جامعة بيرزيت

BIRZEIT UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

COMPUTER SCIENCE DEPARTMENT

COMP233

Discrete Mathematics

CHAPTER 2

THE LOGIC OF COMPOUND STATEMENTS

Outline

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Arguments

- If we have the statement:

If today is Friday, then today is a vacation day.

Today is Friday.

∴ Today is a vacation day.

- We can abstract these statements to:

If p then q
p
$\therefore q$

- This is called an argument.

Terminology

- An argument: a sequence of statements.
- An argument form: a sequence of statement forms.

- Premises (assumptions, hypotheses): all the statements in an argument except the final one.

- Conclusion: The final statement of an argument.

Valid Arguments

- A valid argument is an argument where if all the premises are true, then the conclusion is also true.

- So, to test the validity of the argument we need to do the following:

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. Check the **critical rows**.

Valid Arguments – cont.

If today is Friday, then today is a vacation day. $p \longrightarrow q$

Today is Friday. p

∴ Today is a vacation day. q

p: today is Friday

q: today is a vacation day

		premises		hypothesis
p	q	$p \longrightarrow q$	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

A critical row is a row where all the premises are true.

Valid Arguments – cont.

- A row of the truth table in which all the premises are true is called a **critical row**.

- If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid.

- If the conclusion in ***every*** critical row is true, then the argument form is valid.

Determining Validity or Invalidity

$p \rightarrow q \vee \sim r$
$q \rightarrow p \wedge r$
$\therefore p \rightarrow r$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

Determining Validity or Invalidity

$p \rightarrow q \vee \sim r$
$q \rightarrow p \wedge r$
$\therefore p \rightarrow r$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T			
T	T	F	T	T	F			
T	F	T	F	F	T			
T	F	F	T	T	F			
F	T	T	F	T	F			
F	T	F	T	T	F			
F	F	T	F	F	F			
F	F	F	T	T	F			

Determining Validity or Invalidity

$p \to q \vee \sim r$
$q \to p \wedge r$
$\therefore p \to r$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	
F	F	F	T	T	F	T	T	

Determining Validity or Invalidity

$p \rightarrow q \vee \sim r$
$q \rightarrow p \wedge r$
$\therefore p \rightarrow r$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Determining Validity or Invalidity

$p \rightarrow q \vee \sim r$
$q \rightarrow p \wedge r$
$\therefore p \rightarrow r$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

RULES OF INFERENCING

1. Modus Ponens

An argument consisting of two premises and a conclusion is called syllogism.

The first premise is called the major premise, and the second premise is called the minor premise.

This particular argument form:

If p then q
p
$\therefore q$

is called modus ponens, meaning method of affirming (the conclusion is an affirmation).

The corresponding tautology of modus ponens:

$$((p \longrightarrow q) \wedge p) \longrightarrow q$$

1. Modus Ponens – cont.

If Socrates is a man, then Socrates is mortal.

Socrates is a man

∴ Socrates is mortal.

2. Modus Tollens

- an argument form:

If p then q
$\sim q$
$\therefore\ \sim p$

is called modus tollens, meaning method of denying (the conclusion is a denial), and it is used for proof by contradiction.

- The corresponding tautology of modus tollens:

$$((p \longrightarrow q) \land \sim q) \longrightarrow \sim p$$

2. Modus Tollens – cont.

If Zeus is human, the Zeus is mortal.

Zeus is not mortal.

∴ Zeus is not human.

Because:

- if Zeus were human then by (1) he would be mortal,
- but by (2) he is not mortal; hence, he cannot be human.

3. Generalization

- The following argument forms are valid:

p
$\therefore p \vee q$

and

q
$\therefore p \vee q$

Today is Friday
∴ Today is Friday or it will be sunny tomorrow

This can be any statement I need it to be

4. Specialization (conjunction elimination)

- The following argument forms are valid:

$p \wedge q$
$\therefore p$

Today is Tuesday and it is sunny
∴ Today is Tuesday

and

$p \wedge q$
$\therefore q$

Today is Tuesday and it is sunny
∴Today is sunny

5. Elimination (disjunction elimination)

$p \vee q$
$\sim q$
$\therefore p$

and

$p \vee q$
$\sim p$
$\therefore q$

Today is Tuesday or today is Thursday
Today is not Tuesday
∴ Today is Thursday

6. Transitivity (Chain arguments)

$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$

If 18,486 is divisible by 18, then 18,486 is divisible by 9.

If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

∴ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

7. Division into Cases

$p \vee q$
$p \to r$
$q \to r$
$\therefore r$

You are either a CS student or a CSE students.

If you are a CS student, then you have to take COMP233.

If you are a CSE student, then you have to take COMP233.

∴ You have to take COMP233.

A Complex Example

a) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

b) If my glasses are on the kitchen table, then I saw them at breakfast.

c) I did not see my glasses at breakfast.

d) I was reading the newspaper in the living room, or I was reading the newspaper in the kitchen.

e) If I was reading the newspaper in the living room, then my glasses are on the coffee table.

- Where are the glasses?

A Complex Example – cont.

- Let

RK = I was reading the newspaper in the kitchen.

GK = My glasses are on the kitchen table.

SB = I saw my glasses at breakfast.

RL = I was reading the newspaper in the living room.

GC = My glasses are on the coffee table.

A Complex Example – cont.

1. $RK \to GK$ *by (a)*

   $GK \to SB$ *by (d)*

   $\therefore RK \to SB$ *by transitivity*

2. $RK \to SB$ *by (1)*

   $\sim SB$ *by (c)*

   $\therefore \sim RK$ *by modus tollens*

3. $RL \vee RK$ *by (d)*

   $\sim RK$ *by (2)*

   $\therefore RL$ *by elimination*

4. $RL \to GC$ *by (c)*

   RL *by (3)*

   $\therefore GC$ *by modus ponens*

∴ my glasses are on the coffee table.

Exercise (do it at home)

- The famous detective Percule Hoirot was called in to solve a murder mystery. He determined the following facts:

a) Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick

b) Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.

c) If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.

d) If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.

e) If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.

f) If Sara was not in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

- Assume that there was only one cause of death.
- Who killed Lord Hazelton?

Valid Argument Forms – Summary

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \lor q$ $\sim q$ $\therefore p$ **b.** $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \lor q$ **b.** q $\therefore p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \land q$ $\therefore p$ **b.** $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

Note on the Contradiction Rule

$$\sim p \longrightarrow \mathbf{c}$$
$$\therefore p$$

If you can show that the supposition that statement p is false leads logically to a contradiction, then you can conclude that p is true.

In other words;

If an assumption leads to a contradiction, then that assumption must be false.