Problem Set 1

– Solutions –

ENCS 234
Fall 2015

Problem 1

a) It will be sufficient to use 7 bits to represent the given numbers. The leftmost bit represents the sign.

$$x = (37)_{10} = \overset{+ve}{0}100101$$ (+ve numbers are not converted to 2's comp.)

$$y = -(56)_{10} = -(0111000)_2 = \underset{(-ve)}{1}\underbrace{001000}_{\text{2's comp.}}$$

b)

$$\begin{array}{r} 0100101 \\ \phi001000 + \\ \hline 1101101 \end{array}$$

$\underbrace{1101101}_{\text{take 2's comp}}$ ⟵ Result = −19

take 2's comp ⟹ 010011 = $(19)_{10}$

(c) Using 7-bit ASCII, with a bit added to the left for even parity

3	0 | 0110011	O	1 | 1001111
7	1 | 0110111	K	0 | 1001011
-	0 | 0101101	?	0 | 0111111
5	0 | 0110101		
6	0 | 0110110		
=	1 | 0111101		
_	0 | 1011111		
1	1 | 0110001		
9	0 | 0111001		
,	1 | 0101100		
space	1 | 0100000		

Problem 2

$$F(w,x,y,z) = \Sigma(0,4,5,8,11,12,13,15)$$

a) $F = w'x'y'z' + w'xy'z' + w'xy'z + wx'y'z' +$
$(wx'yz) + wxy'z' + wxy'z + (wxyz)$

b) $F = \cancel{(x'+x)}(w'y'z') + \cancel{(w'+w)}xy'z + \cancel{(x'+x)}(wy'z') +$
$\cancel{(x+x')}wyz$

$= \cancel{(w'+w)}y'z' + xy'z + wyz$

$= y'(z' + \cancel{y}xz) + wyz$

$= y'((z'+x)\cancel{(z'+z)}) + wyz$

$= y'z' + xy' + w\cancel{y}z$

c) $F(w,x,y,z) = \Pi(1,2,3,6,7,9,10,14)$

$= (w+x+y+z')(w+x+y'+z)\cdot(w+x+y'+z')\cdot$
$(w+x'+y'+z)\cdot(w+x'+y'+z')\cdot(w'+x+y+z')\cdot$
$(w'+x+y'+z)\cdot(w'+x'+y'+z)$

Problem 2 (continued)

d) $F = (\cancel{ww'} + x + y + z') \cdot (\cancel{ww'} + x + y' + z) \cdot$

$\cancel{(w+x')} (\cancel{xx'} + w + y' + z') \cdot (\cancel{ww'} + x' + y' + z)$

$= (x+y+z') \cdot \underline{(x+y'+z)} \cdot (w+y'+z') \cdot \underline{(x'+y'+z)}$

$= (x+y+z') \cdot (w+y'+z') \cdot (\cancel{xx'} + y' + z)$

$= (x+y+z') \cdot (wy' + wz + \cancel{y'y} + y'z + z'y' + \cancel{z'z}) \cdot$

$= (x+y+z') \cdot (wz + y' \cdot (\underbrace{1 + w + z + z'}_{1}))$ (absorption)

$= (x+y+z') \cdot (wz + y')$

$= xwz + xy' + wyz + \cancel{yy'} + \cancel{wzz'} + z'y'$

$= \boxed{wxz} + xy' + wyz + z'y'$

(*) Comparing this answer to part (b), we find an extra term, namely "wxz". However, this extra term is redundant. We can conclude this using K-map ~~or~~ Tabulation, but it would be hard to conclude it algebraically.

Problem 3:

a) $F = C'D' + BC' + ABD' + AB'CD$

$= C'D'\cdot(A+A')\cdot(B+B') + BC'\cdot(A+A')\cdot(D+D') +$

$ABD'\cdot(C+C') + AB'CD$

$= \underset{0}{A'B'C'D'} + \underset{4}{A'BC'D'} + \underset{8}{AB'C'D'} + \underset{12}{ABC'D'} +$

$\cancel{A'BC'D'} + \underset{5}{A'BC'D} + \cancel{ABC'D'} + \underset{13}{ABC'D} +$

$\underset{14}{ABCD'} + \cancel{ABC'D} + \underset{11}{AB'CD}$

$= \Sigma(0, 4, 5, 8, 11, 12, 13, 14)$

b) $F = C'D' + BC' + ABD' + AB'CD$

$= (C' + \cancel{BC'} + ABD' + AB'\cancel{C}D) \cdot$ absorption

$(D' + BC' + \cancel{ABD'} + AB'C\cancel{D})$ ex. 2.2 part 2

$= (C' + ABD' + AB'D)\cdot(D' + BC' + AB'C)$

$= (C' + A + \cancel{AB'D})\cdot(C' + B + A\cancel{B'}D)\cdot(C' + D' + AB'\cancel{D})\cdot$

$(D' + B + A\cancel{B'}C)\cdot(D' + C' + AB'\cancel{C})$

$= (C' + A)\cdot(C' + B + AD)\cdot(C' + D' + AB')\cdot$

$(D' + B + AC)\cdot(D' + C' + AB')$

$= (C' + A)\cdot(C' + B + A)\cdot(C' + B + D)\cdot(C' + D' + A)\cdot$

$(C' + D' + B')\cdot(D' + B + A)\cdot(D' + B + C)\cdot\cancel{(D' + C' + A)}\cdot$

$\cancel{(D' + C' + B')}$

Problem 3 (continued)

$$= (A + BB' + C' + DD') \cdot (A + B + C' + DD') \cdot$$
$$(AA' + B + C' + D) \cdot (A + BB' + C' + D') \cdot$$
$$(AA' + B' + C' + D') \cdot (A + B + CC' + D') \cdot$$
$$(AA' + B + C + D')$$

$$= {}_2(A + B + C' + D) \cdot (A + B' + C' + D) \cdot {}_6$$
$${}_3(A + B + C' + D') \cdot (A + B' + C' + D') \cdot {}_7$$
$${}_2\cancel{(A + B + C' + D)} \cdot \cancel{(A + B + C' + D')} \cdot {}_3$$
$${}_2\cancel{(A + B + C' + D)} \cdot (A' + B + C' + D) \; {}_{10}$$
$${}_3\cancel{(A + B + C' + D')} \cdot \cancel{(A + B' + C' + D')} \cdot {}_7$$
$${}_7\cancel{(A + B' + C' + D')} \cdot (A' + B' + C' + D') \cdot {}_{15}$$
$${}_1(A + B + C + D') \cdot \cancel{(A + B + C' + D')} \cdot {}_3$$
$${}_1\cancel{(A + B + C + D')} \cdot (A' + B + C + D') \; {}_9$$

$$= \prod(1, 2, 3, 6, 7, 9, 10, 15)$$