

Birzeit University
Department of Electrical and Computer Engineering

ENCS 234 – Digital Systems

First Semester 2015/2016

Midterm Exam

8/12/2015, 3.30 pm

Instructions:

- 1) This exam is closed-book, closed-notes.
- 2) Exam duration is 90 minutes.
- 3) Answer the questions on this exam paper.

Student name: Laith Mubarak Student Number: 1140962

Choose Section:

- Section 1: SMW 10am (Abdel Salam Sayyad)
- Section 2: SMW 12pm (Adnan Yahya)
- Section 3: TR 12.30pm (Abdellatif Abu-Issa)
- Section 4: TR 11am (Abdellatif Abu-Issa)
- Section 5: SMW 11am (Abdel Salam Sayyad)

Question	Full Mark	Your Mark	ABET outcome
1	20	20	a
2	20	20	e
3	20	17	c
4	20	18	c
5	20	20	e
Total	100	95	a, c, e

Question 1: (20 marks)

Given the 8-bit binary numbers in signed two's complement representation:

$X = 10001110$, and $Y = 01100011 \Rightarrow = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
 $= 1 + 2 + 32 + 64 = 99$ in Decimal

a) What are the decimal values equivalent to X and Y?

$X = 10001110$
 $\rightarrow 01110001$ 1st complement
 sign "negative" $(01110001)_2$ 2nd complement

$X = -114$
 $Y = 99$ in Decimal

$= 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
 $= 2 + 16 + 32 + 64$
 $= -114$ in Decimal

b) Perform the binary addition $X + Y$, then convert the result to decimal.

$X + Y \Rightarrow$
 10001110
 01100011
 +

11110001
 00001110 1st complement
 $(00001111)_2$ 2nd complement
 the result

$\Rightarrow (00001111)_2 = 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$
 $+ 0 \times 2^4 + 0 \times 2^5 + 0 \times 2^6 + 0 \times 2^7$
 $= 1 + 2 + 4 + 8$
 $= -15$ in Decimal

~~10001110
 01100011
 01110001
 10001110
 11110001~~

~~01100011
 $64 32$
 96~~

Question 2. (20 marks)

Given the Boolean function:

$$F(A,B,C,D) = \sum(0, 1, 4, 5, 10, 12, 14)$$

a) Write the function F as a sum of minterms. [6 marks]

$$\begin{aligned}
 F(A,B,C,D) &= \sum(0, 1, 4, 5, 10, 12, 14) \\
 &= m_0 + m_1 + m_4 + m_5 + m_{10} + m_{12} + m_{14} \\
 &= \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}B\overline{C}D + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D \\
 &\quad + \overline{A}B\overline{C}D
 \end{aligned}$$

b) Use Boolean Algebra to minimize (simplify) the expression that you obtained in (a). [8 marks]

$$\begin{aligned}
 F(A,B,C,D) &= \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}B\overline{C}D \\
 &\quad + \overline{A}B\overline{C}D + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D \\
 &= \overline{A}ACD'(B+B) + \overline{A}AB \\
 &\quad + \overline{A}B\overline{C}'(D+D) + \overline{A}B\overline{C}'(D+D) \\
 &\quad + \overline{A}B\overline{C}D \\
 &= \overline{A}CD' + \overline{A}B\overline{C}' + \overline{A}B\overline{C}' + \overline{A}B\overline{C}D
 \end{aligned}$$

celp ds,
any

$F = \overline{A}C' + \overline{A}CD' + \overline{A}BD'$

A	0	1	0	1
B	0	0	1	1
C	0	1	0	0
D	0	1	1	0

011
101
111
001
100
010
110
000
101
111
001
101
111

c) Write the function F as a product of maxterms. [6 marks]

$$\begin{aligned}
 F(A,B,C,D) &= \prod(2, 3, 6, 7, 8, 9, 11, 13, 15) \\
 &= (\overline{A} + \overline{B} + \overline{C} + \overline{D}) \cdot M_2 \cdot M_3 \cdot M_6 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{11} \cdot M_{13} \cdot M_{15} \\
 &= (A + B + \overline{C} + \overline{D}) \cdot (A + B + \overline{C} + D) \cdot (A + \overline{B} + \overline{C} + D) \cdot (A + \overline{B} + C + D) \cdot \\
 &\quad (\overline{A} + B + C + D) \cdot (\overline{A} + \overline{B} + C + D) \cdot (\overline{A} + \overline{B} + \overline{C} + D) \cdot (\overline{A} + \overline{B} + C + \overline{D}) \\
 &\quad (\overline{A} + \overline{B} + \overline{C} + D)
 \end{aligned}$$

Q.2 part b

$$\Rightarrow F = \Sigma(0, 1, 4, 5, 10, 12, 14)$$

$$= \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}D + \overline{A}B\overline{C}\overline{D} +$$

$$\overline{A}B\overline{C}D + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D$$

$$= \overline{A}\overline{B}D'(C+C') + A\overline{C}D'(B+B')$$

$$+ \overline{A}\overline{B}\overline{C}D' + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D}$$

$$= \overline{A}\overline{B}D' + A\overline{C}D' + A\overline{C}'(D\overline{B}' + \overline{B}D + \overline{B}D' + \overline{B}D)$$

$$= \overline{A}\overline{B}D' + A\overline{C}D' + A\overline{C}'(B'(D+D') + B(D+D'))$$

$$= \overline{A}\overline{B}D' + A\overline{C}D' + A\overline{C}'(\overline{B}' + B)$$

$$= \overline{A}\overline{B}D' + A\overline{C}D' + A\overline{C}' \cdot 1$$

$$= \overline{A}\overline{B}D' + A\overline{C}D' + A\overline{C}'$$

the minimize
expression of the
function!

$$F = \overline{A}\overline{C}' + \overline{A}\overline{B}D' + A\overline{C}D'$$

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11				1
10	1			1

Student Name: _____

Student Number: _____

Section? 1, 2, 3, 4, 5

Question 3: (20 marks)

Given the Boolean function:

$F(A,B,C,D) = \Sigma(0, 1, 4, 5, 10, 12, 14)$

the final result \Rightarrow ~~EA~~

$F(A,B,C,D) = \boxed{ACD' + A'C'} + \cancel{BCD} + \cancel{ABD}$

← essential

A) Using the Tabulation method, find all the equally-minimal Boolean expressions for the function F. [12 marks]

$F(A,B,C,D) = \Sigma(0, 1, 4, 5, 10, 12, 14)$

... Prime Implicant

- 0
- 1
- 4
- 5
- 10
- 12
- 14

- ✓ (0,1) (1)
 - ✓ (0,4) (4)
 - ✓ (1,5) (4)
 - ✓ (4,5) (1)
 - (4,12) (8)
 - (10,14) (4)
 - (12,14) (2)
- (0,1,4,5) (1,4)
(0,4,5) (4,5) repeated

	3	4	2	1	
	A	B	C	D	
(4,12) (8)	✓	1	0	0	} prime Implicant
(10,14) (4)	1	✓	1	0	
(12,14) (2)	1	1	✓	0	
(0,1,4,5) (1,4)	0	✓	0	✓	

⇓ ⇓

essential

	0	1	4	5	10	12	14
BCD'			✓			⊗	
ACD'					✓		✓
ABD'						⊗	✓
$A'C'$	✓	✓	✓	✓			
	✓	✓	✓	✓	✓		✓

$F(A,B,C,D) = \underbrace{ACD' + A'C'}_{\text{essential}} + \underbrace{BCD' + ABD}$

Student Name: _____ Student Number: _____ Section? 1, 2, 3, 4, 5

B) Use the K-map method to find a minimal expression for:

$$F(A,B,C,D) = \sum(0, 1, 4, 5, 10, 12, 14)$$

With the don't-care conditions:

$$d(A,B,C,D) = \sum(3,8)$$

[8 marks]

-3

$$F(A,B,C,D) = \overline{E}D + A'C' + AD$$

* here every circle is essential

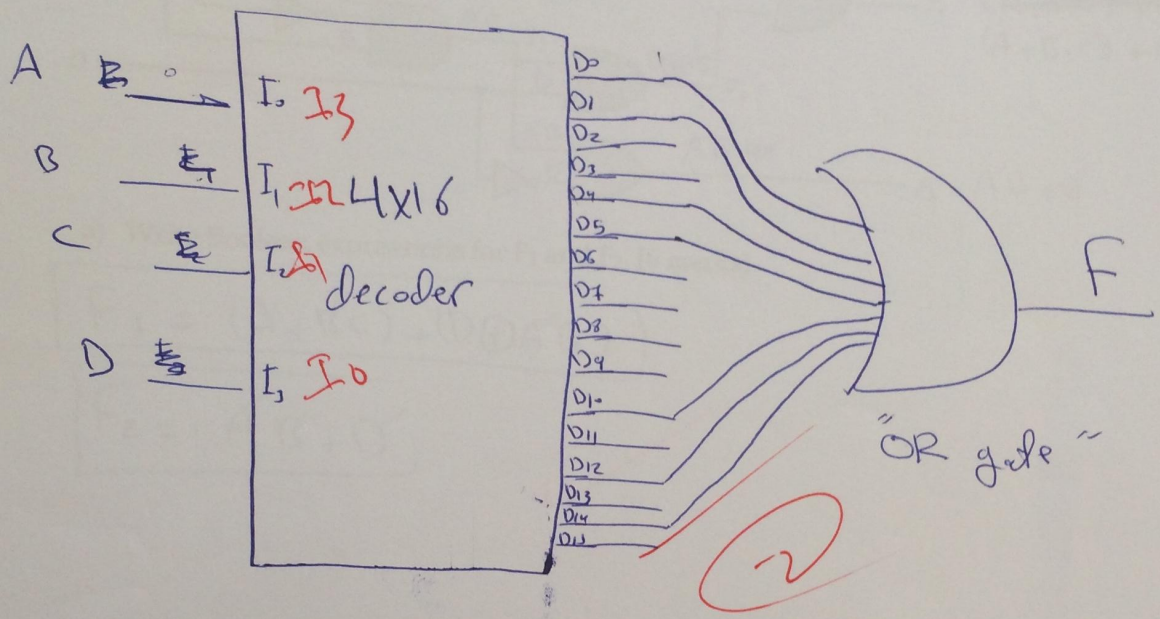
CD \ AB	00	01	11	10
00	1	1	X	
01	1	1		
11	1			1
10	X			1

Question 4: (20 marks)

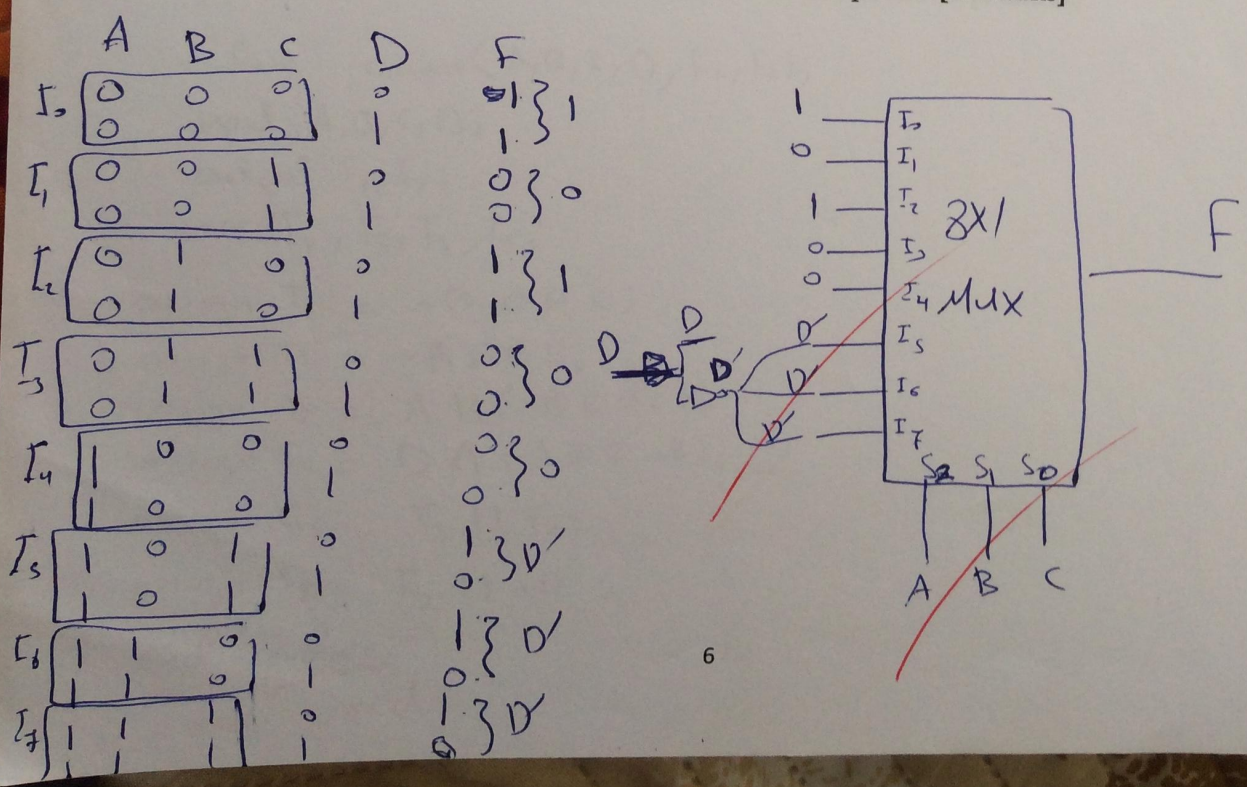
Given the Boolean function:

$$F(A,B,C,D) = \sum(0, 1, 4, 5, 10, 12, 14)$$

- a) Implement the function F using a 4X16 decoder plus any necessary external gates. [10 marks]

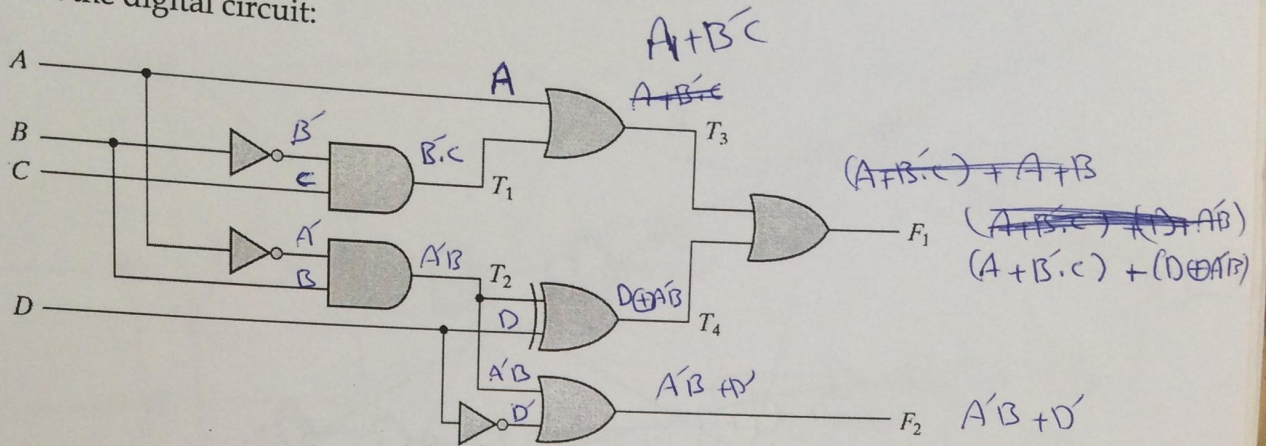


- b) Implement the function F using an 8X1 Multiplexer. [10 marks]



Question 5: (20 marks)

Given the digital circuit:



a) Write Boolean expressions for F_1 and F_2 . [6 marks]

$$F_1 = (A + B'C) + (D + A'B)$$

$$F_2 = A'B + D'$$

b) Write Verilog HDL code to describe the circuit above. [8 marks]

```

module circuit(A,B,C,D,F1,F2);
input A,B,C,D;
output F1,F2;
wire T1,T2,T3,T4;
assign T1 = ~B && C;
assign T2 = ~A && B;
assign T3 = A || (~B && C);
assign T4 = D && (A && ~B);
assign F1 = T3 || T4;
assign F2 = T2 || ~D;
end module;
    
```

one word

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c) Re-implement the circuit using NAND gates only. [6 marks]

