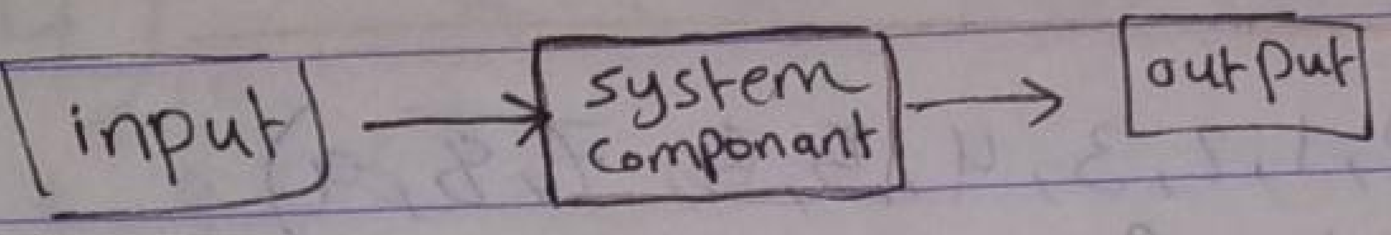
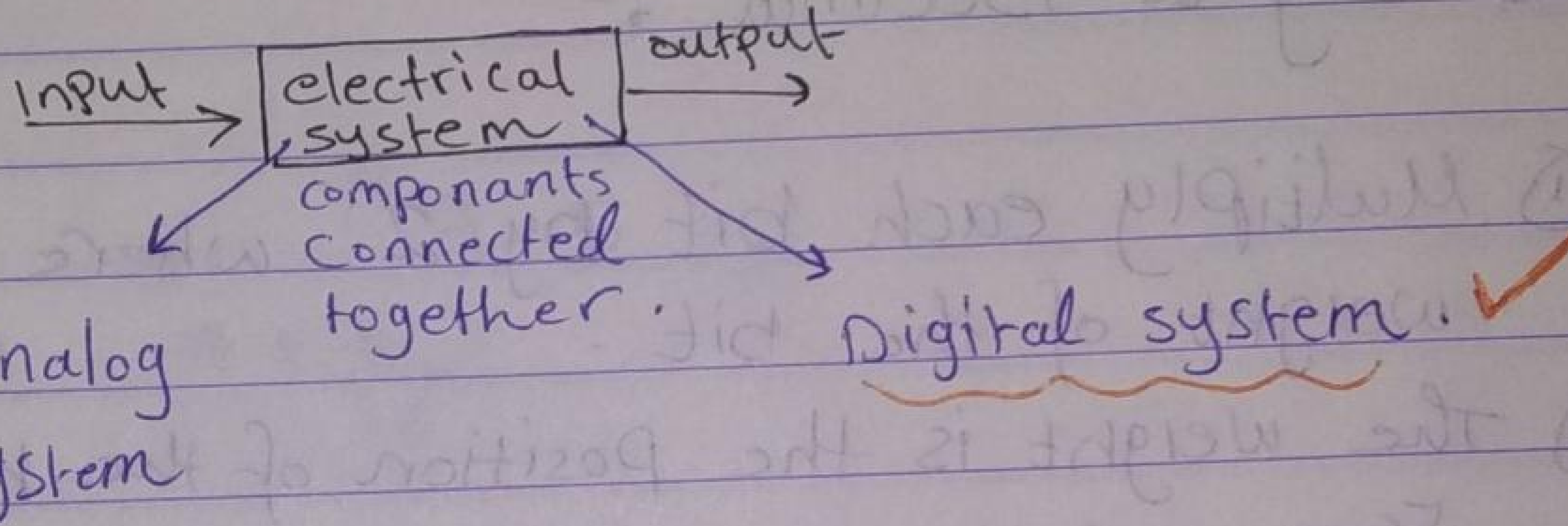
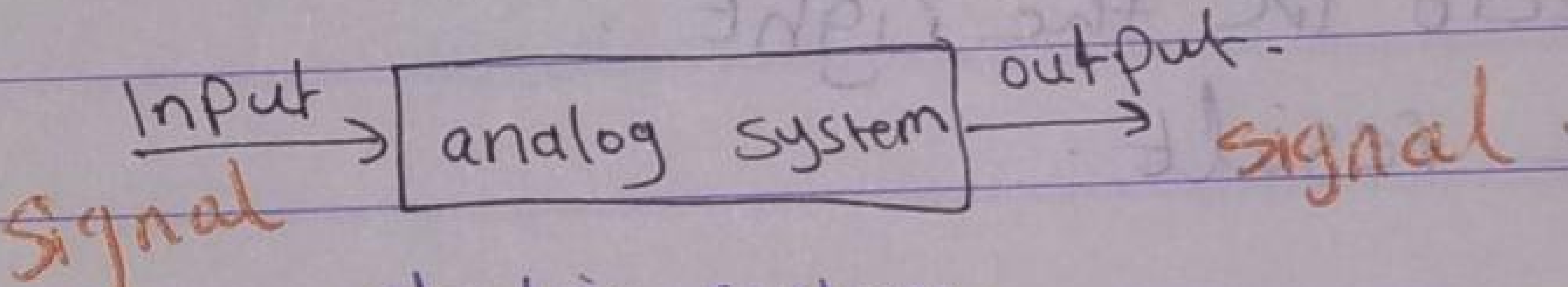
Digital systems :-

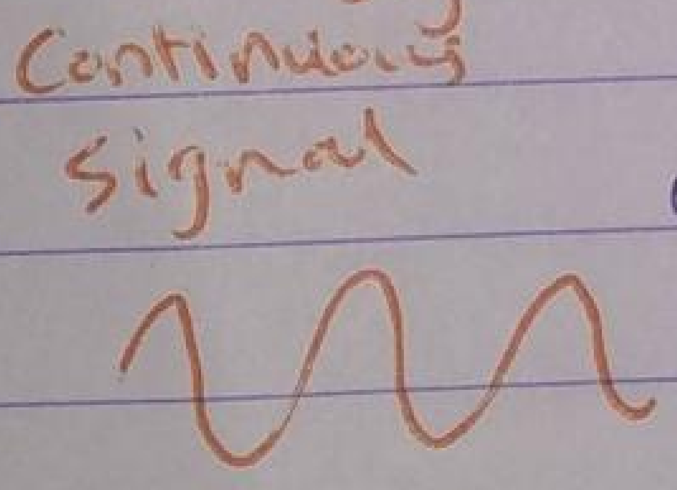
Digital

Systems :- set of things working together as apart of mechanism.

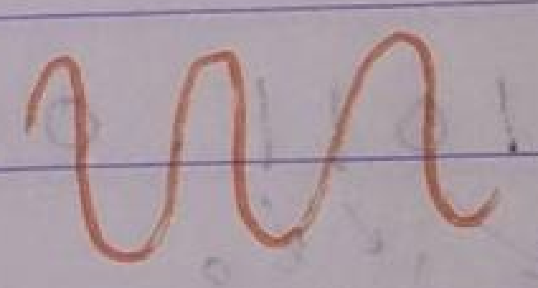




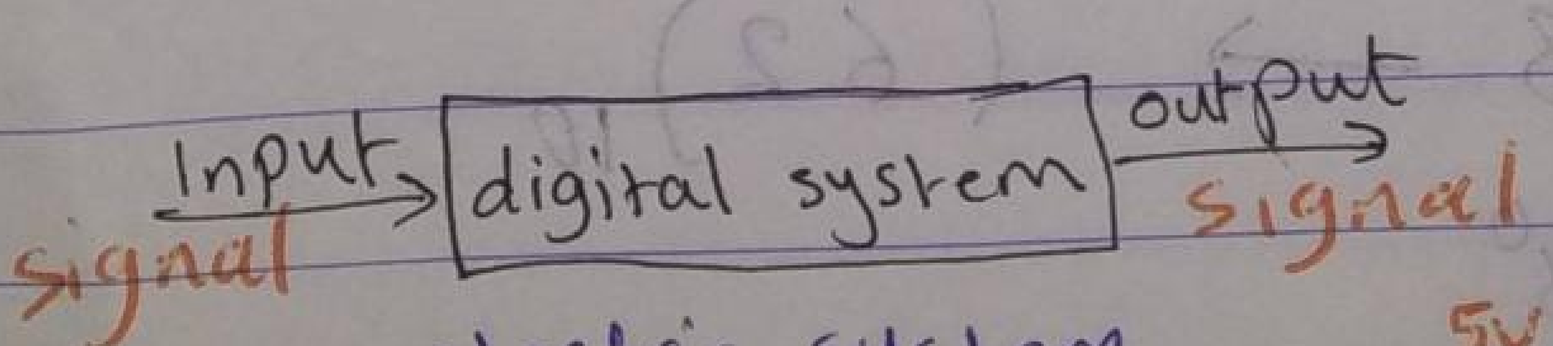


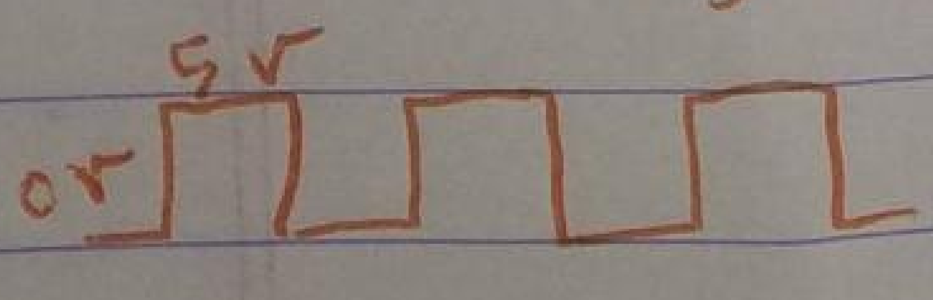


electric system with continous signal variable

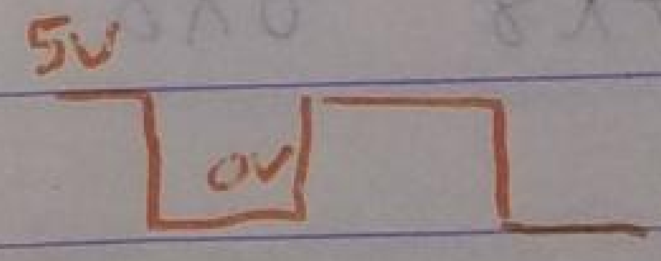


In electronics :- asignal is an or electromagnasim current that is aset for carrying data from one device ~~or~~ to another.

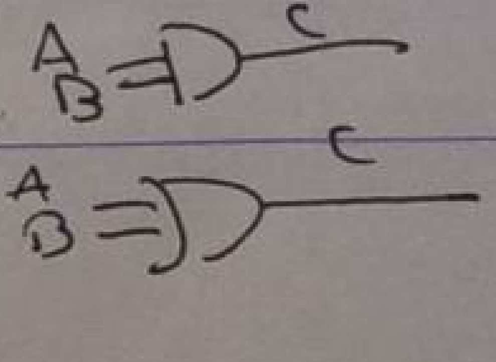




electric system operate using discrete signal



.(بوابات) logic system



Numbering systems.

Decimal system : Base = 10.

There are 10 different digits in the system.

0	$(10)_{10}$	20	- - - 100.
1	$(11)_{10}$	21	
2	$(12)_{10}$		
3			
4			
5			
6			
7			
8			
9			

$$\begin{array}{ccccc} 5 & 7 & 0 & 2 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \end{array}$$

$= 5*10^4 + 7*10^3 + 0*10^2 + 2*10^1 + 1*10^0.$

$= 50000 + 70000 + 0 + 20 + 1 = 57021$

Binary system :- Base = 2.

0	10	100	1000
1	11	101	1001
		110	1010
		111	1011
			1100
			1101
			1110
			1111

Octal system : Base = 8 .

0	(10)8	(20)8	(100)8 .
1	11		
2	12		
3	13		
4	14		
5	15		
6	16		
7	17	. 77	

Hexadecimal (Hex) : Base = 16 .

0	(10)16	(20)16 .	A	1A
1	11			
2	12		B	1B
3	13			
4	14		C	1C
5	15			
6	16		D	1D
7	17		E	1E
8	18			
9	19		F	1F .

معناها 10
بال decimal

exp Base = 4 ·

0	$(10)_4$	$(20)_4$	$(30)_4$	--
1	11	21	..	
2	12	22	..	
3	13	23	..	

التحويل :-

$(1011101)_2$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$1 * 2^6 + 0 * 2^5 + 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 .$

$= 64 + 0 + 16 + 8 + 4 + 0 + 1 = (93)_{10} .$

$(3702)_8 = 3 * 8^3 + 7 * 8^2 + 0 * 8^1 + 2 * 8^0 = (\quad\quad)_{16}$

$8^3 \quad 8^2 \quad 8^1 \quad 8^0$

$(20\text{Ⓐ}4)_{16} = 2 * 16^3 + 0 * 16^2 + 10 * 16^1 + 4 * 16^0 = ($

$16^3 \quad 16^2 \quad 16^1 \quad 16^0$

عند التحويل نضع قيمهن بالأرقام .

Conversion :-

Binary → decimal. ↲

$(\overset{2^4}{1}\overset{2^3}{1}\overset{2^2}{0}\overset{2^1}{1}\overset{2^0}{1})_2 = 2^4 + 2^3 + 0 + 2^1 + 2^0$

$= 16 + 8 + 0 + 2 + 1 = (27)_{10}$.

decimal → Binary.

we devide by 2.

$(38)_{10} \overset{?}{=} (\quad)_2$.

$(38)_{10} = (100110)_2$.

(الباقي)

Reminder	÷
0	38
1	19
1	9
0	4
0	2
1	1
	0

اتجاه الكتابة. ↑

left → 1

Octal → decimal.

$(\overset{8^2}{2}\overset{8^1}{0}\overset{8^0}{5})_8 = 2 * 8^2 + 0 * 8^1 + 5 * 8^0$

$= 128 + 0 +$ [illegible] $+ 5 = (133)_{10}$.

decimal → Octal.

we divide by 8.

$(95)_{10} = (\quad)_8$.

	Reminder	$\div 8$
	7	95
	3	11
left →	1	1
		0

$(95)_{10} = (137)_8$.

Decimal → Hex.

$(93)_{10} = (\quad)_{16}$.

we divide by 16.

	Reminder	$\div 16$
Ⓓ =	13	93
left →	5	5
		0

$(93)_{10} = (5D)_{16}$.

اذا الرقم بنقدر نحوله ل16 (بالحروف) بنعكسو .

Hex → decimal.

$(2A1) = 2 * 16^2 + 10 * 16^1 + 1 * 16^0$.

$16^2 \; 16^1 \; 16^0$

$= 512 + 160 + 1 = (673)_{10}$.

Octal Binary. $\Rightarrow$ $8 = 2^3$

Base = 8 Base = 2

Hex Binary $\Rightarrow$ $16 = 2^4$.

Base = 16 Base = 2

Result :-

① - Each Octal digit is equal 3 binary digit.

② - Each Hex digit is equal 4 binary digit.

Octal	Binary.
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal → Binary.

we replace every octal digit with it's equivilant 3 binary digit

$(5024)_8 = (101000010100)_2$.

$\downarrow$

$(\quad)_{10} = (\quad)_{10}$.

Binary → octal.

- we group from the right every 3 binary digit and score the equivalent octal digit.

$$\underbrace{001}\underbrace{010}\underbrace{100}\underbrace{111}\underbrace{010}.$$

$$(1\ \ 2\ \ 4\ \ 7\ \ 2)_8.$$

Hex → Binary.

- Replace every Hex digit with 4 binary digit.

0	0000	A	1010
1	0001	B	1011
2	0010	C	1100
3	0011	D	1101
4	0100	E	1110
5	0101	F	1111
6	0110		
7	0111		
8	1000		
9	1001		

$$(\underbrace{0011}\underbrace{0010}\underbrace{1111}\underbrace{1010})_2 = (32FA)_{16}.$$

$$(2B9)_{16} = (001010111001)_2.$$

Binary Addition :-

$$\begin{array}{r} {}^{1\,1\,0\,1} \\ 110110 \\ 11110 + \\ \hline 1010100 \end{array} \qquad \begin{array}{r} {}^{1\,0\,1} \\ 111011 \\ 11100 + \\ 11000 \\ \hline 1101111 \end{array}$$

Fraction in Binary :-

Recall : In decimal.

$$\underset{10^1\ 10^0\ 10^{-1}\ 10^{-2}}{52.23} = 5*10^1 + 2*10^0 + 2*10^{-1} + 3*10^{-2}.$$

$$=$$

In Binary :-

$$\underset{2^3\,2^2\,2^1\,2^0\ \ 2^{-1}\,2^{-2}}{1011.11} = 2^3 + 0 + 2^1 + 1 + \underset{\frac{1}{2}}{2^{-1}} + \underset{\frac{1}{4}}{2^{-2}}.$$

$$= 11.75.$$

Fractions in Binary Number

(Floating point Number).

$$\overbrace{1\ 0\ 1\ 1}^{\text{integer part}} \cdot \overbrace{1\ 0\ 1\ 1}^{\text{fraction}}$$

$$\underbrace{2^{3}\ 2^{2}\ 2^{1}\ 2^{0}}_{\Downarrow} \quad \underbrace{2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}}_{\Downarrow}$$

$$2^{3}+2^{1}+2^{0} \cdot 2^{-1}+0+2^{-3}+2^{-4}.$$

$$11 \cdot 0.5+ 0.1250+ 0.0625.$$

$$11.6875.$$

أمثلة :~

Floating point number → Binary.

25 . 375

11001 . 011

	÷ 2
1	25
0	12
0	6
1	3
→ 1	1
	0

	x 2
من هون → 0	0.375
1	$\overset{0}{\cancel{0}}$.75
1	$\overset{0}{\cancel{1}}$.5
	$\overset{0}{\cancel{1}}$.0 → stop

بنعكس

0.0 ↗

١ ١
٣٧٥
٢
———
.٧٥٠ .

25 . 2

$11001.\overline{0011}$

	x 2
→ 0	0.2
0	0.4
1	0.8
1	1.6
	1.2

→ رح يضل يتكرر

25 . 3

$11001.0\overline{1001}$

	x 2
→ 0	0.3
1	0.6
0	1.2
0	0.4
1	0.8
	1.6

→ رح يضل يتكرر

How do Computers Represent Digits ?

CPU :- Central processing Unit . وحدة المعالجة المركزية

↳ Consist of

① ALU : Arithmetic logic Unit . ← Addition + , Sub - , Mult × , div ÷

Comperision

$x > y$

② CU : Control Unit

③ Registers : Memory

Note There is No subtraction operation in Computer

⟹ Subtraction Must be Changed to addition.

Complements :- To change subtraction to addition

= M - N = M + complement N .

الانواع

Complements :- ① Diminshed complement. (r-1) comp
② Radix complement. (r comp).

Diminshed compl :- what the 9's compl of 6. Answer:- 3 (3+6 = 9). (r = 10).

- what is the 9 compl of 66 → (r = 10)

Answer: $99 - \underbrace{66}_{2 \text{ digits}} = 33$.

Given a number N in base r having n digits the (r-1)'s Comp of N is $(r^n - 1)$ N.

ⓔⓧ what is the First comp of 1011 in base 2.

N = 1011 n = 4 r = 2.

$(2^4 - 1) - 1011$

15 - 1011

1111 - 1011 = $\boxed{0100}$.

ⓔⓧ what is the 9's comp of 5555 in base 10

n = 4 r = 10

$(10^4 - 1) - 5555$

9999 - 5555 = 4444

Radix compl :- The r's comp of number N with n digit is defined $(r^n - N)$ For $N \neq 0$.

ⓔⓧ what is 10's comp of 666 in base = 10

$10^3 - 666 = 1000 - 666$

334.

10's complement = 9's comp + 1.

ex What is the 2's compl 1011 in base = 2

2's compl = 1st comp + 1.

0100 + 1. = 0101.

M - N = M + Compl N.

① M > N → end Carry must be discarded.

ex 99 - 79 = 99 + 10's comp $\overline{79}$.

99 + $\underline{21}$ ← $10^2 - 79$.

①
$$\begin{array}{r} 99 \\ +\ 21 \\ \hline \boxed{\cancel{1}}20 \end{array} = 20.$$

ex 222 - 122 = 222 + 10's comp 122.

$10^3 - 122.$

= 878.

$$\begin{array}{r} \overset{1}{2}\overset{1}{2}2 \\ +\ 878 \\ \hline \boxed{\cancel{1}}100 \end{array} = 100$$

ex 1111 - 0111 r = 2.

M > N

1111 + 2's comp 0111

$$\begin{array}{r} 1111 \\ +\ 1001 \\ \hline \cancel{1}1000 \end{array} = 1000.$$

2's comp 0111

1st comp + 1

1000 + 1

1001.

② $M < N$ → No end carry and the result will be negative so to obtained the final answer the r's comp of the summation

ex 79 - 99 = 79 + 10's comp 99.
$M < N$ = 79 + 1 = 80.
final answer = 10's comp 80 = -20.

ex 122 - 222 = 122 + 10's comp 222.
= 122 + 778 ← 10^3 - 222
= 900.

final answer = 10'comp 900 = -100.

ex 0111 - 1111 = 0111 + 2's comp 1111
0111 + 0001 = 1000 1's comp + 1
0000 + 1 = 0001
final answer = 2's comp 1000 = 0111 + 1 = -1000

أسئلة :—

① 2220 - 1220 = 2220 + 10's 1220

↕ 10's comp

2220 + 8780 $10^4 - 1220$

$= \cancel{1}1000 = \boxed{1000}$

② 1220 - 2220

1220 + 10's comp 2220

1220 + 7780 = 9000

final answer = 10's 9000 = $\boxed{-1000}$

③ 10000 - 01111 $10000 + \underbrace{2\text{'s comp } 01111}_{1\text{'st} + 1}$

↕ 2's comp

$$\begin{array}{r} ①10000 \\ +\ 10001 \\ \hline \cancel{1}00001 \end{array} = \boxed{00001}$$

10000 + 1 = 10001

④ 01111 - 10000

01111 + 2's 10000

$$\begin{array}{r} 01111 \\ +\ 10000 \\ \hline 11111 \end{array}$$

$$\begin{array}{r} 01111 \\ +\ 1 \\ \hline 10000 \end{array}$$

final answer = 2's 11111 = 00000 + 1 = $\boxed{-00001}$

Signed Binary Number :-

- There is no sign way in computer.

left most bit → sign.

آخر رقم من الجهة اليسار .

اذا كانت الإشارة (-) الرقم 1 .

اذا كانت الاشارة (+) الرقم 0 .

①

Note :- There is one way to represent the positive numbers.

ex $+9 \rightarrow \underset{\text{للإشارة}}{\underline{0}}\underbrace{1001} \rightarrow 9$ هاد رقم

ex +15 [signed Binary number].

01111

+15 representation by 4 bits? → Overflow.

⇒ I can't

⇒ minimum 5 bits.

مع ال signed number.

ex $+13 \rightarrow 01011$ 5 bits | 4 bits → overflow.

ex Signed number → decimal.

$\underset{+}{\boxed{0}}\underbrace{1110}_{14} \rightarrow +14.$

Carry → unsignd number. (M > N discared).

ex +41

2	41
1	20
0	10
0	5
1	2
0	1
1	0

↑

[0101001].

7 bits.

إذا عن طريق 8bits ←

00101001

$\underline{0}$ = للإشارة.

$\underline{0}$ زيادة.

② There are three ways to represent negative number.

① signed magnitude. -7 → $\underbrace{1}_{-}\underbrace{111}_{-7}$

→ -15 → 11111

$\underline{1}$ = للإشارة.

→ -14 representation (4 bits) → overflow.

signed magnitude. $\underbrace{1\,111}0$ 5 bits.

10110 6 bits.

② First Complement.

→ -7 First Complement 8 bit.

00000111 $\xrightarrow[\text{Comp}]{\text{First}}$ 11111000 → -7

③ Two's Complement.

→ −7 Two Complement.

00000111

11111000 ← First Complement.

$\quad\quad\quad 1+$

11111001 ← Two complement.

اذا حطالك هاد اي رقم ممثل بال decimal

بنقلب الصفر واحد والواحد صفر وبنجمع 1 بيعطينا (-7)

ex $\overset{+}{⓪}11111$ signed magnitude decimal.

$1\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4 = +31$

ex −14 First complement 6 bits.

14 → 001110

1st comp → 110001

2's comp → $\quad 1+$

110010

ex A = −15 B = 25 do the following operations using signed two complement in 6 bits representation.

15 → 001111

2's comp → 110001

25 → 011001

2's comp → 100110

$\quad 1+$

100111

⟹

① A + B   -15 + 25 .

متشابهات $\boxed{1}$ 1 1 0 0 0 1 +   اذا آخر two carry

0 1 1 0 0 1   متشابهين ← no overflow

$\cancel{\boxed{1}}$ 0 0 1 0 1 9 ✓   carry X

② A - B   -15 - 25 .

مختلفات $\boxed{1}$ 1 1 0 0 0 1   Final answer :-

↕ 1 0 0 1 1 1 +   0 1 0 0 1 1 1

overflow. $\boxed{1}$ 0 1 1 0 0 0   1 +

$\boxed{-40}$ = (-) 0 1 0 1 0 0 0

$40 = 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

Binary Cods :-

suppose we have language with 4 symbols

	A	B	C	D
Code	00	01	10	11

ABC
000110

language 8 symbols.

A	B	C	D	0	1	2	3
000	001	010	011	100	101	110	111

ABC 000001010

ASCII Code. 8 bits.

A B - - Z (26).
a b - - z (26)
01 - - - 9 (10)
?! : , ; + - - (100).

162

$128 < 162 < 256$

$2^7 \qquad 2^8$

(13)d → (1101) conversion.

مثلاً ؛ مش لشرط الرقم صح .

(13)d ASCII (0000111100 0011110) له Coding.

BCD Code (Binary coded decimal).

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10-11-12-13-14-15	XXXX [un used).

(10)d → (00010000)BCD

(10)d → $(1010)_2$

(185)d → (000110000101)BCD

Addition with BCD

1 0 0 0
4 → 0 1 0 0
4 + 0 1 0 0
$\underbrace{1\,0\,0\,0}$ valid (لأنه أقل من 10)

1 0 1 0
5 + 0 1 0 1
5 0 1 0 1 +
$\underbrace{1\,0\,1\,0}_{10}$ valid no →

1 1
1 0 1 0
0 1 1 0 + بجمعله 6
$\underbrace{0\,0\,0\,1}_{1}\ \underbrace{0\,0\,0\,0}_{0}$ عشان يصير Valid

1 0 0
4 0 1 0 0
5 + 0 1 0 1 +
$\underbrace{1\,0\,0\,1}_{9}$ valid.

1 1
6 + 0 1 1 0 +
6 0 1 1 0
$\underbrace{1\,1\,0\,0}$ valid no

1 1 0 0
0 1 1 0 ← 6
$\underbrace{0\,0\,0\,1}_{1}\ \underbrace{0\,0\,1\,0}_{2}$

ex BCD

185 → 0001 1000 0101

925 + → 1001 0010 0101

1110

	1011	1011	1010
	0110	0110	0110
00 01	0001	0001	0000
1	1	1	0

ex BCD

99 + → 1001 1001

99 → 1001 1001

198

	1 0011	10010
	0110	0110
1	1001	1000
1	9	8

Other Decimal Code.

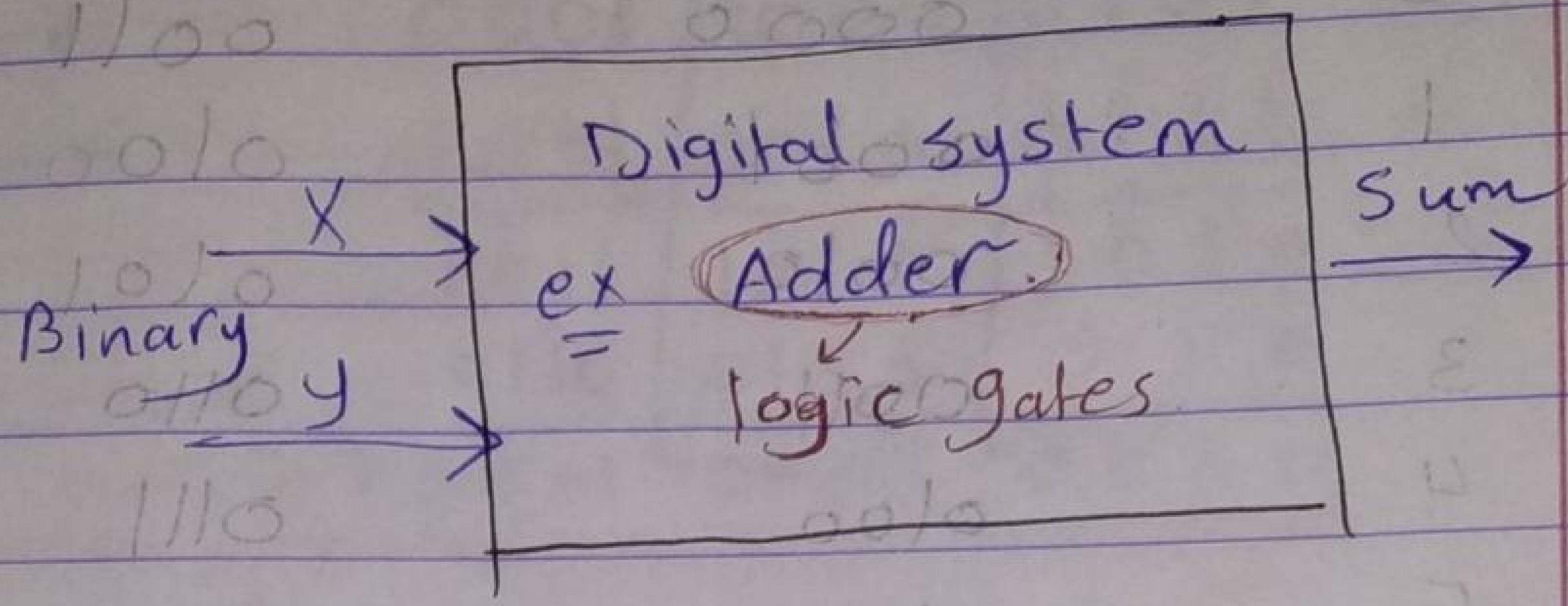
Decimal	BCD	exceis-3
0	0000	0011
1	0001	0100 ← 1+3 = ④
2	0010	0101 ← 2+3 = ⑤
9	1001	1100 ← 12
10	XXXX	1101 ← 13

Gray Code.

0	0000	one بتغير digit.
1	0001	
2	0011	

Binary logic gates

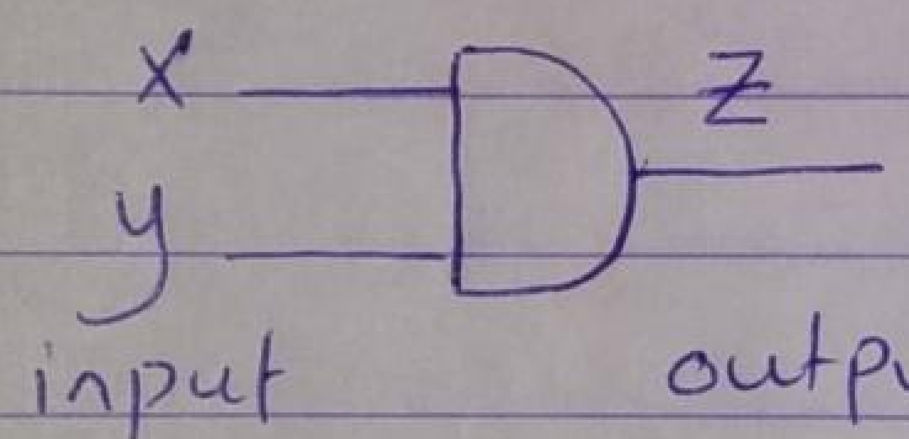
Goal



Fundamental components in

① And gate / has 2 or more inputs.

2 input



$Z = X \text{ AND } y$

$Z = x \cdot y$

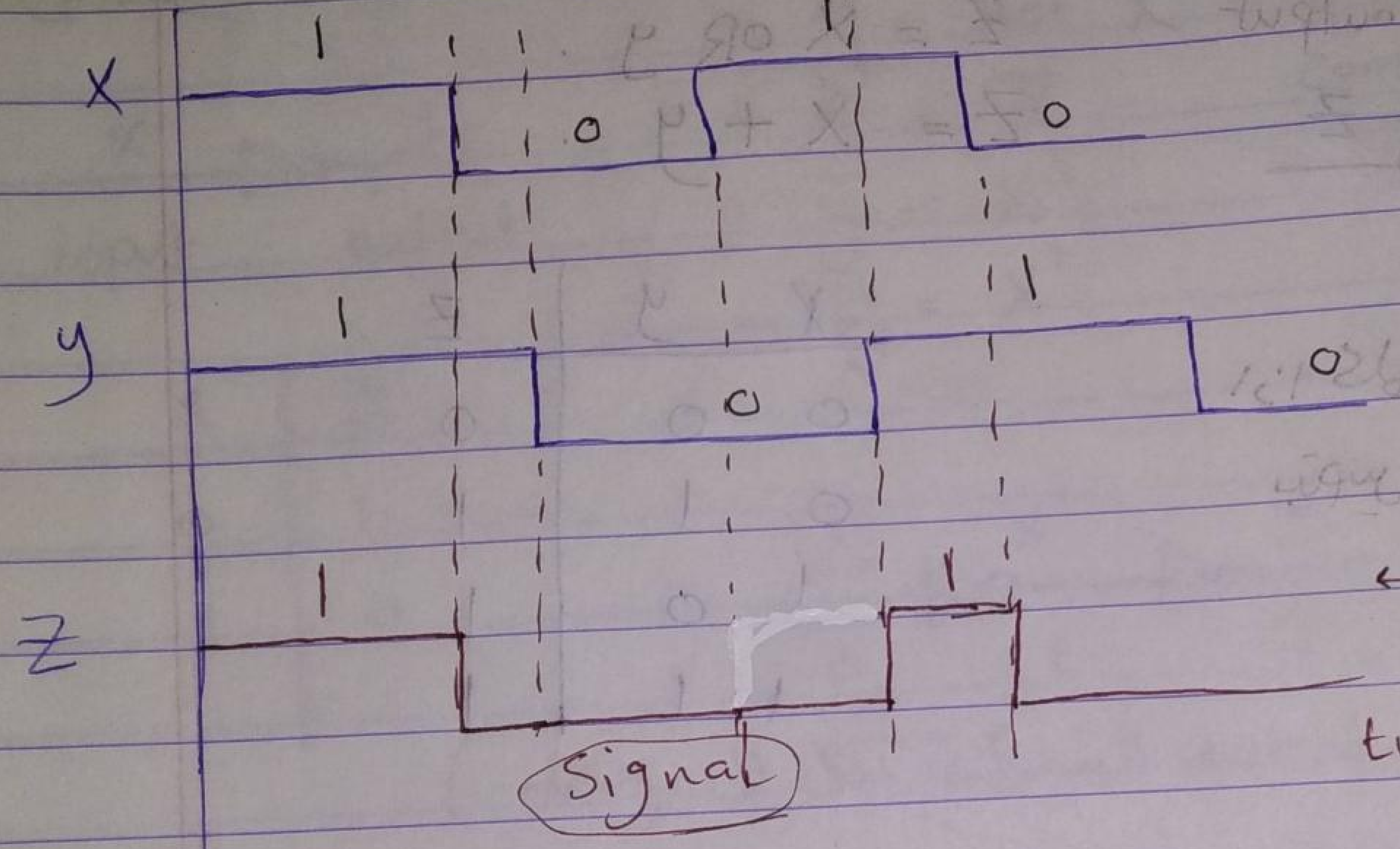
input is defined by user.

output is defined by the system.

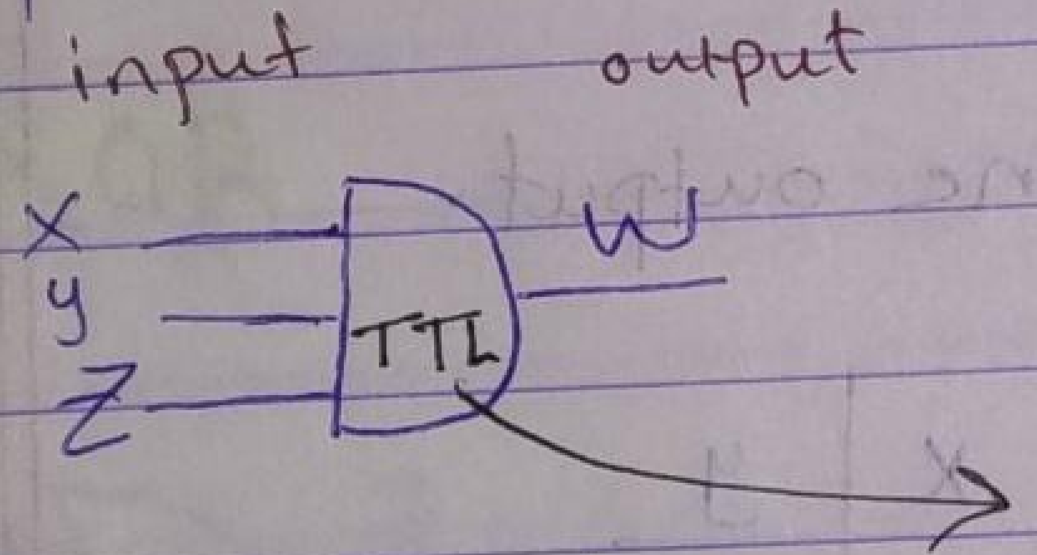
X	y	Z
0	0	0
0	1	0
1	0	0
1	1	1

Truth table.

Z equal 1 when all input equal 1.



← بتساوي 1
لم يكون ال
two inputs = 1



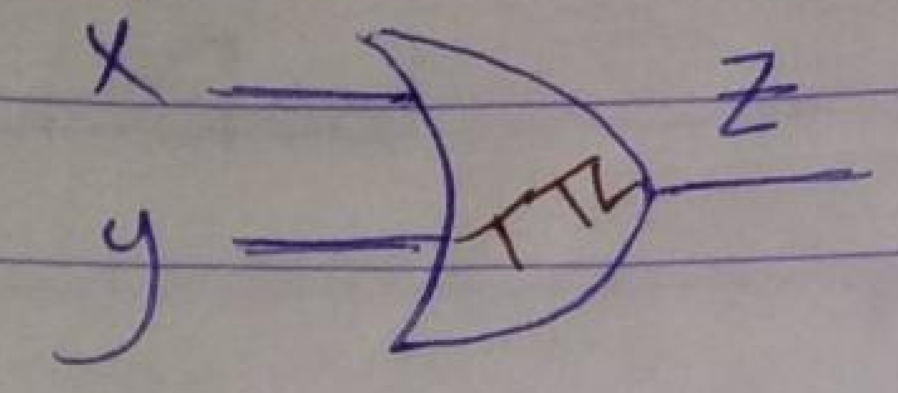
$W = X \cdot y \cdot Z$

Transistor — Transistor level.
group of Transistors.

x	y	Z	W
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

② OR gate

input output



$Z = X \text{ OR } y$.

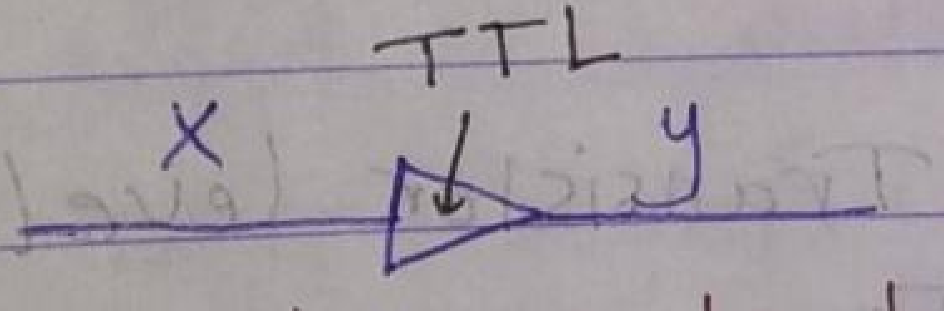
$Z = X + y$.

X	y	Z
0	0	0
0	1	1
1	0	1
1	1	1

اذا كلها Zero

بتعبر open circit

③ Buffer. / one input one output.

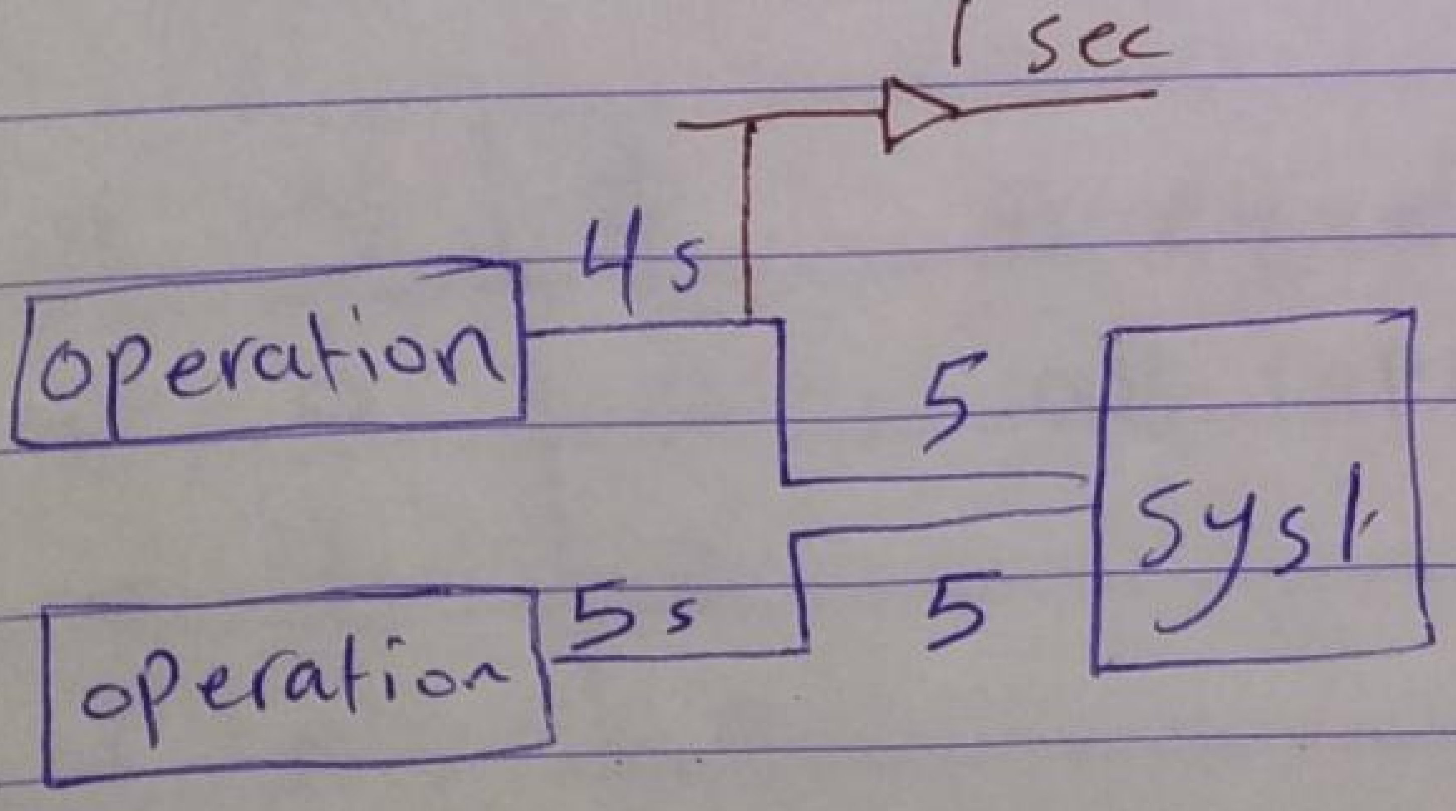


input output

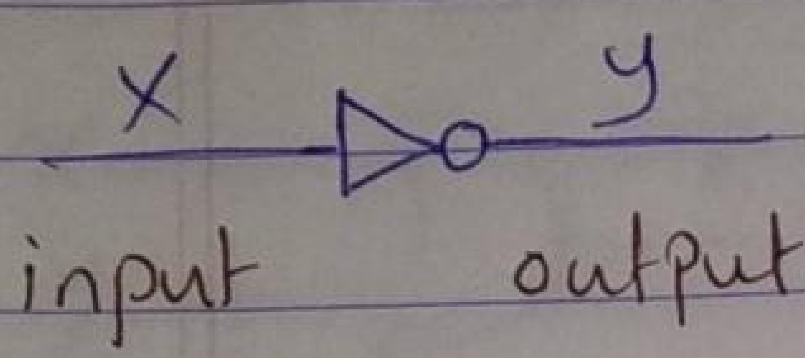
كاندثو

dely تأخير

X	y
0	0
1	1



④ Inverter (not)



$y = \text{not } x$

$y = \overline{x}$

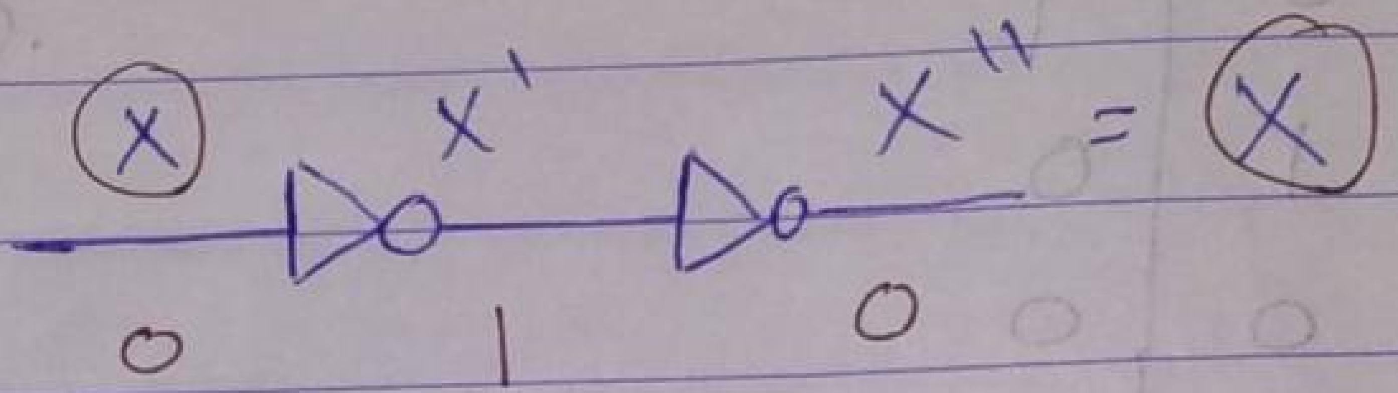
$y = x'$

$y = x'$

complement X

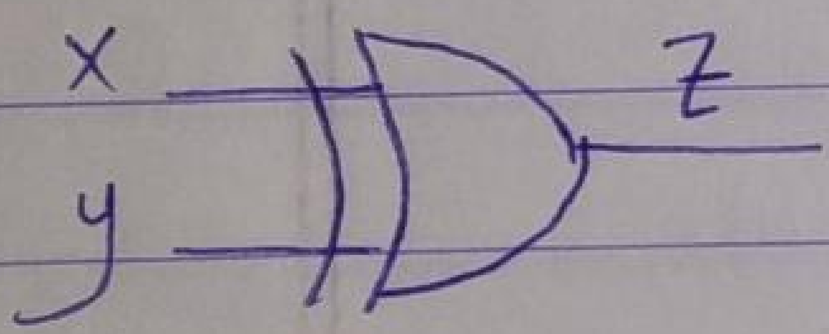
نفي X .

x	y
0	1
1	0



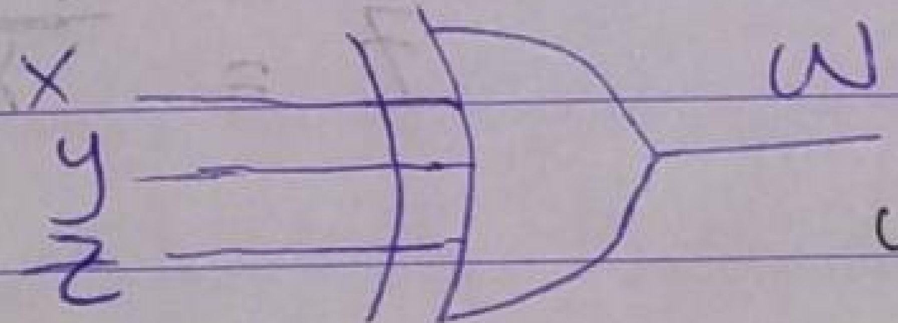
(كلما صغرنا حجم السيركت بقل ال dely .)

⑤ XOR



$z = x \text{ XOR } y$

$z = x \oplus y .$



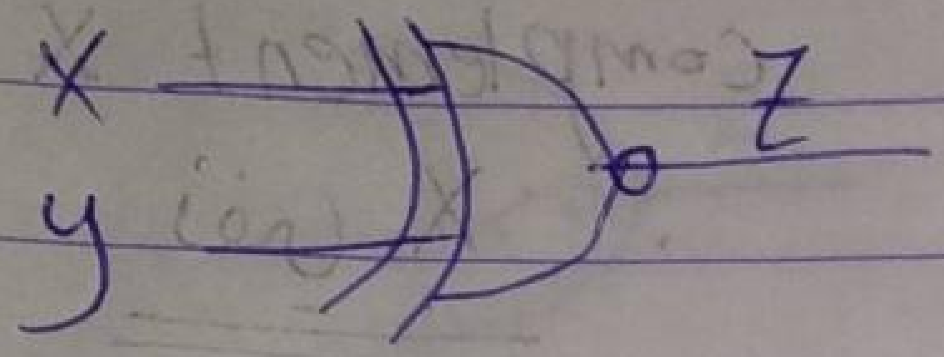
$w = x \oplus y \oplus z$

x	y	z
0	0	0
0	1	1
1	0	1
1	1	0

	x	y	z	w
	0	0	0	0
→	0	0	1	1
→	0	1	0	1
	0	1	1	0
odd Function →	1	0	0	1
	1	0	1	0
	1	1	0	0
→	1	1	1	1

لما عدد الواحدات odd
بتكون فقط .

⑥ XNOR gate / not XOR (even)



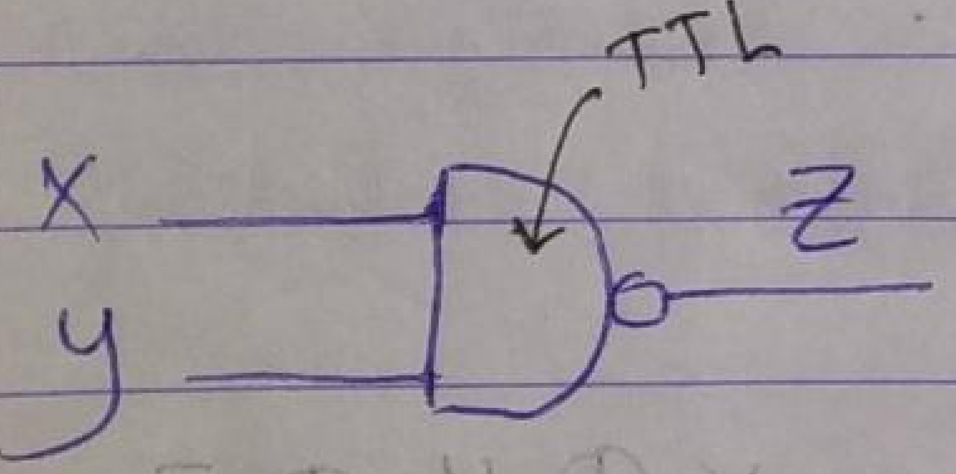
$Z = \overline{X \oplus y}$

$Z = (X \oplus y)'$

x	y	Z
0	0	1
0	1	0
1	0	0
1	1	1

لما عدد الواحدات even
بتحتوي.

⑦ NAND gate / Not AND



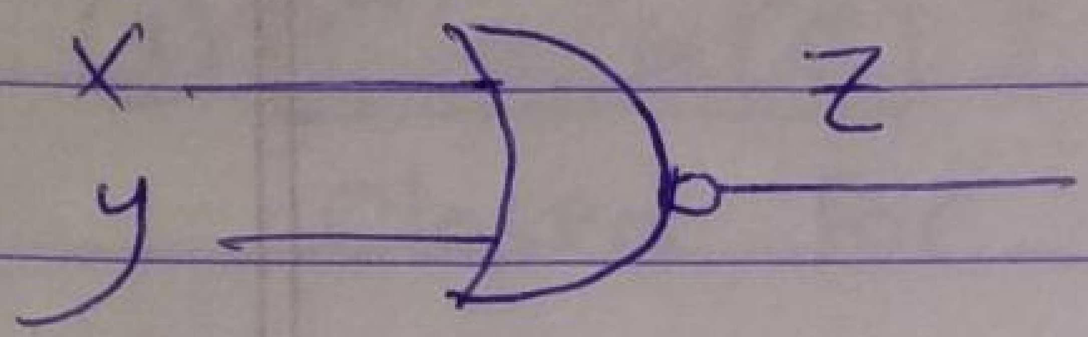
$Z = (X \cdot y)'$

$Z = \overline{X \cdot y}$

x	y	Z
0	0	1
0	1	1
1	0	1
1	1	0

عدد ال transistor في ال NAND
أقل من ال AND.

⑧ NOR gate / not OR.



$$Z = \overline{x+y}$$

$$Z = (x+y)'$$

x	y	z
0	0	1
0	1	0
1	0	0
1	1	0

Decimal	Bcd	Exess-3	Gray code
0	0000	0011	0000
1	0001	0100	0001
2	0010	0101	0011
3	0011	0110	0010
4	0100	0111	0110
5	0101	1000	
6	0110	1001	
7	0111	1010	
8	1000	1011	
9	1001	1110	
10	xxxx	1101	
11	xxxx	1110	
12	xxxx	1111	
13	xxxx	xxxx	
14	xxxx	xxxx	
15	xxxx	xxxx	

2421
0000
0001
0010
0011
0100
0101
0110
0111
1110
1111