جامعة بيرزيت
BIRZEIT UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

COMPUTER SCIENCE DEPARTMENT

COMP233
Discrete Mathematics

CHAPTER 9

Counting and Probability

Counting and Probability

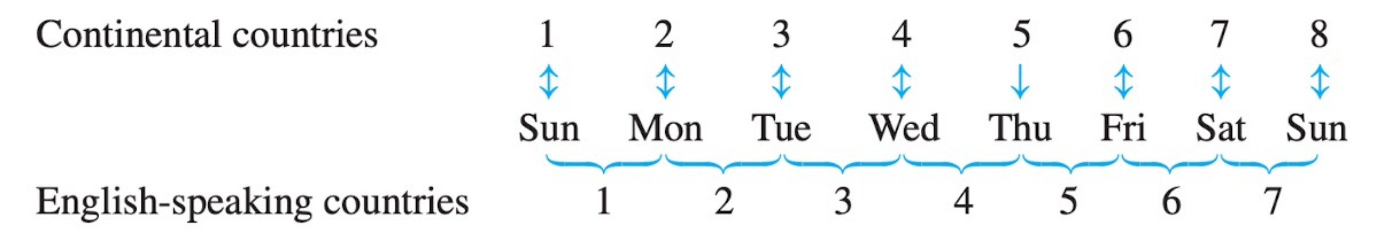
Counting and Probability

Motivation

"It's as easy as 1–2–3."

But is counting as easy as it sounds?

- What does it mean if you tell your friend "We will meet in three days"?
- Counting is different in different countries.



- The issue is not how to count but rather what to count.

Introduction to Counting and Probability

- Introduction
- Counting the Elements of a List

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- Counting the Elements of a List

Introduction

- Imagine tossing two coins and observing whether 0, 1, or 2 heads are obtained.
- What would your guess be for how often each of these events occur?

0 heads

1 head

1/3 chance of each of the events? No.

2 heads

Introduction

- Imagine tossing two coins and observing whether 0, 1, or 2 heads are obtained.
- What would your guess be for how often each of these events occur?

0 heads

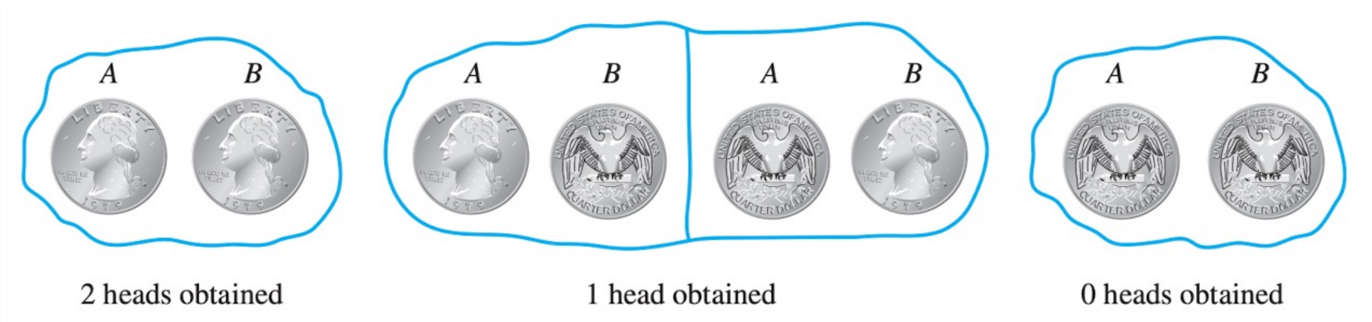
1 head

2 heads

There are two ***different*** events that would produce one head.

Introduction

- Imagine tossing two coins and observing whether 0, 1, or 2 heads are obtained.
- What would your guess be for how often each of these events occur?



There is a 1 in 4 chance of obtaining two heads and a 1 in 4 chance of obtaining no heads.

The chance of obtaining one head, however, is 2 in 4 because either A could come up heads and B tails or B could come up heads and A tails.

Introduction

- Imagine tossing two coins and observing whether 0, 1, or 2 heads are obtained.
- In practice, the frequency of each event might look like this table:

Event	Tally	Frequency (Number of times the event occurred)	Relative Frequency (Fraction of times the event occurred)
2 heads obtained	~~IIII~~ ~~IIII~~ I	11	22%
1 head obtained	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ II	27	54%
0 heads obtained	~~IIII~~ ~~IIII~~ II	12	24%

Randomness

- To say that a process is **random** means that when it takes place, one outcome from some set of outcomes is sure to occur, but it is impossible to predict with certainty which outcome that will be.

- For example, if an ordinary person performs the experiment of tossing an ordinary coin into the air and allowing it to fall flat on the ground, it can be **predicted with certainty** that the coin will land either heads up or tails up, but it is **not known for sure** whether heads or tails will occur.

Sample Spaces and Events

- A sample space is the set of all possible outcomes of a random process or experiment.
- An event is a subset of a sample space.

- For example, tossing a coin twice has the following sample space:

$$S = \{HH, HT, TH, TT\}$$

- The event of obtaining one head is:

$$E = \{HT, TH\}$$

Probability of an Event

- For any finite set A, $N(A)$ denotes the number of elements in A.

- Equally Likely Probability Formula:

If S is a finite sample space in which all outcomes are equally likely and E is an event in S, then the probability of E, denoted $P(E)$, is

$$P(E) = \frac{the\ number\ of\ outcomes\ in\ E}{the\ total\ number\ of\ outcomes\ in\ S} = \frac{N(E)}{N(S)}$$

Probabilities for a Deck of Cards

- Terminology:

An ordinary deck of cards contains 52 cards divided into four ***suits***.

The red suits are ***diamonds*** (♦) and ***hearts*** (♥) and the black suits are ***clubs*** (♣) and ***spades*** (♠).

Each suit contains 13 cards of the following denominations: 2, 3, 4, 5, 6, 7, 8, 9, 10, J (jack), Q (queen), K (king), and A (ace).

The cards J, Q, and K are called ***face cards***.

Probabilities for a Deck of Cards

- Imagine that the cards in a deck have become so thoroughly mixed up that if you spread them out face down and pick one at random, you are as likely to get any one card as any other.

a. What is the sample space of outcomes?

The outcomes in the sample space S are the 52 cards in the deck.

b. What is the event that the chosen card is a black face card?

$$E = \{J\clubsuit, Q\clubsuit, K\clubsuit, J\spadesuit, Q\spadesuit, K\spadesuit\}$$

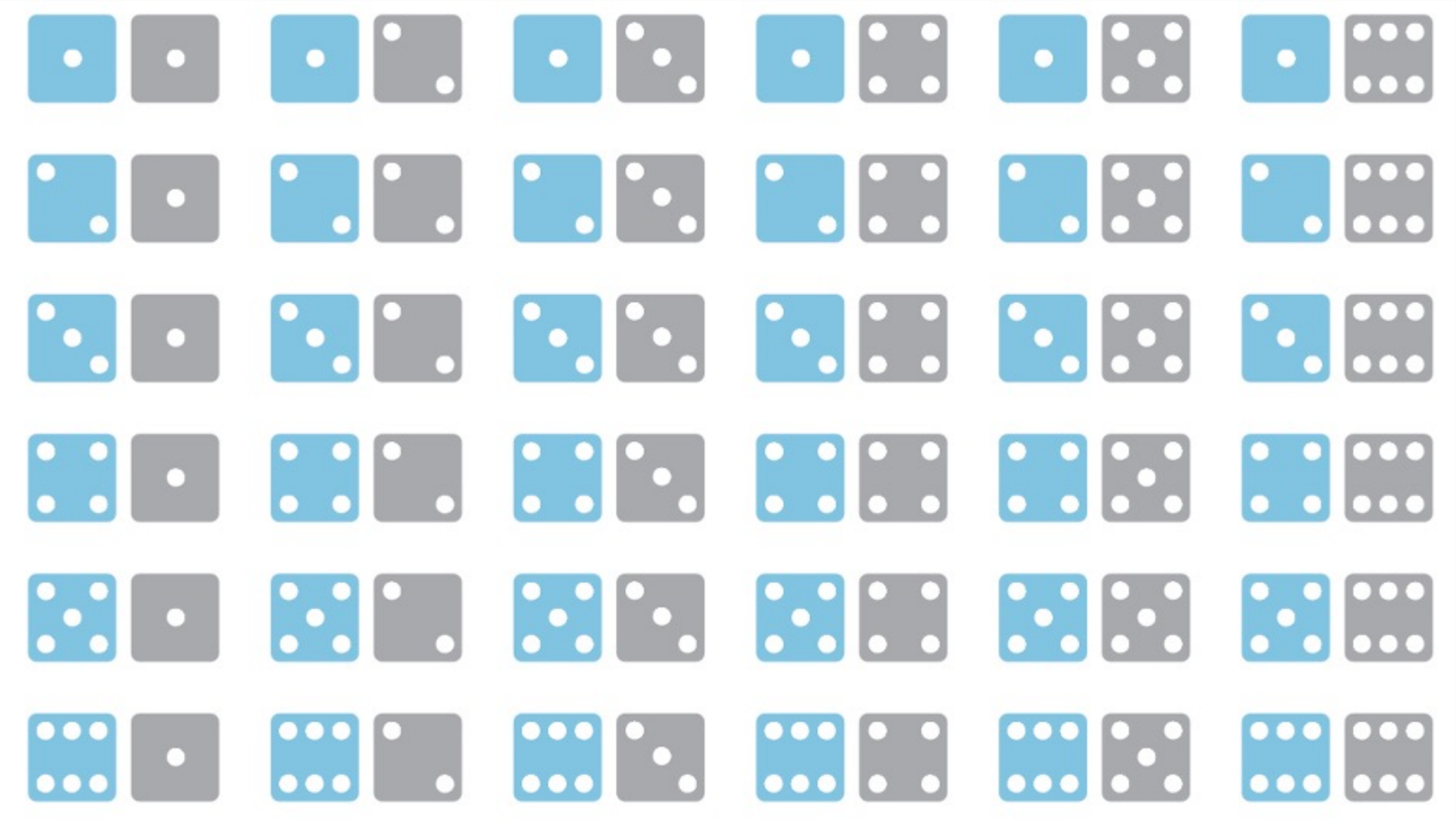
c. What is the probability that the chosen card is a black face card?

$$P(E) = \frac{N(E)}{N(S)} = \frac{6}{52} \cong 11.5\%$$

Rolling a Pair of Dice

- A die is one of a pair of dice.
- It is a cube with six sides, each containing from one to six dots, called pips.
- Suppose a blue die and a gray die are rolled together, and the number of dots that occur face up on each are recorded. The possible outcomes can be listed next, where in each case the die on the left is blue and the one on the right is gray.

Rolling a Pair of Dice



Rolling a Pair of Dice

- Notation: We usually use the notation 24 to refer to this combination 

And the notation 53 to refer to this combination  and so forth.

a. Use the compact notation to write the sample space S of possible outcomes.

$$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$$

b. Use set notation to write the event E that the numbers showing face up have a sum of 6

$$E = \{15, 24, 33, 42, 51\}$$

c. Find the probability of the event E.

$$P(E) = \frac{N(E)}{N(S)} = \frac{5}{36}$$

Introduction to Counting and Probability

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Counting the Elements of a List

- How many integers are there from 5 through 12? 8

list:	5	6	7	8	9	10	11	12
	↕	↕	↕	↕	↕	↕	↕	↕
count:	1	2	3	4	5	6	7	8

- More generally, if m and n are integers and $m \le n$, how many integers are there from m through n? $(n-m)+1$

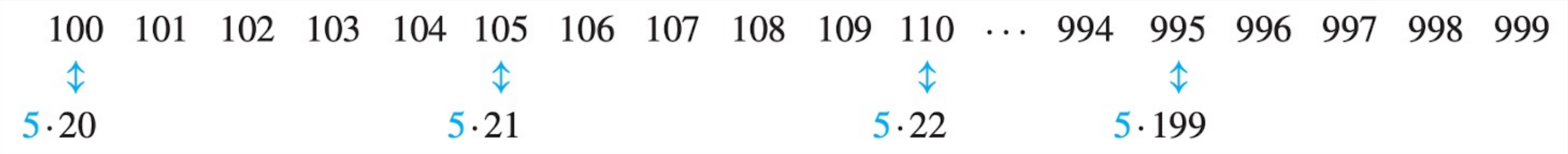
list:	$m (= m + 0)$	$m + 1$	$m + 2$	$\ldots$	$n (= m + (n - m))$
	↕	↕	↕		↕
count:	1	2	3	$\ldots$	$(n - m) + 1$

The Number of Elements in a List

- If m and n are integers and $m \le n$, then there are $n - m + 1$ integers from m to n *inclusive*.

Counting the Elements of a Sublist

- How many three-digit integers (integers from 100 to 999 inclusive) are divisible by 5?



There are as many three-digit integers that are multiples of 5 as there are integers from 20 to 199 inclusive.

According to the previous theorem, the number of integers between 20 and 199 will be

$$199 - 20 + 1 = 180$$

This means that there are 180 three-digit integers that are multiple of 5.

Counting the Elements of a Sublist

- What is the probability that a randomly chosen three-digit integer is divisible by 5?

According to the previous theorem, the total number of integers from 100 through 999 is $999 - 100 + 1 = 900$.

According to the previous question, 180 of these numbers are divisible by 5.

Hence, the probability that a randomly chosen three-digit integer is divisible by 5 is

$$180/900 = 1/5$$