جامعة بيرزيت

BIRZEIT UNIVERSITY

ANSWER BOOKLET

Student: Digital		Number: 4	
Course:	Department:	Number:	
	Division:	Instructor:	
Date: Day	Month	Year	

Dr. Abdellatif Abu Issa

For Instructor's Use

Question	Grade
1	
2	
3	
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8	
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10	
11	
12	
Total	

The Tabulation Method

Determination of Prime Implicants

Ex: $F(w, x, y, z) = \Sigma(0, 1, 2, 8, 10, 11, 14, 15)$

Step 1

	(a) w x y z	(b)		(c)	
0	0 0 0 0 ✓	(0,1)	0 0 0 –	(0,2,8,10)	– 0 – 0
		(0,2)	0 0 – 0 ✓	(0,8,2,10)	– 0 – 0
1	0 0 0 1 ✓	(0,8)	– 0 0 0 ✓		
2	0 0 1 0 ✓			(10,11,14,15)	1 – 1 –
8	1 0 0 0 ✓	(2,10)	– 0 1 0 ✓	(10,14,11,15)	1 – 1 –
		(8,10)	1 0 – 0 ✓		
10	1 0 1 0 ✓				
		(10,11)	1 0 1 – ✓		
11	1 0 1 1 ✓	(10,14)	1 – 1 0 ✓		
14	1 1 1 0 ✓				
		(11,15)	1 – 1 1 ✓		
15	1 1 1 1 ✓	(14,15)	1 1 1 – ✓		

⇒ Prime implicants are

(0,1) : 0 0 0 – → $w'x'y'$

(0,2,8,10) : – 0 – 0 → $x'z'$

(10,11,14,15) 1 – 1 – → wy

⊛ using decimal notations only for determination of prime implicants

(a)	(b)		c	
0 ✓	0,1	(1)	0,2,8,10	(2,8)
—	(0,2)	(2) ✓	0,2,8,10	(2,8)
1 ✓	(0,8)	(8) ✓		
2 ✓	—		10,11,14,15	(1,4)
8 ✓	2,10	(8) ✓	10,11,14,15	(1,4)
—	8,10	(2) ✓		
10 ✓	—			
—	(10,11)	(1) ✓		
11 ✓	10,14	(4) ✓		
14 ✓	—			
—	11,15	(4) ✓		
15 ✓	14,15	(1) ✓		

prime implicants are

Decimal	Binary w x y z	Term
~~0,2~~		
0,1 (1)	0 0 0 —	$w'x'z'$
~~0,2 (2)~~	~~0 0~~	
0,2,8,10	— 0 — 0	$x'z'$
10,11,14,15	1 — 1 —	wy

* Selection of prime implicants that minimize the function

Term	Decimal	0	1	2	8	10	11	14	15
~~$w'x'y'$~~									
~~$x'z'$~~									
$w'x'y'$	0, 1 (1)	X	X						
	~~0, 2, 8, 10~~								
$x'z'$	0, 2, 8, 10 (2, 8)	X		X	X	X			
wy	10, 11, 14, 15 (1, 4)					X	X	X	X
		✓	✓	✓	✓	✓	✓	✓	✓

all of them are essential prime implicants $\Rightarrow$

$$F = w'x'y' + x'z' + wy$$

Ex. $F(w, x, y, z) = \Sigma(1, 4, 6, 7, 8, 9, 10, 11, 15)$

	(a)	(b)	(c)
0001	1 ✓	1,9 (8)	8,9,10,11 (1,2)
0100	4 ✓	4,6 (2)	8,9,10,11 (1,2)
1000	8 ✓		
	—	8,9 (1) ✓	
0110	6 ✓	8,10 (2) ✓	
1001	9 ✓	—	
1010	10 ✓	6,7 (1)	
	—	9,11 (2) ✓	
0111	7 ✓	10,11 (1) ✓	
1011	11 ✓	—	
	—	7,15 (8)	
1111	15 ✓	11,15 (4)	

⇒ Prime implicants

~~1,9 (8)~~

Decimal	Binary w	x	y	z	Term
1,9 (8)	–	0	0	1	$x'y'z$
4,6 (2)	0	1	–	0	$w'xz'$
6,7 (1)	0	1	1	–	$w'xy$
7,15 (8)	–	1	1	1	xyz
11,15 (4)	1	–	1	1	wyz
8,9,10,11 (1,2)	1	0	–	–	wx'

Selection of prime implicants

			1	4	6	7	8	9	10	11	15
✓	$x'y'z$	1,9	X					X			
✓	$w'xz'$	4,6		X	X						
	$w'xy$	6,7			X	X					
	xyz	7,15				X					X
	wyz	11,15								X	X
✓	wx'	8,9,10,11					X	X	X	X	
			✓	✓	✓		✓	✓	✓	✓	

Essential prime implicants

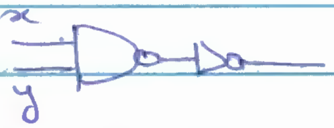
$x'y'z$, $w'xz'$, wx'

take xyz from the prime implicants for the simplest form

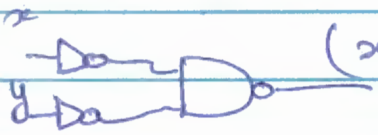
$$\Rightarrow F = x'y'z + w'xz' + wx' + xyz$$

NAND AND NOR Implementations

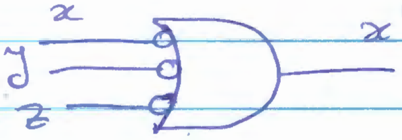
2 level implementation

- AND 

xy

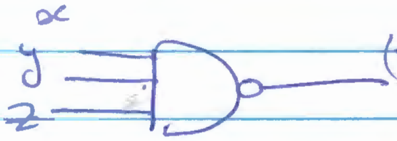
- OR 

$(x'y')' = x+y$



$x'+y'+z' = (xyz)'$

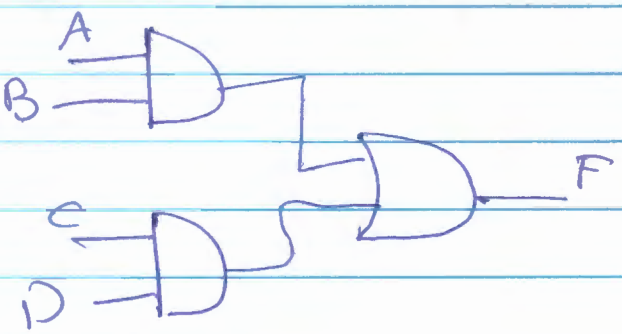
$\equiv$



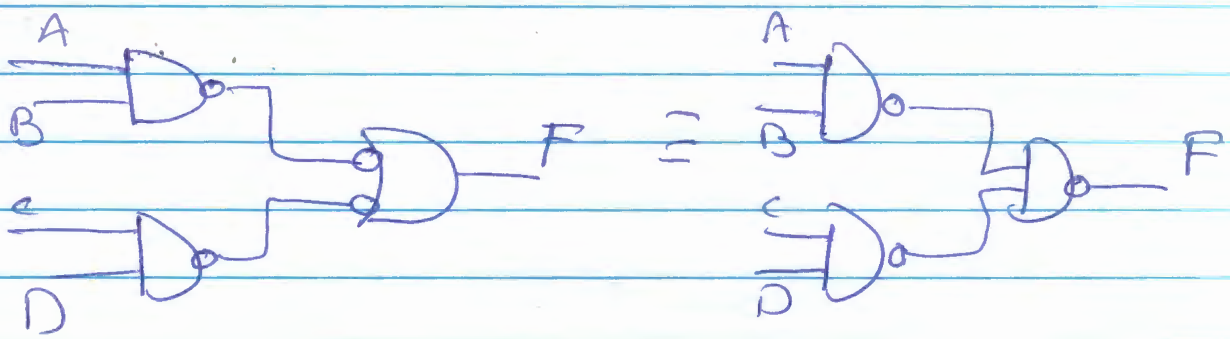
$(x'+y'+z') = (xyz)'$

Two level Implementation

Ex. $F = AB + CD$



(a)

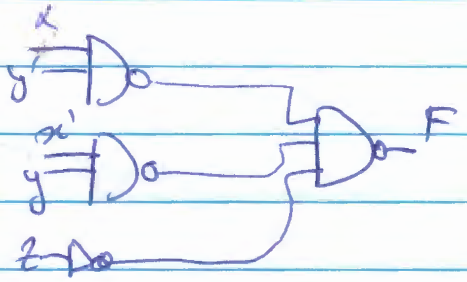


Ex.

$F(x,y,z) = \Sigma(1,2,3,4,5,7)$

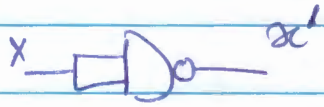
$\Rightarrow$ using k-map $\Rightarrow F = xy' + x'y + z$

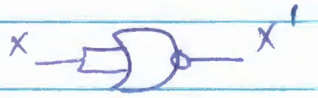
$\Rightarrow$



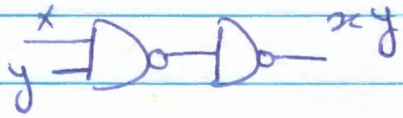
④ How NAND and NOR are functional complete

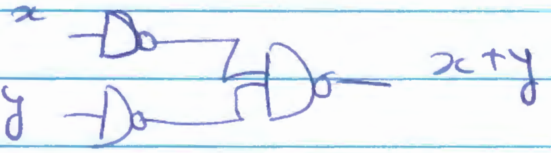
① NOT





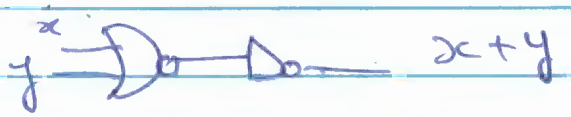
② AND

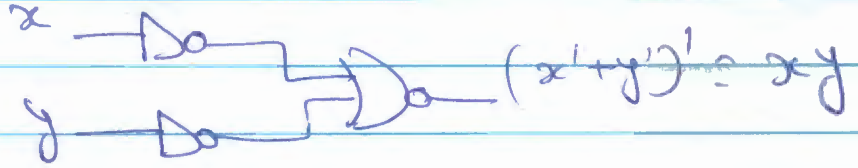




$(x'y')' = x+y$

⊛ NOR

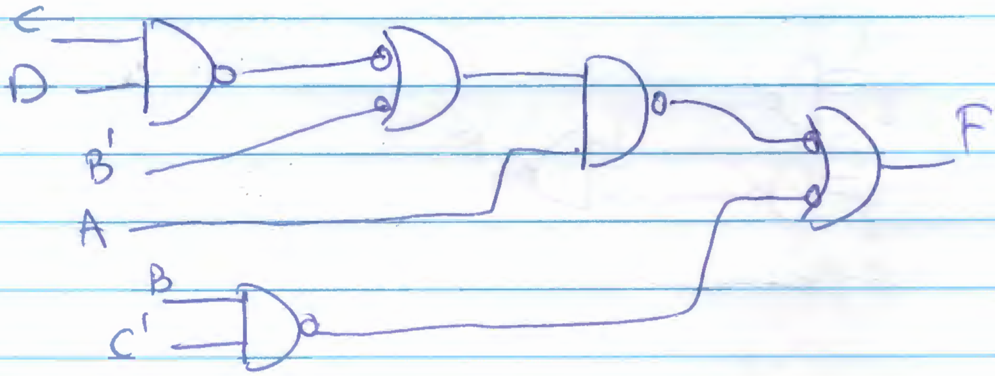




⊛ Multilevel NAND circuit

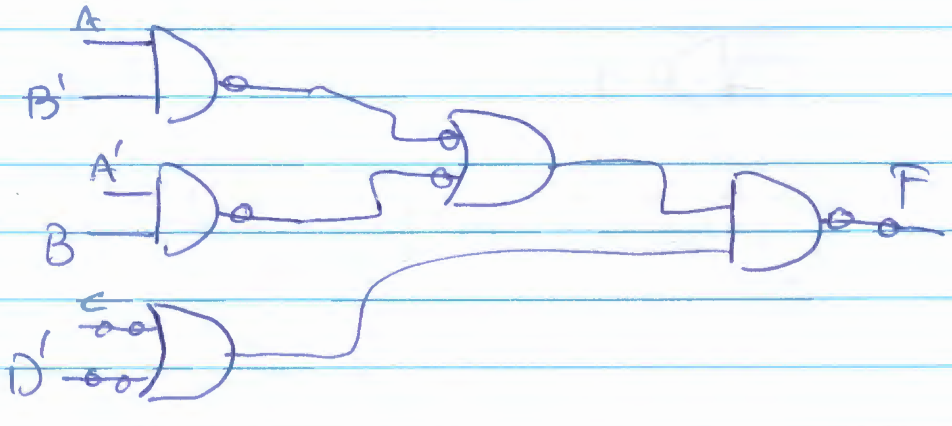
Ex.

$F = A(CD+B) + BC'$



Ex.

$F = (AB' + A'B)(C+D')$

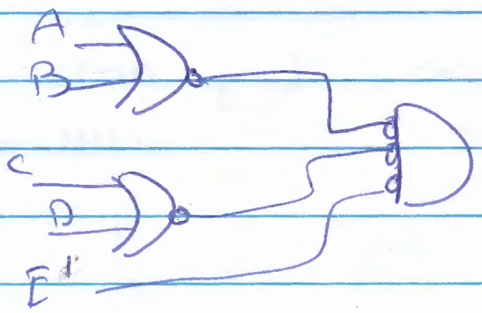


NOR

NOR (OR-invert) or NOR (Invert-AND)

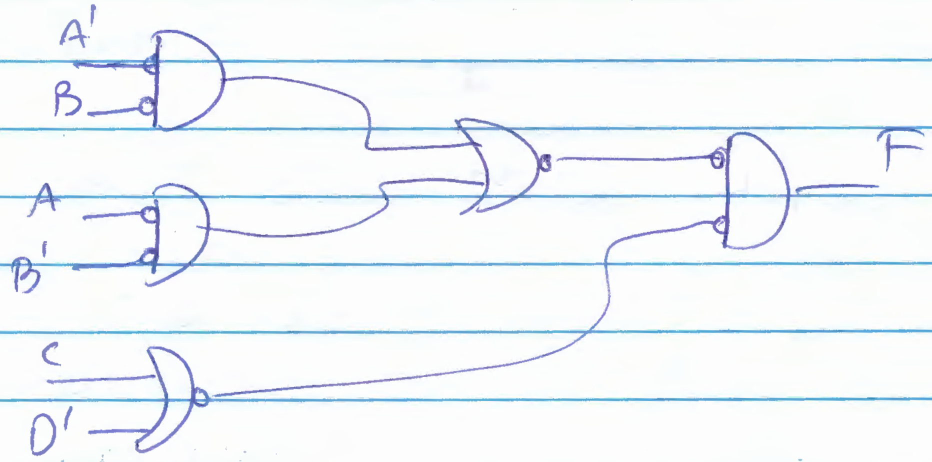
Ex.

$F = (A+B)(C+D)E$



Ex.

$F = (AB' + A'B)(C+D')$



⊛ Wired Logic

⊛ Nondegenerate form

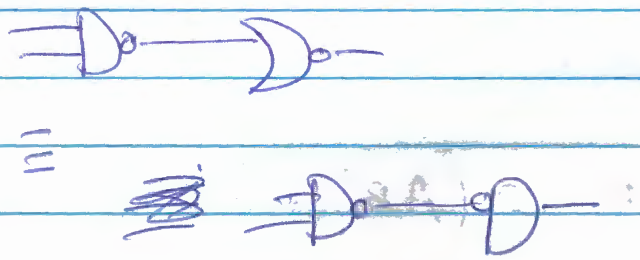
AND
OR
NAND
NOR

} 2 level → 16 forms

- 8 degenerate form (e.g. AND - AND
  NAND - NOR

AND - AND

NAND - NOR



≡ AND - AND

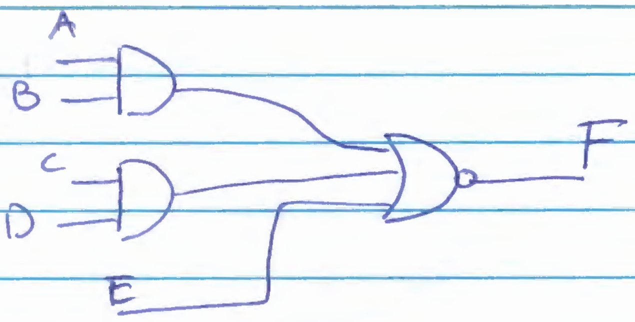
- Non degenerate form (8 forms)

AND - OR	OR - AND
NAND - NAND	NOR - NOR
NOR - OR	NAND - AND
OR - NAND	AND - NOR

dual for each other

Ex. $F = (AB + CD + E)'$

AND - OR - invert



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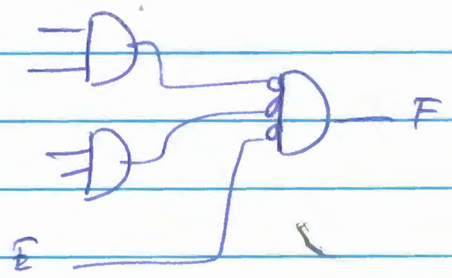
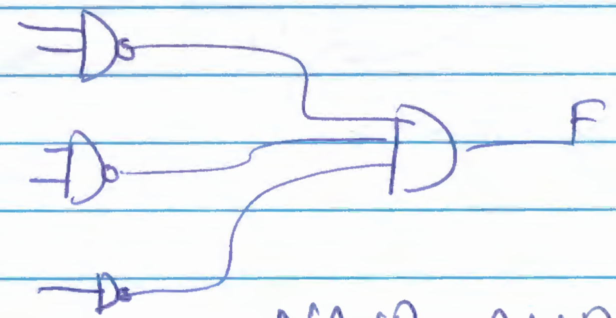


AND - NOR

AND - NOR

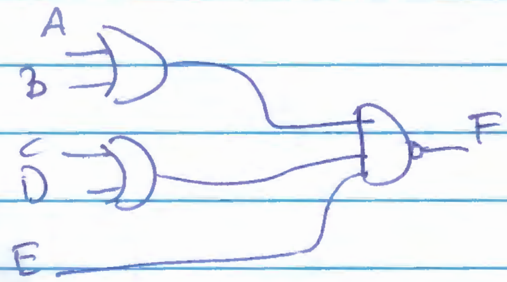
=



NAND - AND

Ex. $F = [(A+B)(C+D)E]'$

OR - AND - invert



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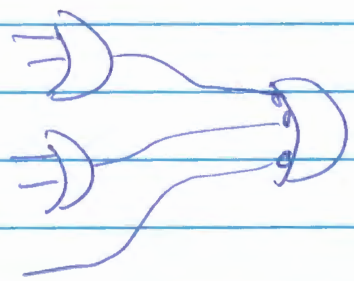
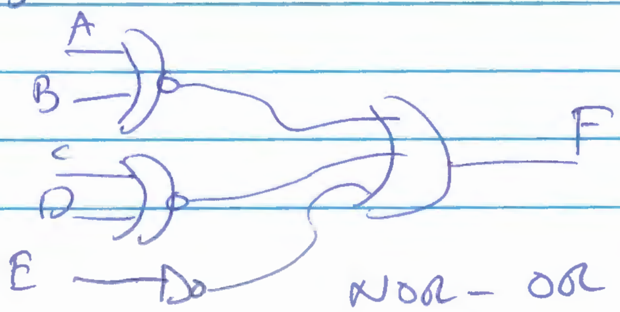


OR - NAND

=



8-9 and

NOR - OR

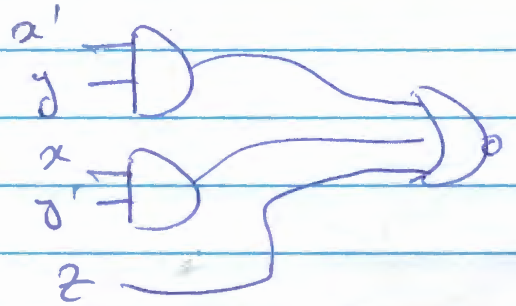
④ In the last 4 cases, we look to simplify f', then use inverter to get f.

$F(x,y,z)\Sigma(0,6)$ → Page 93

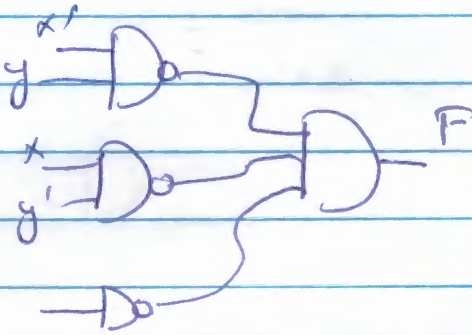
ex. $f' = x'y + xy' + z$

$F = (x'y + xy' + z)'$

① AND - NOR



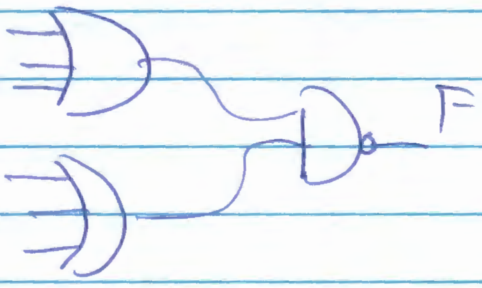
② NAND - AND



③ OR - NAND $(F = x'y'z' + xyz')$

$\rightarrow F' = (x+y+z)(x'+y'+z)$

$\rightarrow F = \{(x+y+z)(x'+y'+z)\}'$



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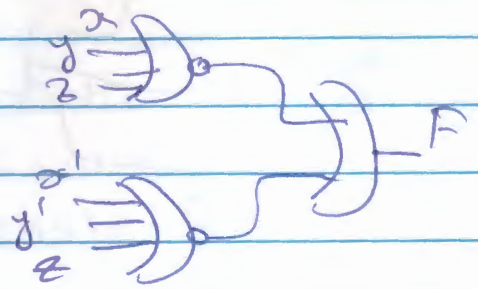
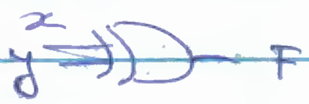


Exclusive-OR function:-

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0



$F = x \oplus y$

$x \backslash y$	0	1
0		1
1	1	

$F = xy' + x'y$

$(x \oplus y)' = xy + x'y'$

* The following are some properties of XOR

$x \oplus 0 = x$

$x \oplus 1 = x'$

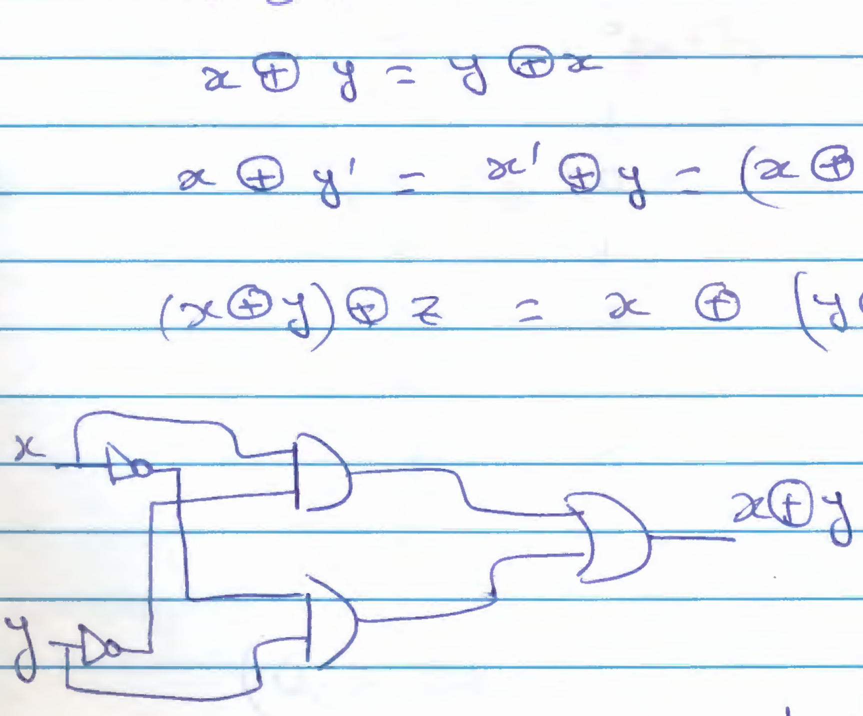
$x \oplus x = 0$

$x \oplus x' = 1$

$x \oplus y = y \oplus x$

$x \oplus y' = x' \oplus y = (x \oplus y)'$

$(x \oplus y) \oplus z = x \oplus (y \oplus z) = x \oplus y \oplus z$



(also using nand-nand).

* XOR is odd function

ex $A \oplus B \oplus C = (A \oplus B) \oplus C$

$$= (AB' + A'B)C' + (AB + A'B')C$$

$$= AB'C' + A'BC' + ABC + A'B'C$$

$$= \Sigma(1, 2, 4, 7)$$

A \ BC	00	01	11	10
0		1		1
1	1		1	

$\Rightarrow F = (A \oplus B \oplus C)'$ is an even function

i.e

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

* The same thing for $f(A, B, C, D)$

$$f(A, B, C, D) = A \oplus B \oplus C \oplus D$$

$= (AB' + A'B) \oplus (CD' + C'D)$

$= \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$

* Parity Generation and Checking

- A parity bit is an extra bit included with a binary message to make the number of 1's either odd or even.

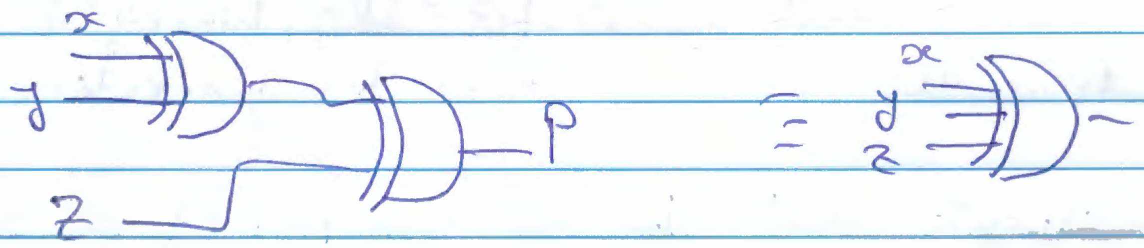
- the circuit that generates the binary bit in the transmitter is called a parity generator.

- The circuit that checks the parity in the receiver is called a parity checker.

Ex: 3 bits x, y, z to be transmitted with an even parity $\Rightarrow$

Three-Bit Message			Parity bit
x	y	z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$P = x \oplus y \oplus z$$



- These 3 bits together with the parity bit are transmitted to their destination
- If the number of 1's in the receiver is odd → error occured during transmission.