Assignment #9 E & CE 223

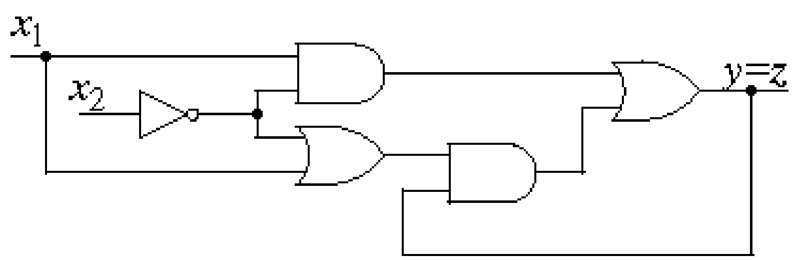
E&CE 223
Assignment 9 - Solutions

1. Mano 9.3

$$Y = x_1x_2' + (x_1+x_2')y$$

$$Z=y$$

(a)



(b)

y \ x_1x_2	00	01	11	10
0	(0)	(0)	(0)	1
1	(1)	0	(1)	(1)

Y

y \ x_1x_2	00	01	11	10
0	0	0	0	0
1	1	1	1	1

z

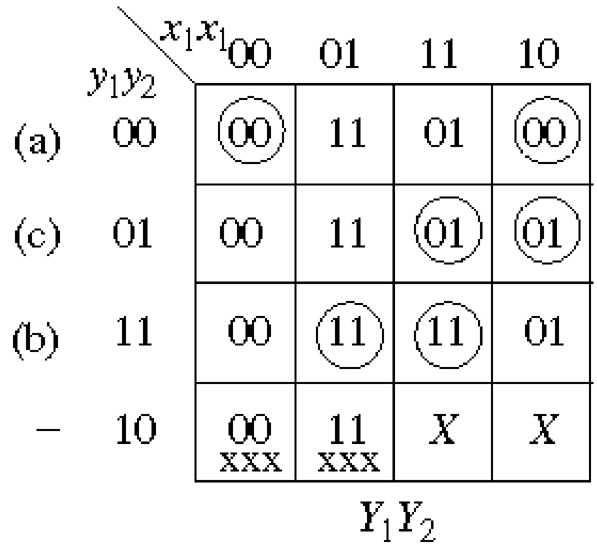
(c)

present state	x_1x_2 00	01	11	10
a	(a),0	(a),0	(a),0	b ,–
b	(b),1	a,–	(b),1	(b),1

(d) For input $x_1x_2 = 01$, output becomes 0
For input $x_1x_2 = 10$, output becomes 1
For the other two input combinations, the output retains its previous value.

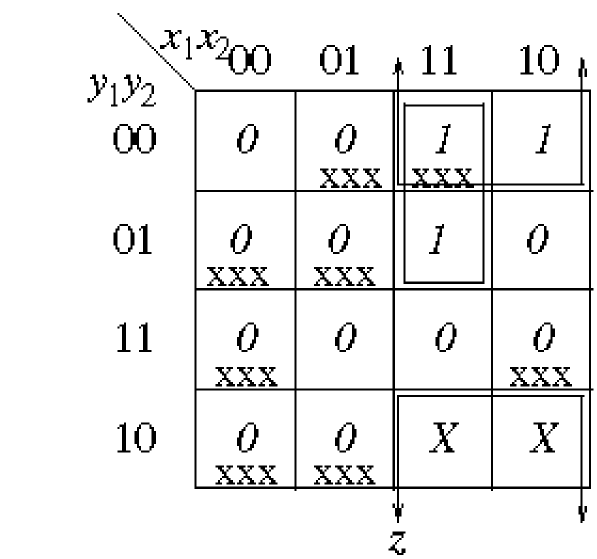
2. Mano 9.5

(a)



xxx - to avoid critical races

(b)



xxx - to avoid dynamic hazards on outputs

(c)

y_1y_2 \ x_1x_2	00	01	11	10
00		1		
01		1		
11		1	1	
10		1	X	X

Y_1

y_1y_2 \ x_1x_2	00	01	11	10
00		1	1	
01		1	1	1
11		1	1	1
10		1	X	X

Y_2

$$Y_1 = x_1'x_2 + y_1x_2$$

$$Y_2 = x_2 + y_2x_1$$

$$z = y_2'x_1 + y_1'x_1x_2$$

Check: Don't care cells are 11 and 10. Ok, no traps.

3. Mano 9.6

y_1y_2 \ x_1x_2	00	01	11	10
00	10	**(00)**	11	10
01	**(01)**	00	10	10
11	01	00	**(11)**	**(11)**
10	11	00	**(10)**	**(10)**

row 00 x_1x_2=01→11 y_1y_2=00→11

critical race:

00→10 stable, or

00→01 tries to go to 10 goes 11 stable

goes 00 oscillator

row 01 x_1x_2=00→10 y_1y_2=10→10
critical race - goes to stable 11 or 10
row 11 x_1x_2=11→01 y_1y_2=11→00
non-critical race
row 10 x_1x_2=11→01 y_1y_2=11→00
non-critical race
There is a cycle in column 00
Starting at y_1y_2=00, x_1x_2=01→00
y_1y_2=00→10→11→01
Cycle can also be entered in rows 11 and 10
when x_1x_2=10→00

4. Mano 9.12

present state	next state and output inputs x_1x_2			
	00	01	11	10
a	(a),00	b,00	–	c,00
b	a,00	(b),00	d,–0	–
c	a,00	–	g,01	(c),00
d	–	e,10	(d),10	f,10
e	a,–0	(e),10	d,10	–
f	a,–0	–	g,—	(f),10
g	–	j,01	(g),01	h,01
h	a,0–	–	g,01	(h),01
j	a,0–	(j),01	d,—	–

5. Mano 9.15

(a)

	00	01	11	10
a	(a),0	b,-	-,-	d,-
b	a,0	(b),1	(b),1	c,-
c	b,0	-,-	b,1	(c),0
d	c,-	(d),1	c,1	(d),1

(b)

	00	01	11	10
a	(a),0	b,0	b,-	(a),0
b	a,0	(b),0	(b),1	c,1
c	b,-	d,1	(c),1	(c),1
d	(d),0	(d),1	c,1	a,0

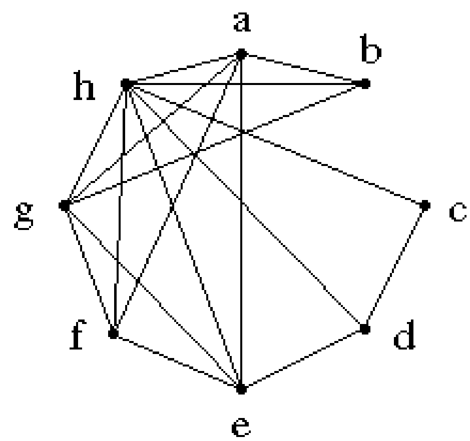
6. Mano 9.18

P9-18(a)

(a)

b	√						
c	b,d e,h	b,d					
d	b,d	×	√				
e	√	c,f	c,f e,h	√			
f	b,g e,h	b,g c,f	d,g	d,e e,h	e,h		
g	b,g	√	d,g	×	√	√	
h	e,h	√	√	√	√	√	√
	a	b	c	d	e	f	g

(b)



(c) Note: This merger diagram has a "gotcha" for the unwary.
The mergers a-f and a-g are only possible if b-g is merged.
The mergers a-h, a-f and f-e are only possible if e-h is merged.
Consequently, the apparent merger (a,h,g,f,e) is not possible since it prevents the b-g merger required to allow the a-f and a-g merges.

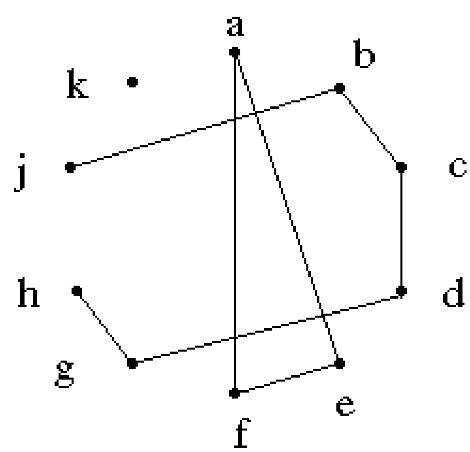
Possible mergers are:
(e,f,g,h) (a,b) (c,d)
(a,b,g) (e,h,f) (c,d)

P9.18b

(a)

	a	b	c	d	e	f	g	h	j
b	a,c e,b								
c	×	✓							
d	×	j,g	✓						
e	✓	×	×	×					
f	✓	×	×	×	✓				
g	d,f e,k	j,g b,k	×	✓	e,k	×			
h	×	c,h b,k	×	×	×	×	✓		
j	e,b	✓	d,f	×	×	j,g	×	×	
k	e,k	×	×	×	×	j,g	×	×	×

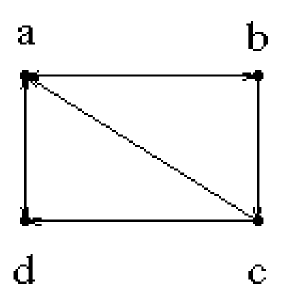
(b)



(c) minimum row solution:
(a,e,f) (b,j) (c,d) (h,g) (k)

7. Mano 9.19

Required transitions:



The assignment
a = 00
b = 01
c = 11
d = 10
would is ok except for the c → a transition. Checking the reduced flow table shows that this transition can be handled by a non-critical race.

y_1y_2 \ x_1x_2	00	01	11	10
(a) 00	(00)	(00)	01	10
(b) 01	00	(01)	(01)	11
(c) 11	00	–	10	(11)
(d) 10	00	00	(10)	(10)

Y_1Y_2

y_1y_2 \ x_1x_2	00	01	11	10
00	*0*	*1*	*X*	*X*
01	*0* xxx	*0*	*0*	*0* xxx
11	*0* xxx	*X*	*X*	*0*
10	*0* xxx	*1* xxx	*1*	*1*

z

Note in the output table xxx indicates "don't care" assignments to prevent dynamic hazards.

$$z = y_2'x_2 + y_2'x_1$$

[no static hazards]

Latch solution:

y_1y_2 \ x_1x_2	00	01	11	10
00	φ	φ	φ	α
01	φ	φ	φ	α
11	β	–	I	I
10	β	β	I	I

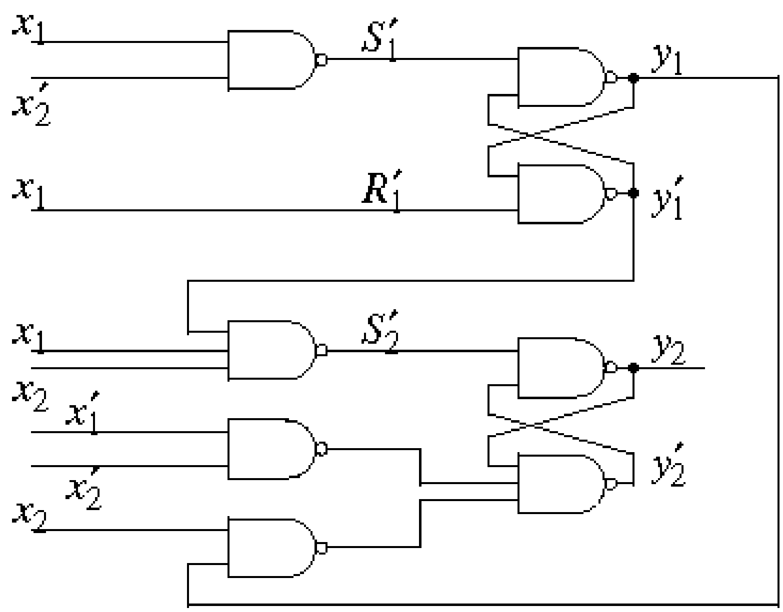
y_1 transitions

y_1y_2 \ x_1x_2	00	01	11	10
00	φ	φ	α	φ
01	β	I	I	I
11	β	–	β	I
10	φ	φ	φ	φ

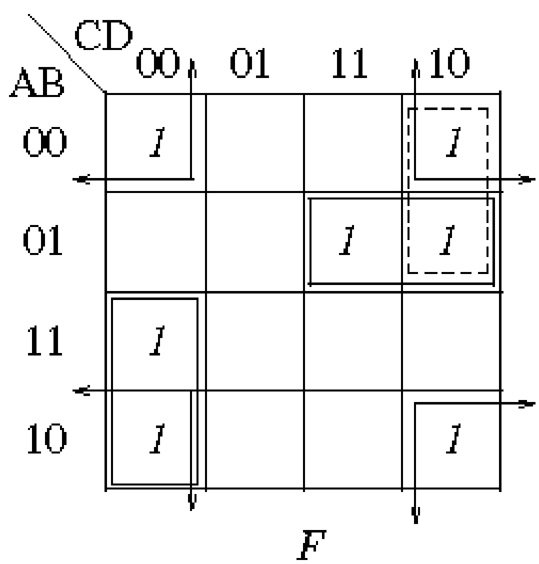
y_2 transitions

$$S_1 = x_1 x_2' \qquad S_2 = y_1' x_1 x_2$$

$$R_1 = x_1' \qquad R_2 = x_1' x_2' + y_1 x_2$$



8. Mano 9.22



$$F = B'D' + AC'D' + A'BC + A'CD'$$

The last term eliminates the hazard.

9. Mano 9.13 and 9.25

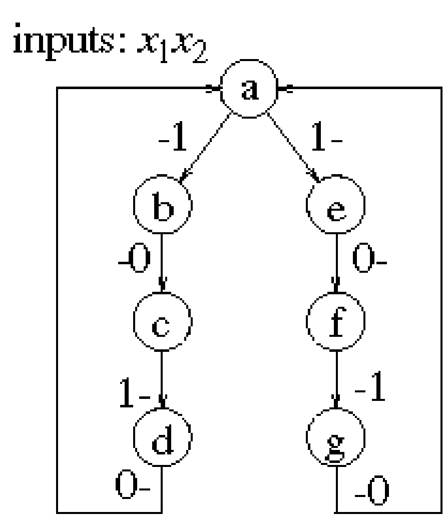
Define states:

state	inputs x_1x_2	light	comment
a	00	0	no train in intersection
b	01	1	train crossing x_1
c	00	1	train in intersection
d	10	1	train crossing x_2
			[in states b,c,d train moving from x_1 towards x_2]
e	10	1	train crossing x_2
f	00	1	train in intersection
g	01	1	train crossing x_1
			[in states e,f,g train moving from x_2 towards x_1]

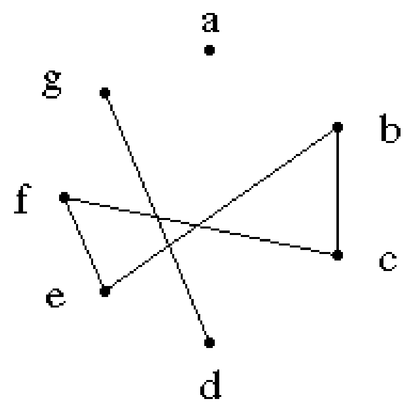
Assumptions:

- single track - only one train in intersection
- train length less than distance between x_1 and x_2
- trains do not stop and reverse
- one train clears intersection before another reaches the intersection.

state	x_1x_2 00	01	11	10
a	(a),0	b,–	–,–	e,–
b	c,1	(b),1	–,–	–,–
c	(c),1	–,–	–,–	d,1
d	a,–	–,–	–,–	(d),1
e	f,1	–,–	–,–	(e),1
f	(f),1	g,1	–,–	–,–
g	a,–	(g),1	–,–	–,–



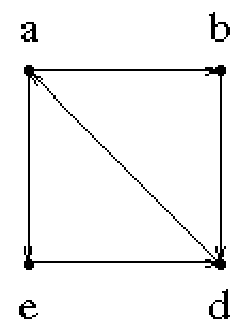
	a	b	c	d	e	f
b	×					
c	×	√				
d	d,e	×	×			
e	×	c,f	c,f d,e	×		
f	×	c,f b,g	√	×	√	
g	b,g	×	×	√	×	√



Two possible 4 row tables:
(a) (b,c) (d,g) (e,f)
(a) (b,e) (d,g) (c,f) [(b,e) only if (c,f)]

Consider (a) (b,c) (d,g) (e,f) merge:

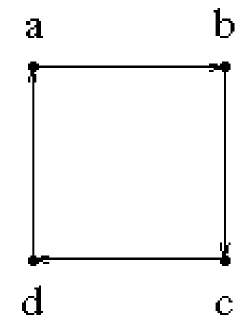
x_1x_2		00	01	11	10
	a	(a),0	b,–	–,–	e,–
(b,c)	b	(b),1	(b),1	–,–	d,–
(d,g)	d	a,–	(d),1	–,–	(d),1
(e,f)	e	(e),1	d,1	–,–	(e),1



Can not arrange rows to avoid a race, and there are not any "don't cares" to construct a cycle.
Therefore would have to add a third secondary variable.

Consider (a) (b,e) (d,g) (c,f) merge:

		x_1x_2 00	01	11	10
	a	a,0	*b*,–	–,–	*b*,–
(b,e)	b	*c*,–	b,1	–,–	b,1
(d,g)	d	*a*,–	d,1	–,–	d,1
(c,f)	c	c,1	*d*,1	–,–	*d*,1



No races if assignment:
a = 00
b = 01
c = 11
d = 10
[Other assignments are also possible.]

y_1y_2 \ x_1x_2	00	01	11	10
(a) 00	00	01	–	01
(b) 01	11	01	–	01
(c) 11	11	10	–	10
(d) 10	00	10	–	10

Y_1Y_2

y_1y_2 \ x_1x_2	00	01	11	10
00	*0*	*X*	*X*	*X*
01	*1*	*1*	*X*	*1*
11	*X*	*1*	*X*	*1*
10	*1*	*1*	*X*	*1*

output z

$z = y_1 + y_2$

Non-latch:

y_1y_2 \ x_1x_2	00	01	11	10
00			X	
01	1		X	
11	1	1	X	1
10		1	X	1

Y_1

y_1y_2 \ x_1x_2	00	01	11	10
00		1	X	1
01	1	1	X	1
11	1		X	
10			X	

Y_2

Y_1 has a hazard going from

y_1y_2=11,x_1x_2=00→01 or 10

Y_2 has a hazard going from

y_1y_2=01,x_1x_2=00→01 or 10

Add extra terms to eliminate hazards.

$$Y_1 = y_2x_1'x_2' + y_1x_1 + y_1x_2 + y_1y_2$$

$$Y_2 = y_2x_1'x_2' + \ + y_1'x_2 + y_1'x_2 + y_1'y_2$$

In both equations the last term removes the hazard.

Latch version:

y_1y_2 \ x_1x_2	00	01	11	10
00	φ	φ	X	φ
01	α	φ	X	φ
11	I	I	X	I
10	β	I	X	I

y_1 transitions

y_1y_2 \ x_1x_2	00	01	11	10
00	φ	α	X	α
01	I	I	X	I
11	I	β	X	β
10	φ	φ	X	φ

y_2 transitions

$S_1 = y_2x_1'x_2'$ $\quad$ $S_2 = y_1'x_1 + y_1'x_2$

$R_1 = y_2'x_1'x_2'$ $\quad$ $R_2 = y_1x_1 + y_1x_2$

NO HAZARDS