



BIRZEIT UNIVERSITY

ANSWER BOOKLET

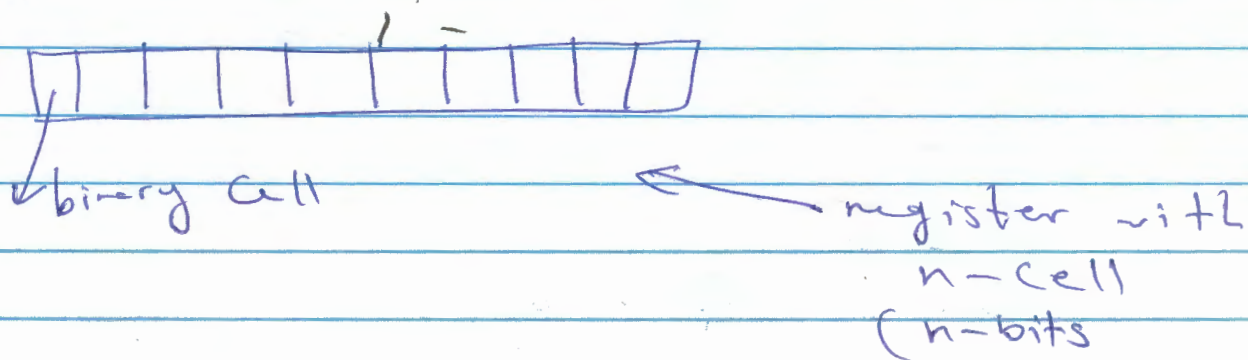
Student: <u>Digital</u> Number <u>2</u>
Course: Department: Number:
Division: Instructor:
Date: Day Month Year

Dr. Abdellatif Abu Issa

For Instructor's Use

Question	Grade
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total	

⊕ Registers.



$\Rightarrow 2^n$ different combinations.

⊕ Binary Logic :

~~Based on logic, can~~

True, false

yes, no

(1, 0) \rightarrow bits

* 3 - basic logical operations:

* AND, OR, NOT

① * AND e.g. $a = b \cdot c$ or $a = bc$

$a = 1$ if $b = 1$ & $c = 1$

otherwise $a = 0$

② OR: $a = b + c$

$a = 0$ if $b = 0$ && $c = 0$

otherwise $a = 1$

③ NOT $a = \bar{b}$

$\rightarrow a = 1$ if $b = 0$

$a = 0$ if $b = 1$

* These are logical operation not arithmetic operations.

$1 + 1 = 10$ (arithmetic).

$1 + 1 = 1$ (logic)

⊕ Logic Gates



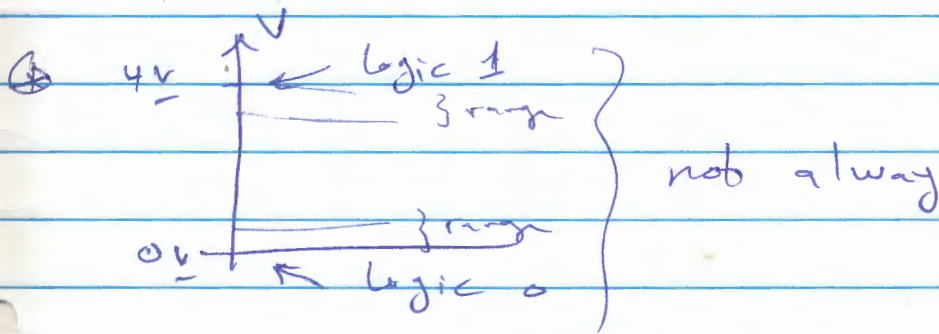
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1



x	y	z
0	0	0
0	1	1
1	0	1
1	1	1



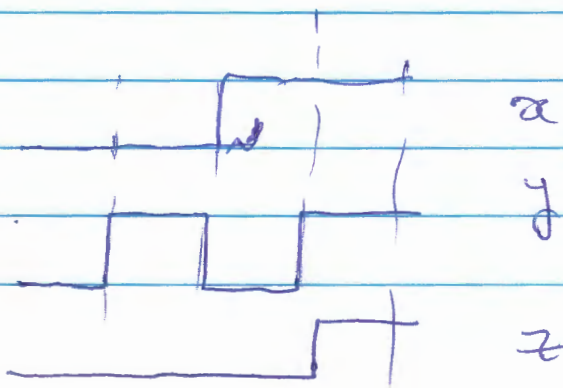
x	z
0	1
1	0



⊕ gates may have multiple inputs (AND + OR)



⊕ signal



AND

$z = x \cdot y$

CHAPTER 2

Boolean Algebra & Logic Gates

- A set of elements is any collection of objects having a common property.

- If S is a set, and x and y are certain objects, then $x \in S$ denotes that x is a member of the set, S , and $y \notin S$ denotes that y is not an element of S .

- The most common postulates used to formulate various algebraic structures are:-

1. Closure: A set S is closed with respect to a binary operator if, for every pair of elements of S , the binary operator specifies a rule for obtaining a unique element of S .

e.g. natural number $N = \{1, 2, 3, \dots\}$

is closed for binary operation (+) because for any $a, b \in N \Rightarrow c = a + b \in N$

eg: it is not closed for (-) since

$$2 - 3 = -1 \notin N \quad 2 - 5 = -3 \notin N$$

2. Associative law: A binary operator $*$ on a set S is said to be associative if

$$(x * y) * z = x * (y * z) \text{ for all } x, y, z \in S$$

e.g. $*$ in \mathbb{N} is associative

3. Commutative law: A binary operator $*$ on a set S is said to be commutative whenever

$$x * y = y * x \text{ for all } x, y \in S$$

in \mathbb{N} x is commutative ($-$ is not).

4. Identity element: A set S is said to have an identity element with respect to a binary operation $*$ on S if there exists an element $e \in S$ with the property

$$e * x = x * e = x \text{ for all } x \in S.$$

e.g. 0 is identity for $+$ on \mathbb{N}

1 is identity for \times on \mathbb{N} .

5. Inverse: A set S having the identity element e with respect to a binary operator $*$ is said to have an inverse whenever, for every $x \in S$, there exists

an element $y \in S$ such that

$$x * y = e.$$

e.g. ~~$+$~~ in $S = \{ \dots, -3, -2, -1, 0, 1, 2, \dots \}$

the inverse of a is $-a$

6- ~~6~~ Distributive law: if $*$ and \circ are two binary operators on a set S , $*$ is said to be distributive over \circ if

$$x * (y \circ z) = (x * y) \circ (x * z)$$

Ex The set of real numbers ($\dots, -1, 0, 1, \dots$) with the binary operator $+$ & \cdot , forms the field of real number. The $+$ operators and postulates have the following meanings:-

(1) $+$ binary operator defines addition-

- closure \checkmark

- Associative $(5+2)+3 = 5+(2+3)$

- Commutative $(5+7) = 7+5$

- Identity element 0 $(5+0=0+5=5)$

- Inverse: a inverse is $-a$

$$5 + (-5) = (-5) + 5 = 0 \text{ (identity)}$$

(2) \cdot binary op defines multiplication

- closure

- Associative

- commutative

- Identity

- Inverse

\rightarrow Distributive law: (\cdot) is distributive over $(+)$

$$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$$

~~$$a + (b \cdot c) \neq (a+b) \cdot (a+c)$$~~

* Axiomatic definition of boolean algebra

- Boolean algebra is an algebraic structure defined by a set of elements, B , together with two binary operators, $+$ and \cdot , provided that the following postulates are satisfied

1- (a) Closure with respect to $(+)$

(b) " " " " (\cdot)

2- (a) An identity element with respect to $+$, designated by 0 : $\forall x \in B, x+0 = 0+x = x$

(b) An identity element with respect to (\cdot) , designated by 1 : $x \cdot 1 = 1 \cdot x = x$

3- (a) commutative with respect to $+$: $x+y = y+x$

(b) " " " " \cdot : $x \cdot y = y \cdot x$

4- (a) (\cdot) is distributive over $+$: $x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

(b) $+$ " " " (\cdot) : $x+(y \cdot z) = (x+y) \cdot (x+z)$

5. for every $x \in B$, there is $x' \in B$ (complement of x) such that

(a) $x+x' = 1$

(b) $x \cdot x' = 0$

(6) There exists at least two elements $x, y \in B$ such that $x \neq y$.

notes

Distributive law for (\cdot) over $(+)$ & $(+)$ over (\cdot) is Boolean algebra (not valid for ordinary algebra).

Boolean algebra doesn't have additive or multiplicative inverses; therefore, there are no subtraction or division operations.

Complement is not available in ordinary algebra.

B is defined as a set with only two elements, 0 & 1, while ordinary algebra deals with the real numbers, which constitute an infinite set of elements.

* Two-Valued Boolean Algebra

* This algebra is defined on a set of two elements, $B = \{0, 1\}$ and two binary operators $(+)$ & (\cdot) .

The following table shows the operations

x	y	$x \cdot y$	$x + y$	x	x'
0	0	0	0	0	1
0	1	0	1	1	0
1	0	0	1		
1	1	1	1		

* As shown in the table, these rules are exactly the same as the AND, OR, and NOT operations, respectively.

The postulates are valid

1- closure: The result is either 0 or 1
 \Rightarrow the result always $\in B$.

2- commutative $0+1 = 1+0 = 1$
 $(0) \cdot (1) = (1) \cdot (0) = 0$

3- identity element

$$0+1 = 1+0 = 1$$

$0+0 = 0 \Rightarrow 0$ is identity element for (+)

~~1+0~~

$$1 \cdot 0 = 0 \cdot 1 = 0$$

$1 \cdot 1 = 1 \Rightarrow 1$ is identity element for (\cdot)

4. distributive $x \cdot (y+z) = x \cdot y + x \cdot z$

x	y	z	y+z	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y) + (x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0
1	1	0	1	1	1	0	1
1	1	1	0	0	0	0	0

also distributive law of $+$ over (\cdot)

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

5- complement

$$\left. \begin{array}{l} x + x' = 1 \\ x \cdot x' = 0 \end{array} \right\} \rightarrow \text{demo by tables}$$

6- two distinct elements 1 & 0 ($1 \neq 0$).

Basic Theorems And Properties of Boolean Algebra

Duality principle

Every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.

eg $0 + 0 = 0$ by duality $1 \cdot 1 = 1$

$0 + 1 = 1$ \longrightarrow $1 \cdot 0 = 0$

& the same thing for variables.

Basic Theorems

1a) $x+x = x$

Proof

$$x+x = (x+x) \cdot 1 \quad (2b)$$

$$= (x+x) \cdot (x+x')$$
 (5a)

~~$$= xx + xx + xx' + xx'$$~~

$$= x + xx' \quad (4b)$$

$$= x + 0 \quad (5b)$$

$$= x$$

1b) $x \cdot x = x$

$$x \cdot x = xx + 0 \quad (2a)$$

$$= xx + xx' \quad (5b)$$

$$= x(x+x') \quad (4a)$$

$$= x \cdot 1 \quad (5a)$$

$$= x$$

Theorem

2a) $x+1 = 1$

$$x+1 = 1 \cdot (x+1)$$

$$= (x+xx') \cdot (x+1)$$

$$= x + x' \cdot 1$$

$$= x + x' = 1$$

2b) $x \cdot 0 = 0$ (by duality)

$$x \cdot 0 = x \cdot (\cancel{xx}) \cdot 0 + (x \cdot 0)$$

$$= (xx') + (x \cdot 0)$$

$$= x(x'+0)$$

$$= xx' = 0$$

$$(3) (x')' = x$$

$$\begin{aligned} (4a) \quad x + (y+z) &= (x+y) + z \\ (4b) \quad x(yz) &= (xy)z \end{aligned} \quad \left. \vphantom{\begin{aligned} (4a) \\ (4b) \end{aligned}} \right\} \text{associative}$$

$$\begin{aligned} (5a) \quad (x+y)' &= x' \cdot y' \\ (5b) \quad (xy)' &= x' + y' \end{aligned} \quad \left. \vphantom{\begin{aligned} (5a) \\ (5b) \end{aligned}} \right\} \text{DeMorgan}$$

$$\begin{aligned} (6a) \quad x + xy &= x \\ (6b) \quad x(x+y) &= x \end{aligned} \quad \left. \vphantom{\begin{aligned} (6a) \\ (6b) \end{aligned}} \right\} \text{absorption}$$

Truth Table proof

$$x + xy = x$$

x	y	xy	x+xy
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$\Rightarrow x + xy = x$

x	y	x+y	x(x+y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

$\Rightarrow x(x+y) = x$

all possible combinations

⊛ Proof that $(xy)' = x' + y'$ using truth table

x	y	xy	$(xy)'$	x'	y'	$x' + y'$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

← identical for all possible cases →

$$\Rightarrow (xy)' = x' + y'$$

⊛ Operator Precedence

- 1) Parentheses
- 2) NOT (complement)
- 3) AND
- 4) OR

e.g. $x'y + z$

- 1) complement x
- 2) make x' & y
- 3) make $x'y + z$

⊕ Boolean Functions :-

- If x, y and z are binary variables (0,1), then $F_1 = x + y'z$ is an example of a Boolean function.

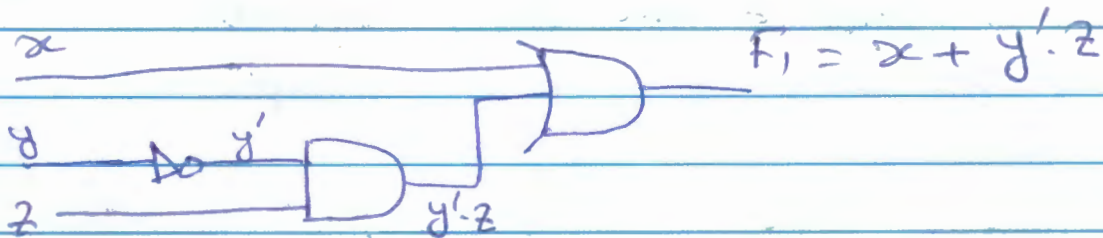
$$F_2 = xyz \quad , \quad F_3 = x'y'z' \quad \dots \text{etc.}$$

- A Boolean function can be represented in a truth table. A truth table is a list of combinations of 1's and 0's assigned to the binary variables and a column that shows the value of the function for each binary combination. The number of rows in the truth table is 2^n , where n is the number of variables (inputs) in the function.

Ex: $F_1 = x + y'z$

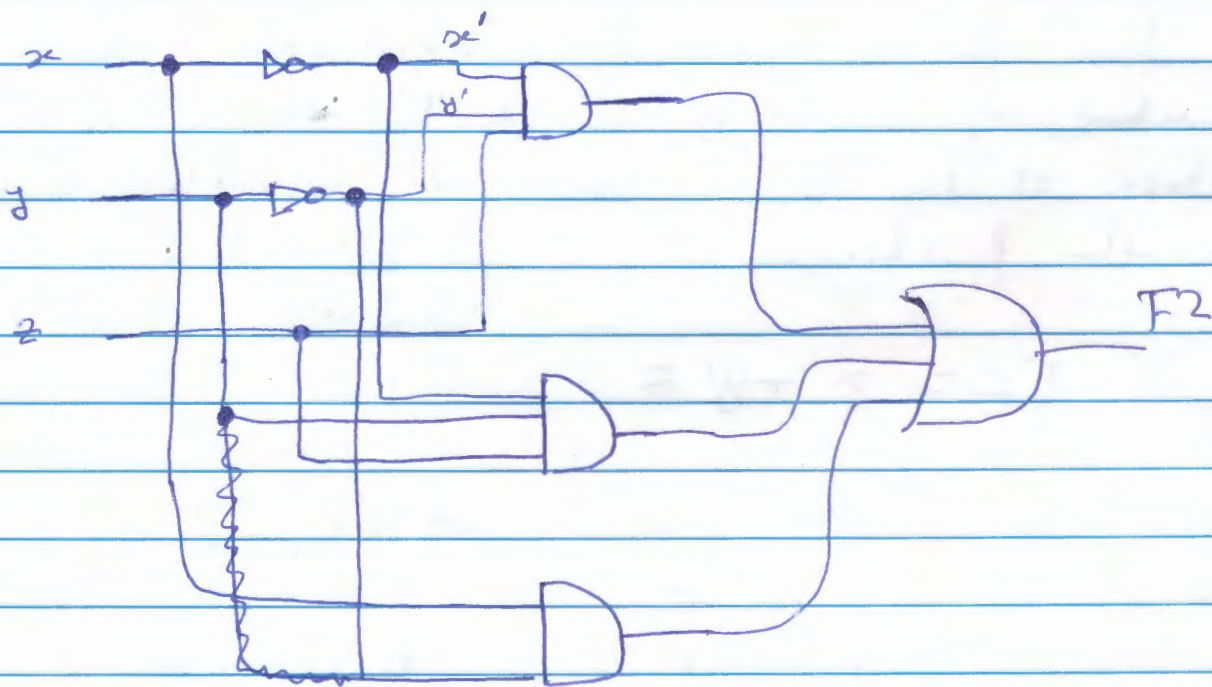
x	y	z	y'	$y'z$	$x + y'z = \underline{\underline{F_1}}$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	0	0	1

- Gate implementation of F_1



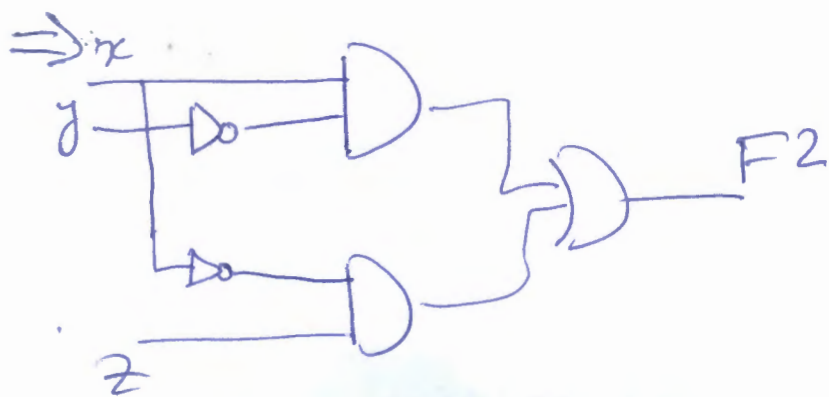
F2

$$F_2 = x'y'z + x'yz + xy'$$



The question: Is it possible to make a simpler implementation??

$$\begin{aligned} F_2 &= x'y'z + x'yz + xy' \\ &= x'z(y + y') + xy' \\ &= x'z + xy' \end{aligned}$$



~~tsim~~

- ⊛ This is a simpler implementation more than the previous one.
- ⊛ Algebraic Manipulation was used to achieve the simple formula.
- ⊛ Other methods for simplification will be described later.