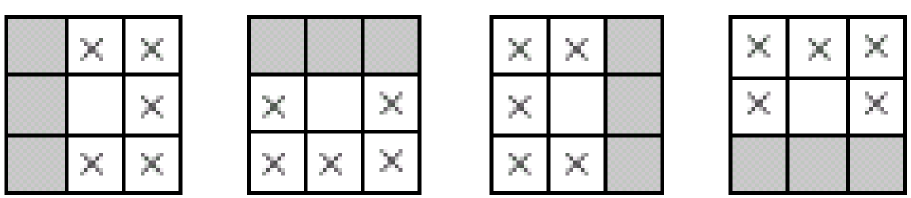
Algorithm:

$$C(A) = \bigcup_{i=1}^{4} D^i$$

where

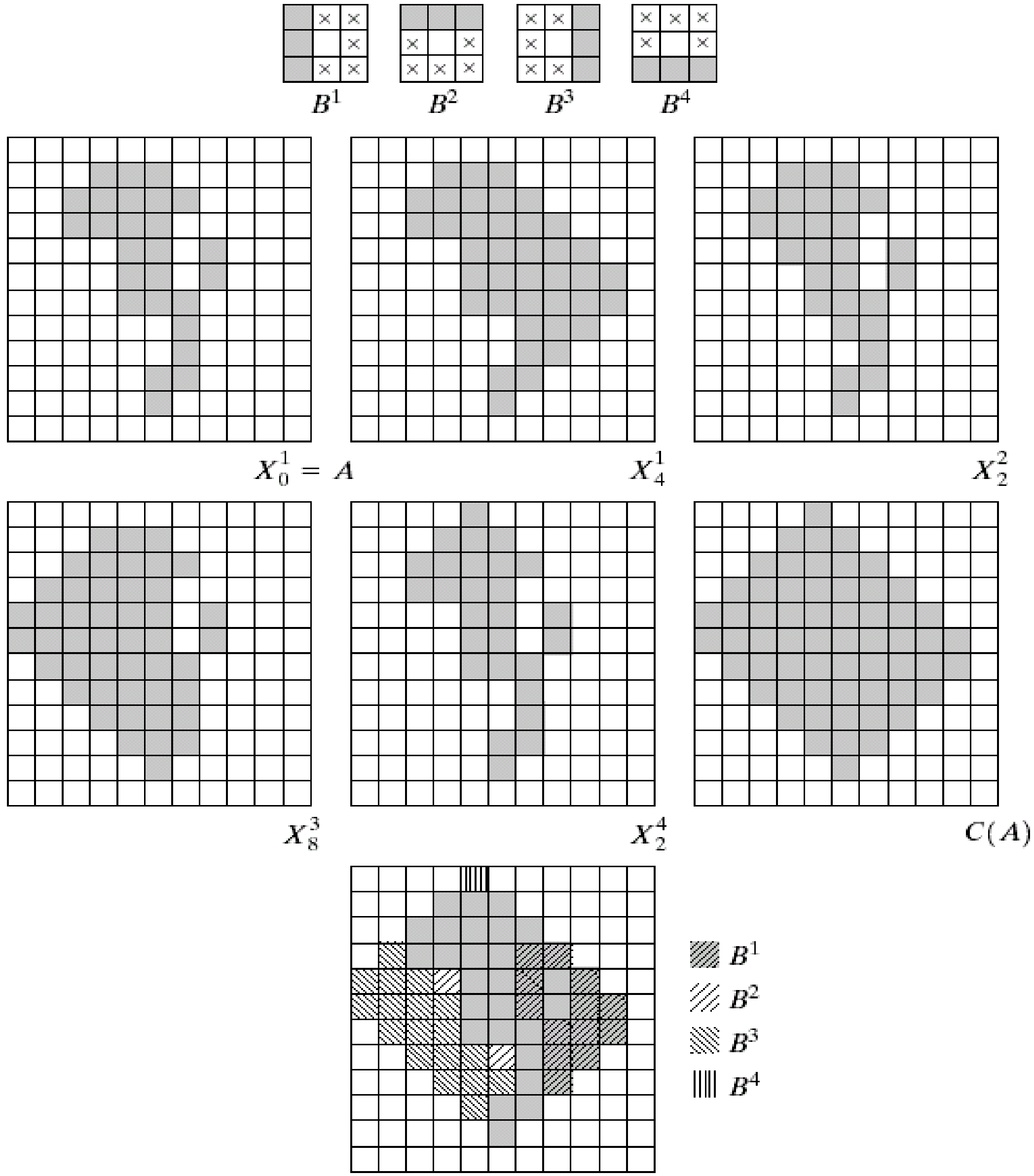
$$D^i = X^i_{conv}$$

$$X^i_k = \left(X_{k-1} \circledast B^i\right) \cup A, \qquad i = 1,2,3,4$$



B^1 B^2 B^3 B^4

Convex Hull



Thinning

- The thinning of a set A by a structuring element B

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

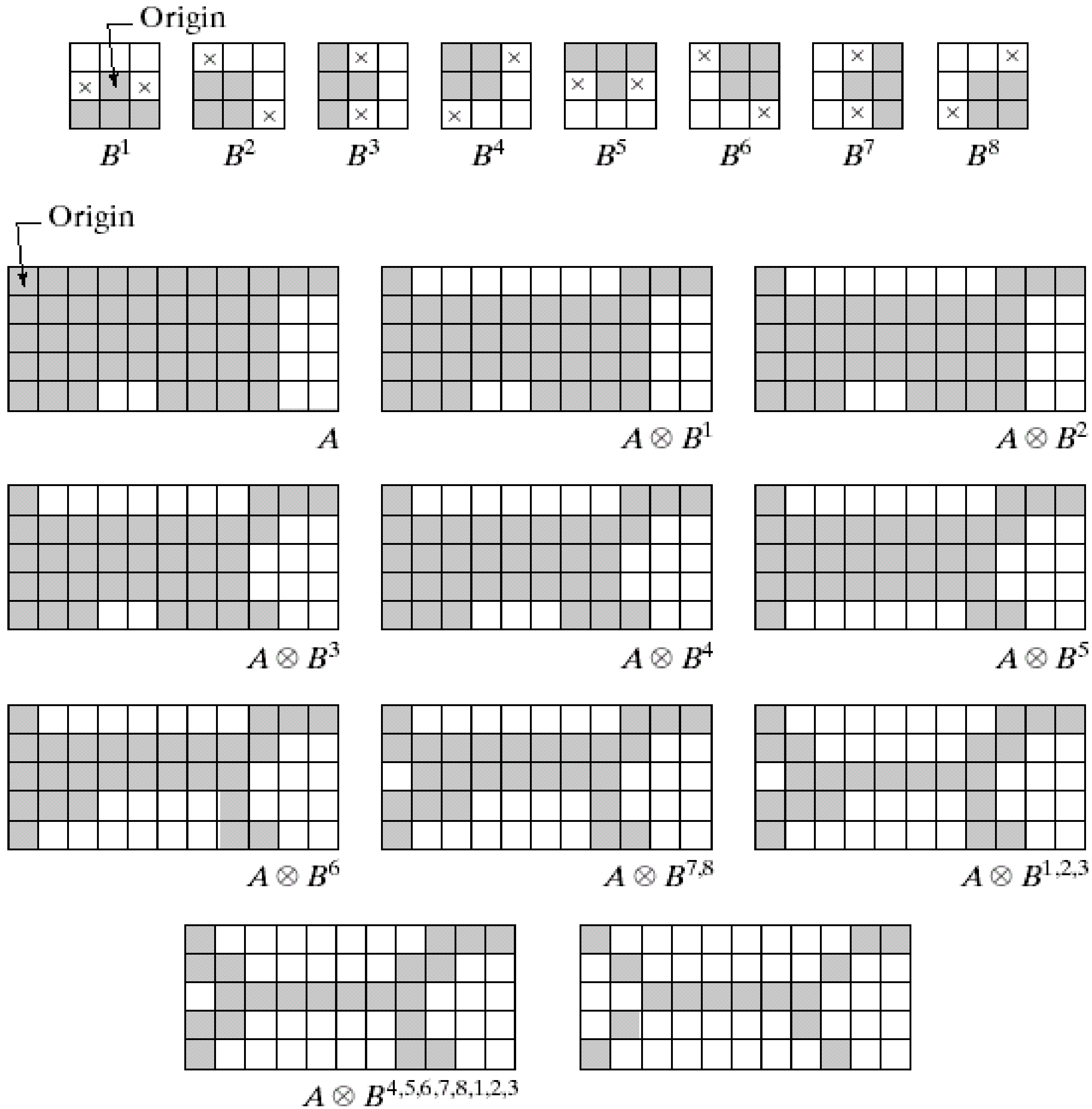
- A more useful expression for thinning based on a sequence of structuring elements

$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

- where B^i is a rotated version of B^{i-1}

$$A \otimes \{B\} = ((\dots((A \otimes B^1) \otimes B^2)\dots) \otimes B^n)$$

Thinning


Origin
B^1
B^2
B^3
B^4
B^5
B^6
B^7
B^8
Origin
A
$A \otimes B^1$
$A \otimes B^2$
$A \otimes B^3$
$A \otimes B^4$
$A \otimes B^5$
$A \otimes B^6$
$A \otimes B^{7,8}$
$A \otimes B^{1,2,3}$
$A \otimes B^{4,5,6,7,8,1,2,3}$

Thickening

- Thickening is the morphological dual of thinning

$$A \odot B = A \cup (A \circledast B)$$

- As in thinning, thickening can be defined as a sequential operation

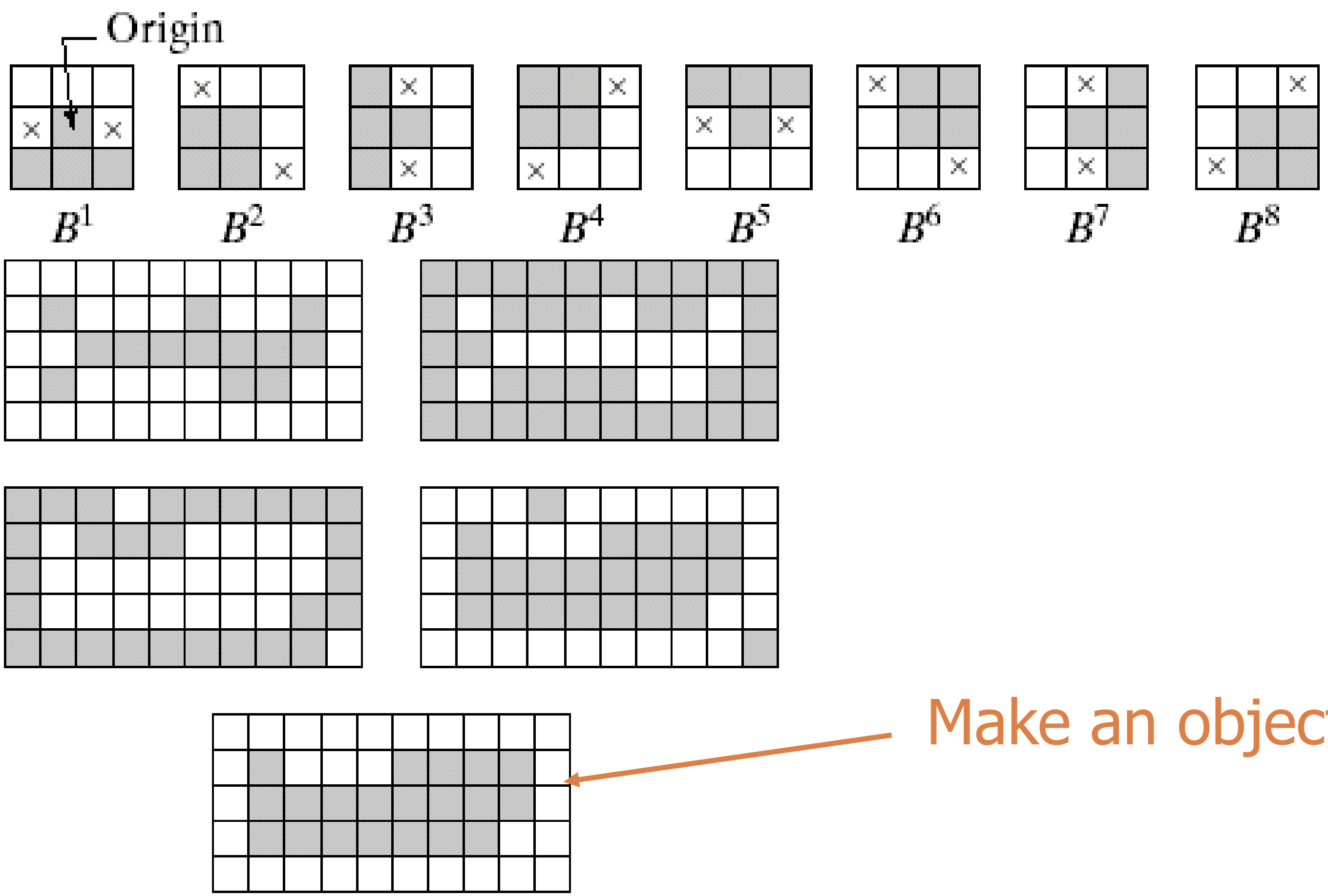
$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

- The structuring elements used for thickening have the same form in thinning, but with all 1's and 0's interchanged
- In general, thickening is accomplished by thinning the background and then taking complement of the result
- The thinned background forms a boundary for the thickening process

Thickening

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$



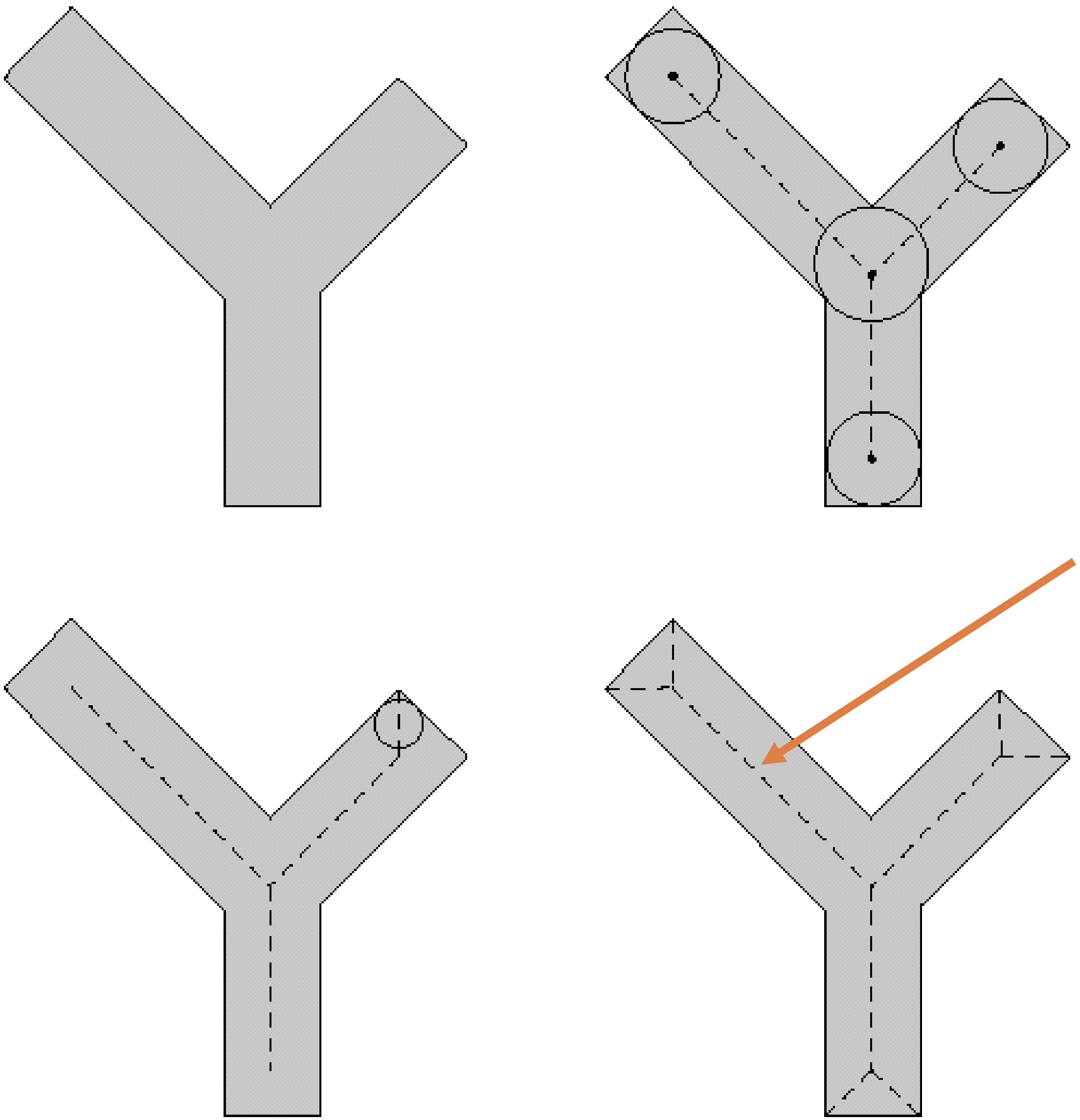
Make an object thicker

Skeletons

- Skeleton, S(A), of a set A
  1. If z is a point of S(A) and (D)z is the largest disk centered at z and contained in A, one cannot find a larger disk containing (D)z and included in A. The disk (D)z is called a maximum disk
  2. The disk (D)z touches the boundary of A at two or more different places

- An inner point belongs to the skeleton if it has at least two closest boundary points

Skeletons



Dot lines are skeletons of this structure

Skeletons

- Morphological Skeleton $$S(A)=\bigcup_{k=0}^{K} S_k(A)$$

  where $$S_k(A)=(A\ominus kB)-(A\ominus kB)\circ B$$
  $$(A\ominus kB)=(\ldots(A\ominus B)\ominus B)\ominus\ldots)\ominus B$$

  k successive erosions
- And K is the last iterative step before A erodes to an empty set
- The set A can be reconstructed by
  $$A=\bigcup_{k=0}^{K}(S_k(A)\oplus kB)$$

  where k successive dilations
  $$(S_k(A)\oplus kB)=((\ldots(S_k(A)\oplus B)\oplus B)\oplus\ldots)\oplus B$$