Chapter 3 SOLUTIONS

1 a) $I_D = \dfrac{V_{in} - 2V_{Don}}{R_1 + R_2}$

$= \dfrac{2.5 - 1.4}{4k\Omega} = \dfrac{1.1V}{4k\Omega}$

$\boxed{I_D = 275\mu A}$

b) $I_S = 10^{-14}A$, $T = 300K$, $V_{Don} = 0.7V$

$I_D = I_S(e^{V_D/\phi_T} - 1)$

where $\phi_T = \dfrac{kT}{q} = 26mV$ @ 300 K

(*) $I_D = \dfrac{V_{in} - 2V_D}{R_1 + R_2} = I_S(e^{V_D/\phi_T} - 1)$

$\dfrac{2.5 - \cancel{1.4}}{4k\Omega} = 10^{-14}(e^{V_D/0.026} - 1)$

iterating on this expression we can obtain.

$\boxed{V_D = 0.628V}$
$\boxed{I_D = 311\mu A}$

c) SPICE

```
exercise 3.1

vin in 0 dc 2.5
r1 in 2 2k
d1 2 3 D
r2 3 out 2k
d2 out 0 D

.options post=2
.op

.model D D level=1 is=1e-14 m=0.5

.end

*****************************
NODE      VOLTAGE
2         1.8750E+00
3         1.2500E+00
OUT       6.2501E-01

CURRENT   3.1249E-04
```

d) $I_S = 10^{-16}A$, $T = 300K$;

again use (*) from b and iterate to find:

$\boxed{V_D = 0.743V}$
$\boxed{I_D = 253\mu A}$

$I_S = 10^{-14}A$, $T = 350K$;

$\boxed{V_D = 0.728V}$
$\boxed{I_D = 261\mu A}$

```
exercise 3.1d
vin in 0 dc 2.5
r1 in 2 2k
d1 2 3 D
r2 3 out 2k
d2 out 0 D

.options post=2
.op

*.temp27
.temp 77
*.model D D level=1 is=1e-16 m=0.5
.model D D level=1 is=1e-14 m=0.5
.end
************************
t=27C°
NODE      VOLTAGE
2         1.9889E+00
3         1.2500E+00
IN        2.5000E+00
OUT       7.3892E-01

ID        2.5554E-04
************************
t=77C°
NODE      VOLTAGE
2         1.7844E+00
3         1.2500E+00
IN        2.5000E+00
OUT       5.3436E-01

ID        3.5782E-04
```

2

a) $I_D = 0$, $V_D = -V_S = 3.3V$

(More exact : $I_D = I_{Rev} = -I_S$

$V_D = -(V_S - I_S R_S)$)

b) Reverse biased

c) $$W_j = \sqrt{\left(\frac{2\varepsilon_{Si}}{q}\frac{N_A+N_D}{N_A N_D}\right)(\phi_0 - V_D)}$$

$q = 1.6\times10^{-19}C$, $V_D = -V_S = 3.3V$

$\varepsilon_{Si} = 11.7\varepsilon_0 = 1.035\times10^{-12} F/cm$

$N_A = 2.5\times10^{16}/cm^3$ $N_D = 7.5\times10^{15} \frac{Atoms}{cm^3}$

$$\boxed{w_j = 110.75\times10^{-6}\ cm}$$

UNIT CHECK: $$\sqrt{\frac{F/cm}{C}\frac{(\frac{1}{cm^3})}{(\frac{1}{cm^3})(\frac{1}{cm^3})}\cdot V}$$

$\sqrt{\frac{F}{C}cm^2 \cdot V} = cm$ ✓

Since 1 coulumb = 1 farad · 1 Volt

d) $$C_j = \frac{\varepsilon_{Si}\cdot A_D}{W_j}$$

w/ $A_D = 120\times10^{-9} cm^2$

$$\boxed{C_j = 1.12 fF}$$

e) $V_{S\,new} = 1.5V < V_{S\,old} = 3.3V$

The new voltage, reduces the reverse bias of the P-N junction, hence the width of the depl. region, w_j, decreases. As ~~you bring the plates of~~ a capacitor together, the capacitance increases.

3 a) $V_{GS} = 2.5V$, $V_{DS} = 2.5V$ sat

$$I_D = \frac{k'}{2}\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$$

$$= \frac{115\times10^{-6}}{2}(2.5-0.43)^2(1+0.06\times2.5)$$

$= 283.3\mu A$

$V_{GS} = -0.5V$, $V_{DS} = -1.25V$ sat

$$I_D = \frac{k'}{2}\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$$

$$= \frac{30\times10^{-6}}{2}(0.5-0.4)^2(1+0.1\times1.25)$$

$= 0.17\mu A$

b) $V_{GS} = 3.3V$, $V_{DS} = 2.2V$ linear.

$$I_D = k'\frac{W}{L}\left((V_{GS}-V_t)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

$$= 115\times10^{-6}\left((3.3-0.43)2.2 - \frac{2.2^2}{2}\right)$$

$= 447.8\mu A$

$V_{GS} = -2.5$, $V_{DS} = -1.8$ linear

$$I_D = k'\frac{W}{L}\left((V_{GS}-V_t)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

$$= 30\times10^{-6}\left((2.5-0.4)1.8 - \frac{1.8^2}{2}\right)$$

$= 64.8\mu A$

c) $V_{GS} = 0.6V$, $V_{DS} = 0.1V$ linear

$$I_D = 115\times10^{-6}\left((0.6-0.43)\times0.1 - \frac{0.1^2}{2}\right)$$

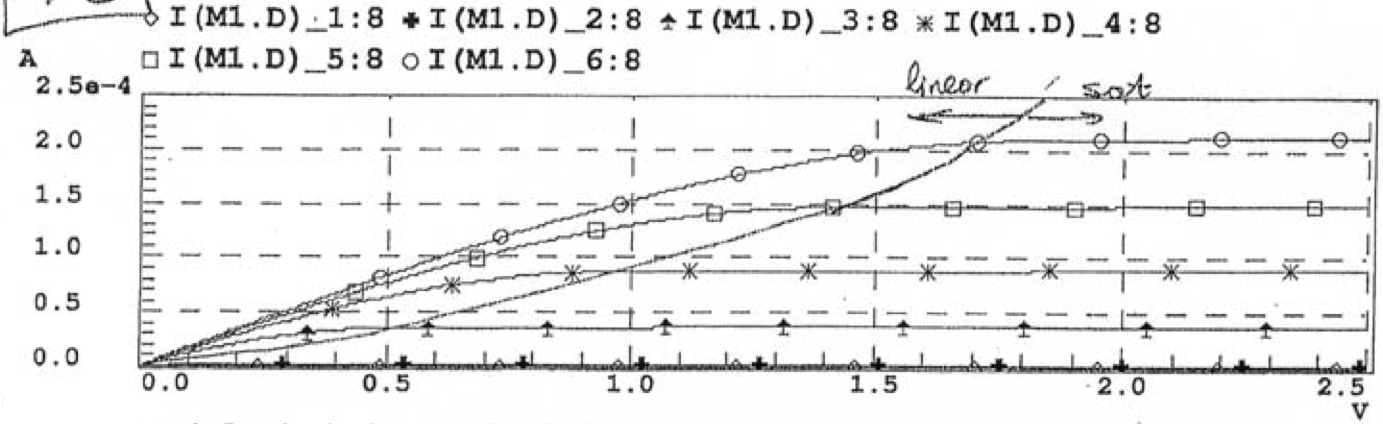
$= 1.38\mu A$

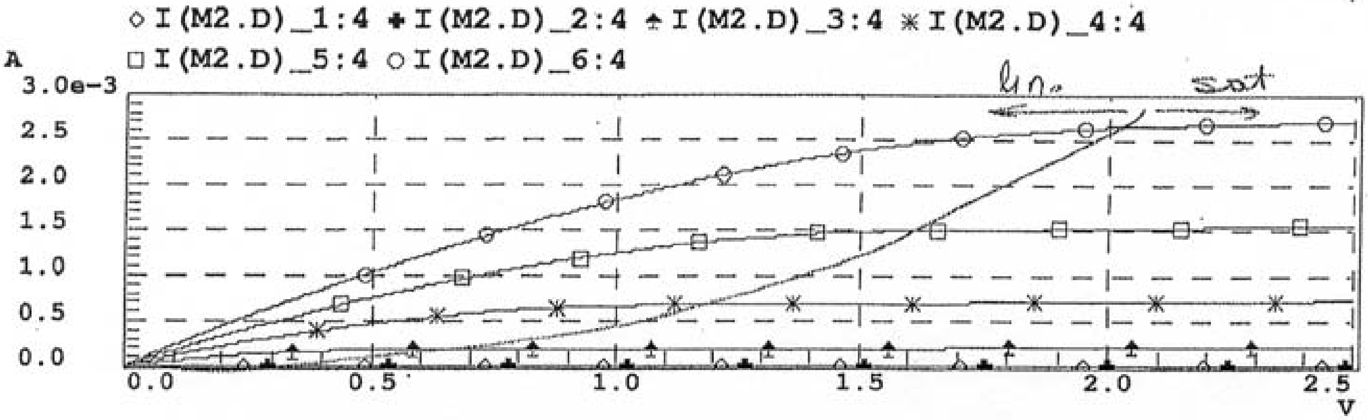
$V_{GS} = -2.5V$, $V_{DS} = 0.7V$ linear

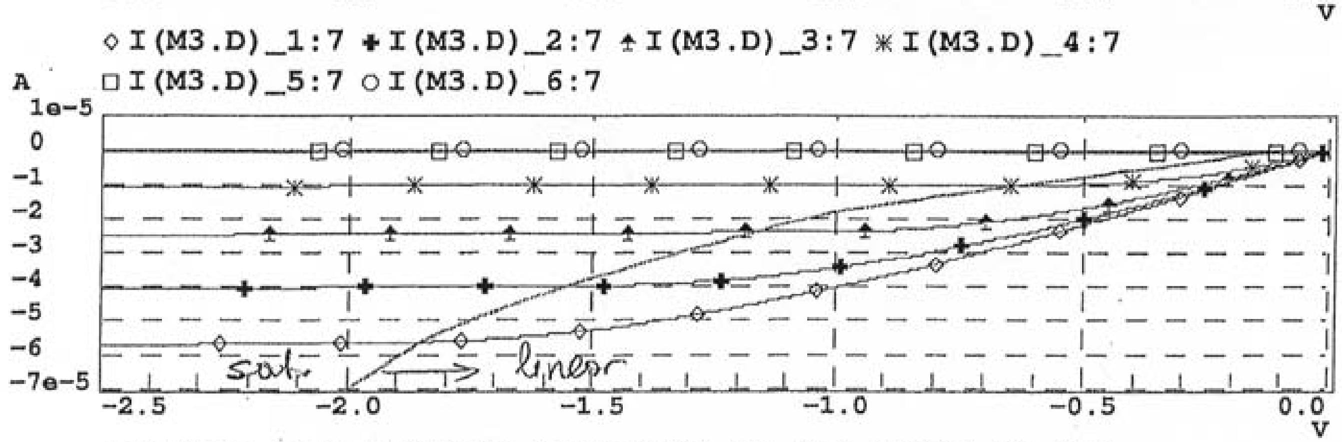
$$I_D = 30\times10^{-6}\left((2.5-0.4)\times0.7 - \frac{0.7^2}{2}\right)$$

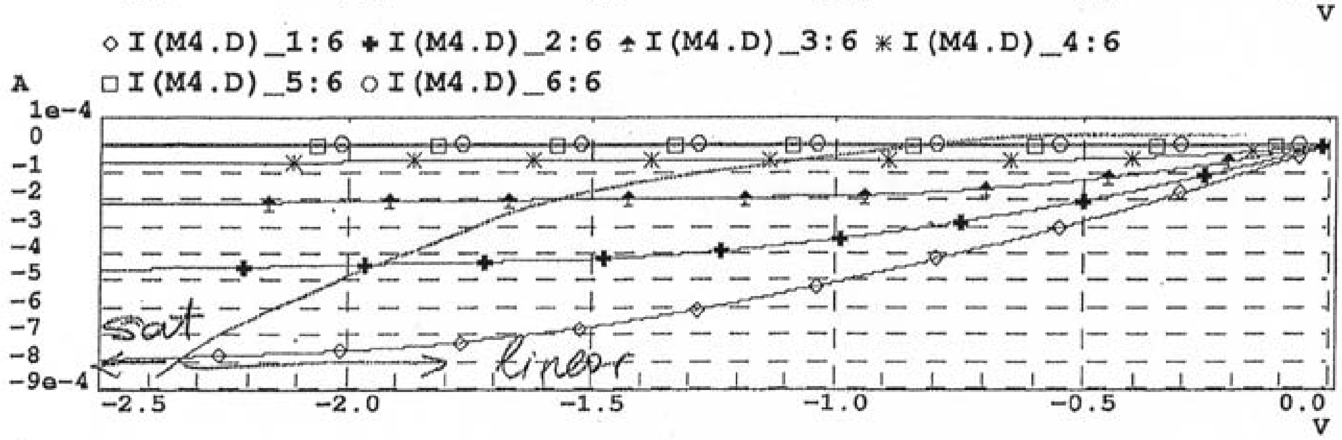
$= 36.75\mu A$

4-5









c) 1 & 3 are in velocity sat, The effect can be seen from the linearly increasing I_D.

6 for a short channel device

$$I_D = k' \frac{W}{L}\left[(V_{GS}-V_t)V_{min} - \frac{V_{min}^2}{2}\right](1+\lambda V_{DS})$$

$$V_{min} = \min\left[(V_{GS}-V_t), V_{DS}, V_{DSAT}\right]$$

To begin with the operation regions need to be determined.

For any of these data to be in saturation. V_T should be ; $V_{GS}-V_T < V_{DSAT}$

$2 - V_T < 0.6 \Rightarrow 1.4 < V_T$

This is a quite high value in our process. Thus, we can assume that all data are taken in velocity saturation. We will check this assumption later.

In velocity sat:

$$I_D = k' \frac{W}{L}\left[(V_{GS}-V_t)V_{DSAT} - \frac{V_{DSAT}^2}{2}\right](1+\lambda V_{DS})$$

using 1 & 2.

$$I_D = k' \frac{W}{L}\left[(2.5-V_{t_0})\,0.6 - \frac{0.6^2}{2}\right](1+\lambda 1.8) = 1812$$

$$I_D = k' \frac{W}{L}\left[(2-V_{t_0})\,0.6 - \frac{0.6^2}{2}\right](1+\lambda .18) = 1297$$

$$\frac{1812}{1297} = \frac{(2.5-V_{t_0})\,0.6 - \frac{0.6^2}{2}}{(2-V_{t_0})\,0.6 - \frac{0.6^2}{2}} \Rightarrow \boxed{V_{t0} = 0.44\,V}$$

$[V_{t_0} < 1.4\,V]$ so 1,2,3 are in vel. sat.

using 2&3

$$\frac{1297}{1361} = \frac{1+\lambda 1.8}{1+\lambda 2.5} \Rightarrow \boxed{\lambda = 0.08\,V^{-1}}$$

using 2&4

$V_t = 0.587V$ (1)

using 2 & 5 : $\Rightarrow V_t = 0.691\,V$ (2)

both these values satisfy $V_t < 1.4V$ so all the data in our table were taken in velocity saturation.

$$V_t = V_{t0} + \gamma\left(\sqrt{|V_{SB}| + |2\phi_f|} - \sqrt{|2\phi_f|}\right)$$

(1)&(2) can be used along with $V_{t0} = 0.44V$ to conclude

$\boxed{|2\phi_f| = 0.6V}$ and $\boxed{\gamma = 0.3V^{1/2}}$

also using 2nd set of data

$$I_D = 1297\,\mu A = k' \frac{W}{L}\left[(V_{GS}-V_t)V_{DSAT} - \frac{V_{DSAT}^2}{2}\right]$$

$$\boxed{\frac{W}{L} = 15}$$

7 a) This is a PMOS device

b) using measurements 1&4 $V_{t0} = 0.5V$

c) Using 1&5 : $\gamma = 0.538\,V^{1/2}$

d) Using 1&6 : $\lambda = 0.05\,V^{-1}$

e) 1- Vel. Sat, 2- cut off, 3-saturation, 4-5-6 - Vel. Sat., 7- linear

8 i) When $R = 10k$, $V_D = V_{DD} - IR$

$\Rightarrow V_D = 2.5 - 50\times10^{-6}\times10^4 = 2.5 - 0.5 = 2V$

assume the device is in saturation: (needs to be verified eventually.)

$$I_D = \frac{k'}{2}\frac{W}{L}(V_{GS}-V_t)^2 = 50\,\mu A$$

$\Rightarrow V_{GS} - V_t = 0.3\,V \Rightarrow V_{GS} = 0.3 + 0.4 = 0.7V$

$V_S = 2 - 0.7 = 1.3V$

$V_{min} = \min(V_{GS}-V_t, V_{DSAT}, V_{DS}) = \min(0.3, 0.6, 0.7)$

$= V_{GS} - V_t \Rightarrow$ saturation verified.

$\boxed{V_D = 2V \quad V_S = 1.3V}$ $\boxed{\text{saturation operation}}$

b) $V_D = 2.5 - 30\times10^3\times50\times10^{-6} = 2.5 - 1.5$ $\boxed{V_D = 1V}$

assume linear op:

$$I_D = k'\frac{W}{L}\left[(V_{GS}-V_t)V_{DS} - \frac{V_{DS}^2}{2}\right] = 50\,\mu A$$

$$110\times10^{-6}\times10\left[(2 - V_S - 0.4)(1-V_S) - \frac{(1-V_S)^2}{2}\right] = 50\mu$$

$\Rightarrow \boxed{V_S = 0.93V}$

$\min(V_{GS}-V_T, V_{DS}, V_{DSAT}) = \min((2-0.93-0.4), 0.07, 0.$

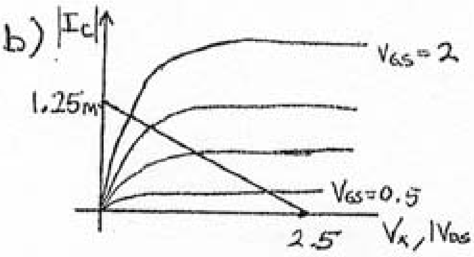
$= V_{DS} \Rightarrow \boxed{\text{linear}}$ verified cont

c) $\boxed{\text{increase.}}$ V_D is fixed due to const. current. $(1+\lambda V_{DS})$ term would try to increase the current more than available $50\mu A$, thus V_{GS} needs to reduce by increasing V_S.

9 Device is always in saturation.

a) $$\frac{-V_x}{R} = \frac{k_p'}{2}\frac{W}{L}\left(V_x - |V_{tp}|\right)^2$$

b)

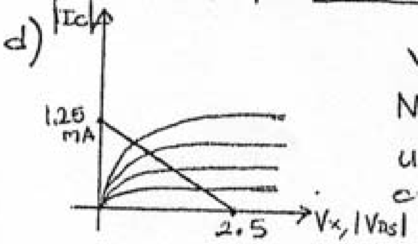


c) $$\frac{1V}{20k\Omega} = \frac{30\times10^{-6}}{2}\left(\frac{W}{L}\right)\times(1.5-0.4)^2$$

$$50\mu A = 15\times10^{-6}\left(\frac{W}{L}\right)1.21$$

$$2.755 = \left(\frac{W}{L}\right) \Rightarrow \boxed{W \approx 0.69\mu m}$$

d)



Voltage @ Node X would go up since the current drive of PMOS is lower.

10 a) $$I_D = \frac{k_n'}{2}\frac{W}{L}(V_i - V_o - V_t)^2$$

$$\sqrt{\frac{2I_D}{k_n'\frac{W}{L}}} = V_i - V_o - V_t$$

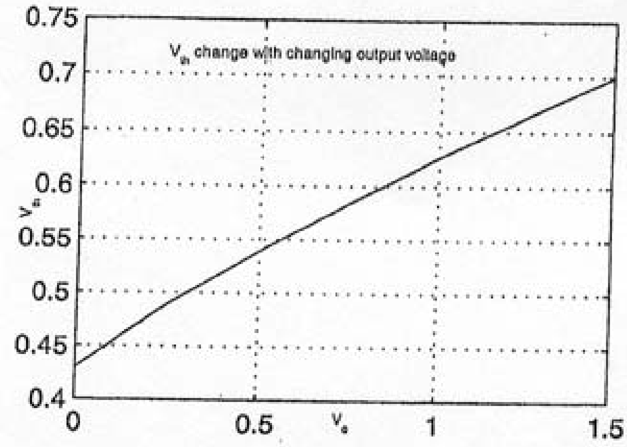
neglecting body effect $V_t = V_{to}$

$$V_i = \underbrace{\sqrt{\frac{2I_D}{k'\left(\frac{W}{L}\right)}} + V_{to}}_{\text{Level Shift (LS)}} + V_o$$

$$LS = V_{to} + \sqrt{\frac{2I_D}{k'\left(\frac{W}{L}\right)}} = 0.43 + \sqrt{\frac{2\times 35\mu A}{115\mu A/V^2 \cdot 3}} = \boxed{0.88V = LS.}$$

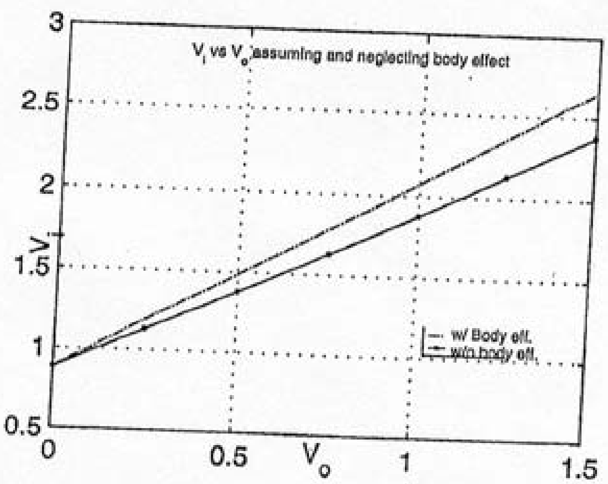
b) $$V_t = V_{to} + \gamma\left(\sqrt{|V_o| + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$$

$$= 0.43 + 0.4\left(\sqrt{V_o + 0.6} - \sqrt{0.6}\right)$$



c) $$V_o = V_i - \sqrt{\frac{2I_D}{k'\frac{W}{L}}} - V_t$$

Plot with 1) $V_t = V_{to}$, 2) $V_t = V_t(V_o)$



11 a) In saturation

$$\frac{V_{DD}-V_{out}}{R} = \frac{k'}{2}\frac{W}{L}(V_{in}-V_t)^2$$

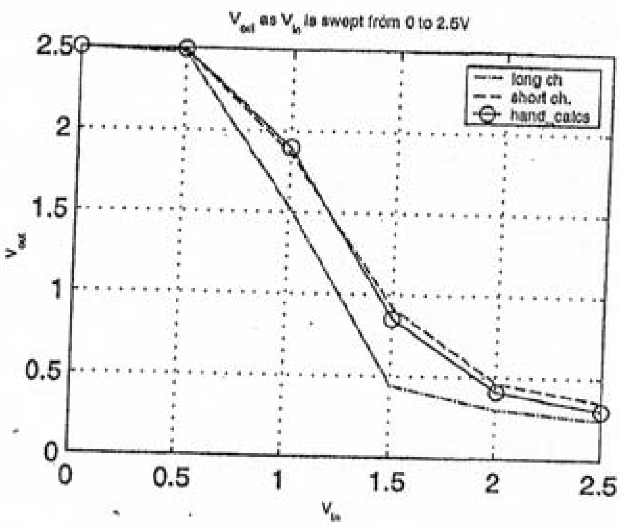
In triode:

$$\frac{V_{DD}-V_{out}}{R} = k'\frac{W}{L}\left[(V_{in}-V_t)V_{out}-\frac{V_{out}^2}{2}\right]$$

$V_{DD}=2.5$, $R=8k\Omega$, $k'=115\,\mu A/V^2$

$V_t = V_{t0} = 0.43V$

b) Results and spice follow




```
exercise 3.12
.lib g25.lib TT

vdd vdd 0 dc 2.5

vin in 0 dc 2.5
r1 vdd out_long 8k
m1 out_long in s1 0 nmos_t l=0.5u w=4u
m2 s1 in 0 0 nmos_t l=0.5u w=4u

r2 vdd out_short 8k
m3 out_short in 0 0 nmos_t l=0.25u w=1u

.probe .dc v(in) v(out) v(out1)
.dc vin 0 2.5 0.1
.options post=2 csdf
.op
.end
************************************
```

Vin	Vout_long	Vout_short	Vout_hand
0.0	2.500	2.50	2.50
0.5	2.470	2.48	2.49
1.0	1.537	1.86	1.90
1.5	0.442	0.92	0.85
2.0	0.301	0.46	0.41
2.5	0.244	0.35	0.30

c) The long device was modeled as two transistors in series. The equivalent transistor has a steeper transition

12

V_{T0} This one should immediately signal you to look at a curve(s) that don't have body-effect. That means $V_{BS} = 0V$. Pick two points, each from different curves that satisfy the no-body-effect condition. Make sure they're in the same operating region too!

Point	V_{GS}	V_{DS}	I_D	Operating Region
A	2.5V	1.8V	300uA	saturation
B	2.0V	1.8V	160uA	saturation

The reason why I chose points with the same V_{DS} will be evident once I work through the math.

$$\frac{I_{D,A}}{I_{D,B}} = \frac{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,A}-V_{T0})^2(1+\lambda\cdot V_{DS,A})}{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,B}-V_{T0})^2(1+\lambda\cdot V_{DS,B})}$$

$$\frac{300}{160} = \frac{(2.5-V_{T0})^2}{(2.0-V_{T0})^2}$$

$V_{T0} = 0.64V$

As you can see, in order for me to isolate V_{T0}, I needed to make sure I can cancel as many variables to be able to solve the equation.

λ We can use the same methodology as above. This time, we want to keep V_{GS} constant.

Point	V_{GS}	V_{DS}	I_D	Operating Region
A	2.5V	2.4V	310uA	saturation
B	2.5V	1.8V	300uA	saturation

$$\frac{I_{D,A}}{I_{D,B}} = \frac{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,A}-V_T)^2(1+\lambda\cdot V_{DS,A})}{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,B}-V_T)^2(1+\lambda\cdot V_{DS,B})}$$

$$\frac{310}{300} = \frac{(1+\lambda\cdot 2.4)}{(1+\lambda\cdot 1.8)}$$

$\lambda = 0.0617V^{-1}$

γ It shouldn't be a surprise, but that leaves us to keep almost everything constant except for V_{SB}.

Point	V_{SB}	V_{GS}	V_{DS}	I_D	Operating Region
A	1.0V	2.0V	1.2V	105uA	saturation
B	0.0V	2.0V	1.2V	150uA	saturation

12 cont'd

$$\frac{I_{D,A}}{I_{D,B}} = \frac{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,A} - V_T)^2(1+\lambda \cdot V_{DS,A})}{\frac{1}{2}k_p\left(\frac{W}{L}\right)(V_{GS,B} - V_{T0})^2(1+\lambda \cdot V_{DS,B})}$$

$$\frac{105}{150} = \frac{(2.0 - V_T)^2}{(2.0 - 0.64)^2}$$

$V_T = 0.862\text{V}$

Now solve for γ using the following equation:

$$V_T - V_{T0} = \gamma\left(\sqrt{|V_{SB} - 2\Phi_F|} - \sqrt{-2\Phi_F}\right)$$

$$0.862 - 0.64 = \gamma\left(\sqrt{|1+0.6|} - \sqrt{0.6}\right)$$

$\gamma = 0.453\text{V}^{1/2}$

13

$V_{in} = 0.2 \Rightarrow$

$I_{DS} = 3\times10^{-8}\text{A}$ 1

or

$V_{in} = 0.2 \Rightarrow$

$I_{DS} = 5\times10^{-9}\text{A}$ 2

$\Delta t = C\,\frac{\Delta V}{I}$

$\Delta t_1 = 1\,\text{pF} \times \frac{1}{3\times10^{-8}}$

$= 33.3\,\mu s$

$\Delta t_2 = 1\text{pF} \times \frac{1\text{V}}{5\times10^{-9}}$

$= 200\,\mu s$

$\boxed{14}$

a) $I_{DS} = Q\nu$ (ν = velocity)

$Q = CV$ (V = voltage)

$C = W\cdot C_{ox}$ $\quad V = V_{GS} - V_t$

$I_{DS} = W C_{ox}(V_{GS} - V_t)\,\nu$

b) $\nu = \mu_n \cdot E$

$E = \frac{(V_{GS} - V_t)}{2L}$

$I_{DS} = W C_{ox}(V_{GS} - V_t)\frac{(V_{GS} - V_t)}{2L}$

$I_{DS} = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

c) $\nu = \nu_{max}$ – constant

$I_{DS} = W C_{ox}(V_{GS} - V_t)\,\nu_{max}$

d)

1) $I_D \propto W$ not $\frac{W}{L}$

2) $I_D \propto (V_{GS} - V_t)^1$

not $(V_{GS} - V_t)^2$

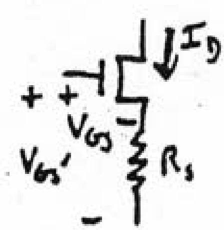
$\boxed{15}$

a) $I_D = \frac{K'}{2}\frac{W}{L}(V_{GS}' - V_t)^2$

$V_{GS}' = V_{GS} - I_D R_s$

$I_D = \frac{K'}{2}\frac{W}{L}\left[(V_{GS} - V_t)^2 - 2(V_{GS} - V_t)I_D R_s + \cancel{(I_D R_s)^2}\right]$

(neglect)



15a cont'd

$$I_D\left[1 + K'\frac{W}{L}(V_{GS}-V_t)R_S\right] = \frac{K'}{2}\frac{W}{L}(V_{GS}-V_t)^2$$

$$\therefore I_D = \frac{1}{1+\frac{K'W}{L}(V_{GS}-V_t)R_S} \cdot \frac{\mu C_{ox}}{2}\frac{W}{L}(V_{GS}-V_t)^2$$

Comparing w/ given short channel equation reveals:

$$\frac{K'W}{L}(V_{GS}-V_t)R_S = \frac{(V_{GS}-V_t)}{E_{sat}L}$$

$$\boxed{R_S = \frac{1}{\mu_n C_{ox} W E_{sat}}}$$

b) $E_{sat} = 1.5\,V/\mu m \quad K' = 20\,\mu A/V^2$

$$R_S = \frac{1}{W(1.5\,V/\mu m)(20\,\mu A/V^2)}$$

$$\boxed{R_S = \frac{33.33\,K\Omega}{W} \quad (W \text{ in } \mu m)}$$

Independent of channel length

16

a) First let us write the resistance as a function of the output voltage

$$R(V) = \frac{V}{I(V)} = \frac{V}{k*V*e^{V/V_0}} = \frac{1}{k*e^{V/V_0}}$$

Then, we need to average this resistance over the voltages of interest. A variant of the formula 3.42 in course notes can be written as

$$R_{eq} = \frac{1}{(V_2-V_1)}\int_{V1}^{V2} R(v)dv$$

plugging the $R(V)$ expression in and carrying out integral, we obtain

$$R_{eq} = \frac{1}{2V_0}\int_0^{2V_0}\frac{1}{k*e^{v/V_0}}dv = \frac{1}{2V_0}\frac{-V_0}{k}\left(e^{-2V_0/V_0}-1\right) = \frac{1}{2k}\left(1-e^{-2}\right) = \frac{0.423}{k}\Omega$$

b) Again we should obtain R(v) by starting from the I-V relation.

Note that the device will be operating in velocity saturation régime. This can be seen by comparing $V_{GS}-V_T = V_{DD}-V_T \approx 2.5-0.4 \approx 2.1$; VDSAT ≈ 0.6 and $V_{DS} > V_{DD}/2 = 1.25$, where $V_{DD} = 2.5$v and V_T and V_{DSAT} from Table 3.2 were used.

In velocity saturation region :

$I = k'W/L[(V_{DD}-V_T)V_{DSAT}-V_{DSAT}^2/2](1+\lambda V_{DS}) = I_{DSAT}(1+\lambda V_{DS})$,

Where, we define $I_{DSAT} = k'W/L\,[(V_{DD}-V_T)V_{DSAT}-V_{DSAT}^2/2]$. Using this I-$V_{DS}$ relation we can write the integral.

$$R_{eq} = -2/(V_{DD}I_{DSAT})\int_{V_{DD}}^{V_{DD}/2} dV_{DS}/(1+\lambda V_{DS})$$. Carrying out this integral we obtain

$R_{eq} = 2/(\lambda * V_{DD} * I_{DSAT}) * \{V_{DD}/2 - 1/\lambda[\ln(1+\lambda V_{DD}) - \ln(1+\lambda V_{DD}/2)]\}$

Now, we will replace the ln(1+x)'s with their respective Taylor expansions.

$\ln(1+\lambda V_{DD}) \approx \{\lambda V_{DD} - (\lambda V_{DD})^2/2 + (\lambda V_{DD})^3/3)\}$ and
$\ln(1+\lambda V_{DD}/2) \approx \{\lambda V_{DD}/2 - (\lambda V_{DD})^2/8 + (\lambda V_{DD})^3/24)\}$.

Subtracting these two expressions we get, $\ln(1+\lambda V_{DD}) - \ln(1+\lambda V_{DD}/2 \approx \{\lambda V_{DD}/2 - 3(\lambda V_{DD})^2/8 + 7(\lambda V_{DD})^3/24\}$.

Now let's insert this expression in the R_{eq} equation to get:
$R_{eq} = 2/(\lambda * V_{DD} * I_{DSAT}) * \{V_{DD}/2 - V_{DD}/2 + 3\lambda V_{DD}^2/8 - 7\lambda^2 V_{DD}^3/24\}$

Bringing the expression $3\lambda V_{DD}^2/8$ outside the curly brackets, we obtain
$R_{eq} = 2/(\lambda * V_{DD} * I_{DSAT}) * 3\lambda V_{DD}^2/8\{1 - 7\lambda V_{DD}/9\} = \underline{(3/4) * (V_{DD}/I_{DSAT})\{1 - 7\lambda V_{DD}/9\}}$

17

17 $C_{ox} = 6fF/\mu m^2$ $L_D = 0.5\mu m$ $W_D = 1\mu m$

	C_g
cut-off	$C_{ox}WL + 2C_oW$
Linear	$C_{ox}WL + 2C_oW$
sat - Vel. sat	$\frac{2}{3}C_{ox}WL + 2C_oW$

Diffusion Capacitance (C_d)

$C_d = C_j L_D W_D + C_{jsw}(2L_D + W_D)$: diffusion cap

$$C_j = \frac{C_{jo}}{\left(1+\frac{V_{DS}}{\phi_D}\right)^{m_j}} \qquad C_{jsw} = \frac{C_{jsw0}}{\left(1+\frac{V_{DS}}{\phi_0}\right)^{m_{jsw}}}$$

a) → $V_{in} = 2.5V$, $V_{out} = 2.5V$

Vel. saturation.

$\boxed{C_g = 1.62fF \quad Q = 4.05fC = 4.05\times10^{-15}C}$

$\boxed{C_d = 0.827fF}$

-) $V_{out} = 0.5V$

Linear region

$\boxed{C_g = 2.12fF \quad Q = 5.3fC = 5.3\times10^{-15}C}$

$\boxed{C_d = 1.263fF}$

-) $V_{out} = 0V$

Linear region

$\boxed{C_g = 2.12fF \quad Q = 5.3fC = 5.3\times10^{-15}C}$

$\boxed{C_d = 1.56fF}$

b) $V_{in} = 0 \Rightarrow$ Cut off

Regardless of V_{DS},

$C_g = C_{ox}WL$

$C_g = 2.12fF$, $Q = 0$

C_d, are the same as part b.

$V_{out} = 2.5 \Rightarrow C_d = 0.827fF$

$V_{out} = 0.5 \Rightarrow C_d = 1.263fF$

$V_{out} = 0 \Rightarrow C_d = 1.56fF$

18 a) The value of C_T can change after $V_g = V_T$

for $\boxed{\begin{array}{l} V_g: 0 \to V_T \Rightarrow C_T = C_T(1) \\ V_g: V_T \to 2V_T \Rightarrow C_T = C_T(2) \end{array}}$

then. $t_1 = C_T(1)\frac{V_T}{I_{in}}$,

$t_2 = C_T(2)\frac{(2V_T - V_T)}{I_{in}}$,

$$\boxed{t = [C_T(1) + C_T(2)]\frac{V_T}{I_{in}}}$$

b) C_{sb}, C_{db} do not contribute to the total gate capacitance. Until, $V_g = V_T$ device is off and only gb makes up C_T. Between $V_T < V_g < 2V_T$ C_{gb} falls down to zero and being in Vel sat, C_{gd} & C_{gs} add up to C_T. Thus,

$$\boxed{\begin{array}{l} 0 < V_g < V_T \Rightarrow C_T = C_T(1) = C_{ox}WL + W(C_{GDO} + C_{GSO}) \\ V_T < V_g < V_T \Rightarrow C_T = C_T(2) = \frac{2}{3}C_{ox}WL + W(C_{GDO} + C_{GSO}) \end{array}}$$

c) This time the device is completely off at all times. C_{GS}, C_{GB}, C_{SB} do not have any connection to drain node Thus they don't contribute to

Only overlap component of C_{gd} & a varying C_{db} make up C_T in this case

$C_T = WC_{GDO} + K_{eq}C_{JO} + K_{eqsw}C_{JSWO}$

$C_{JO} = C_J AD$

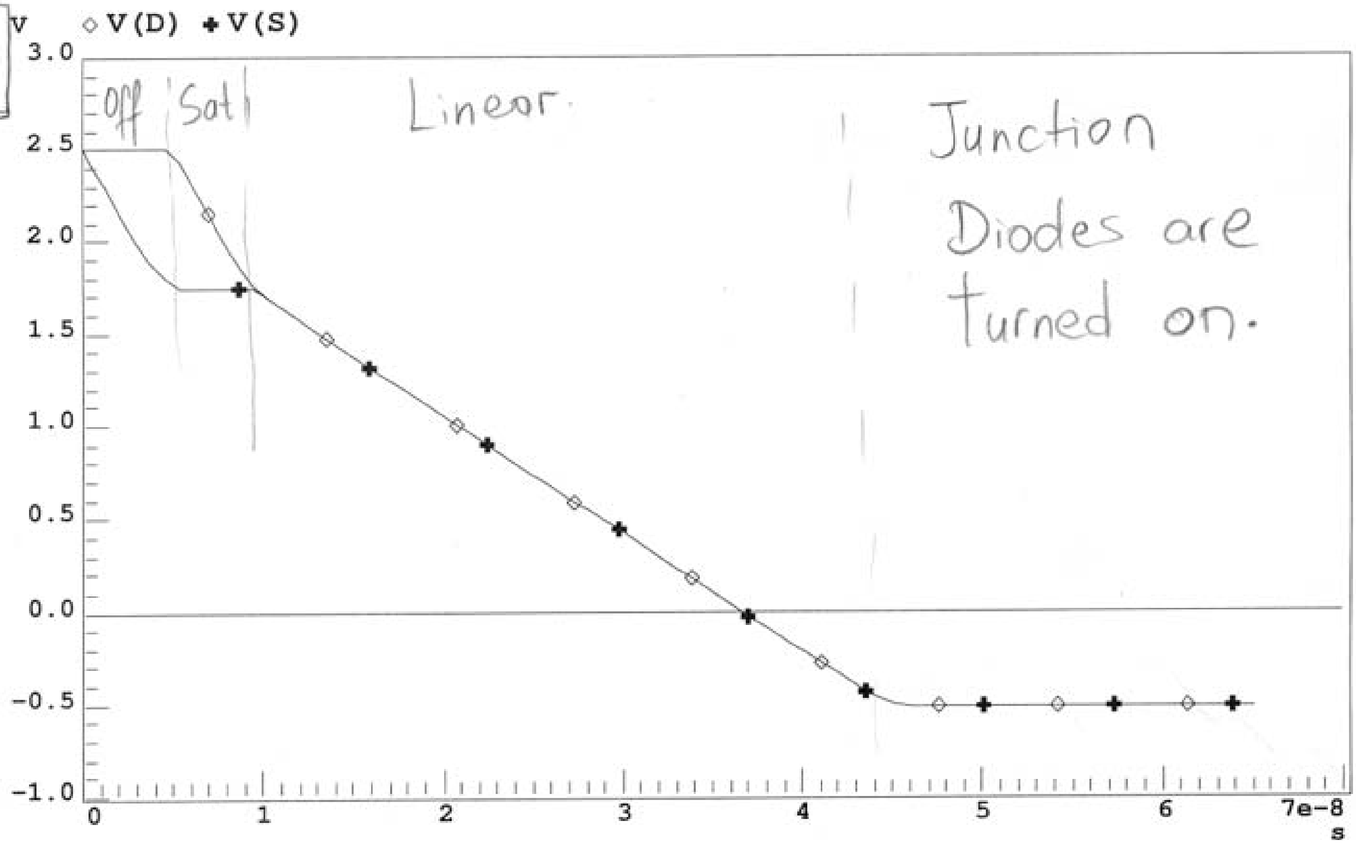
$C_{JSWO} = C_{JSW} PD$

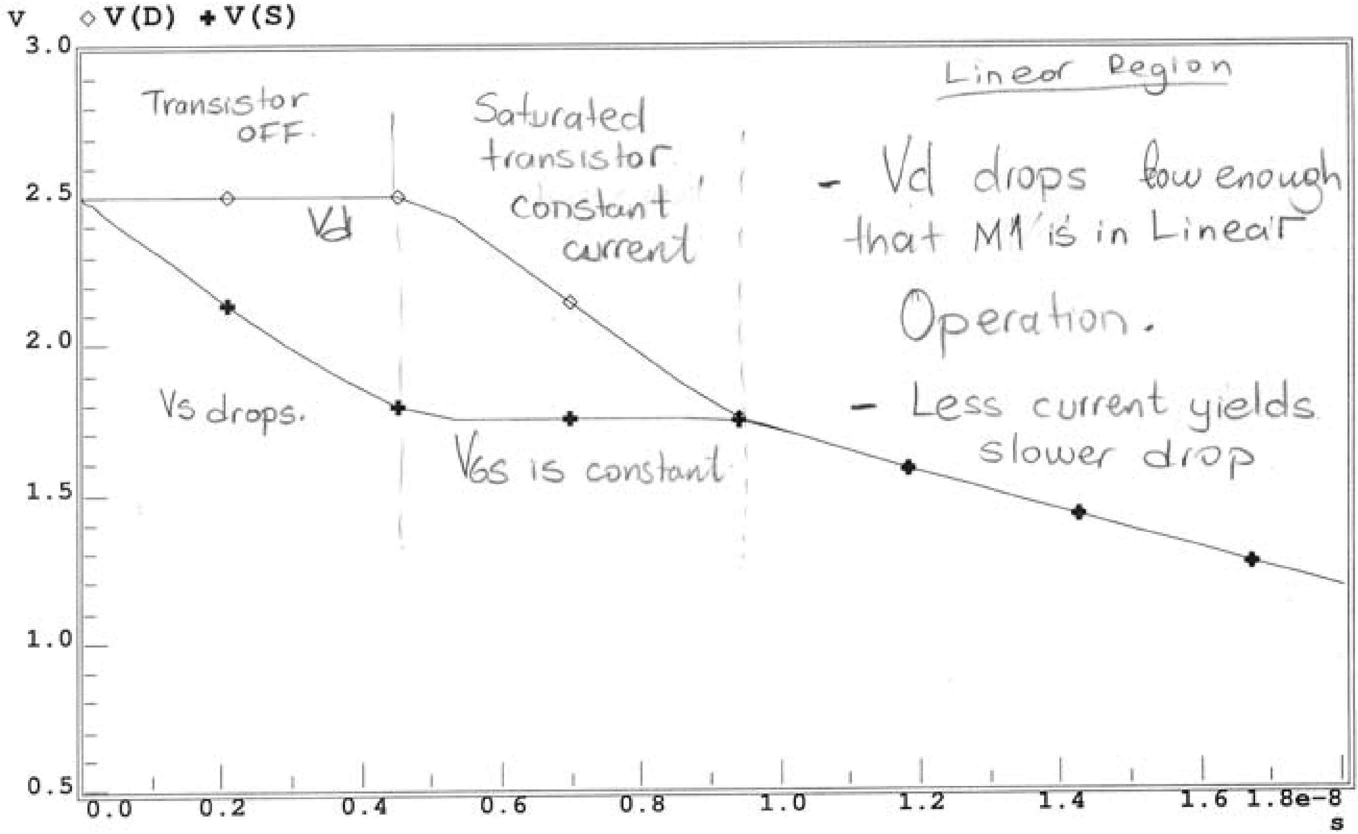
$$\boxed{C_T = WC_{GDO} + K_{eq}C_J AD + K_{eqsw}C_{JSW}PD}$$

$$K_{eq} = \frac{-P_B^{M_J}}{2V_T(1-M_J)}\left[(P_B - 2V_T)^{(1-M_J)} - (P_B)^{(1-M_J)}\right]$$

$$K_{eqsw} = \frac{-P_B^{M_{JSW}}}{2V_T(1-M_{JSW})}\left[(P_B - 2V_T)^{(1-M_{JSW})} - (P_B)^{(1-M_{JSW})}\right]$$

19


V(D) V(S)
v
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0
1
2
3
4
5
6
7e-8
s
Off
Sat
Linear
Junction Diodes are turned on.


V(D) V(S)
v
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8e-8
s
Linear Region
Transistor OFF
Saturated transistor constant current
Vd
Vs drops.
VGS is constant
- Vd drops low enough that M1 is in Linear Operation.
- Less current yields slower drop

20 a) Minimum: $K_n' = 16.66\,\mu A/V^2$
$V_t = 0.765$ $(W/L_{eff}) = (14.7/5.0)$

Nominal:

$K_n' = 19.60\,\mu A/V^2$ $V_t = 0.740V$

$(W/L_{eff}) = (20/4.7)$

Maximum:

$K_n' = 22.54\,\mu A/V^2$ $V_t = 0.715V$

$(W/L_{eff}) = (20.3/4.4)$

$V_{gs} = 0$:

$I_{min} = I_{nom} = I_{max} = 0$

$V_{gs} = 2.5V$ (sat)

$I_{min} = 98.7\,\mu A$

$I_{nom} = 129\,\mu A$

$I_{max} = 165\,\mu A$

$V_{gs} = 5.0V$ (triode)

$I_{min} = 398\,\mu A$

$I_{nom} = 438\,\mu A$

$I_{max} = 471\,\mu A$

b) For $V_{min} \rightarrow I_{max}$, $R = 8800\Omega$

$V_{max} \rightarrow I_{min}$, $R = 7200\Omega$

$V_{gs} = 0 \rightarrow V_{out} = 5V$

(sat) $V_{gs} = 2.5V$; $V_{out} = V_{DD} - IR$

$I_{min} = 99\,\mu A$, $R_{min} \Rightarrow V_{out,max} = 4.29V$

$I_{nom} = 129\,\mu A$, $R_{nom} \Rightarrow V_{out,nom} = 3.97V$

$I_{max} = 165\,\mu A$, $R_{max} \Rightarrow V_{out,min} = 3.55V$

$V_{gs} = 5V$ (triode)

$R_{min} \Rightarrow V_{out,max} = 1.97V$

$R_{nom} \Rightarrow V_{out,nom} = 1.50V$

$R_{max} \Rightarrow V_{out,min} = 1.14$

```
* PROBLEM 2.15
.par w1 = 20u
.par l1 = 5u
vdd vdd 0 dc 5
R  vdd out  R1
mn out in 0 0 nmos w=w1 l=l1
vin in 0 dc 0
.DATA d1
w1 l1 kpn vt0 R1
+19.7u 5.3u 9.20068E-05 0.768469 7200
+20.0u 5.0u 8.00059E-05 0.743469 8000
+20.3u 4.7u 6.80050E-05 0.718469 8800
* SPICE LEVEL 2 Model for MOSIS 1.2 mu Process
.MODEL NMOS NMOS LEVEL=2 LD=0.15U TOX=200.0E-10
+ NSUB=5.36726E+15 VTO=vt0 KP=kpn GAMMA=0.543
+ PHI=0.6 U0=655.881 UEXP=0.157282 UCRIT=31443.8
+ DELTA=2.39824 VMAX=55260.9 XJ=0.25U LAMBDA=0.0367072
+ NFS=1E+12 NEFF=1.001 NSS=1E+11 TPG=1.0 RSH=70.00
+ CGDO=4.3E-10 CGSO=4.3E-10 CJ=0.0003 MJ=0.6585
+ CJSW=8.0E-10 MJSW=0.2402 PB=0.58
* Weff = WDrawn - Delta_W
* The suggested Delta_W is 1.9970E-07
.dc vin 0 5 2.5 sweep data=d1
.print v(out) i(vdd) i(vdd2)
.option post nomod
.end
```

OUTPUT:

Data index#1 :

volt	voltag out	current
0.	5.0000	-10.6299p
2.50000	2.5350	-342.3599u
5.00000	688.5851m	-598.8076u

Data index#2 (nominal):

volt	voltage out	current
0.	5.0000	-10.9104p
2.50000	2.3750	-328.1237u
5.00000	658.0161m	-542.7480u

Data index#3:

volt	voltage out	current
0.	5.0000	-11.3080p
2.50000	2.2784	-309.2724u
5.00000	646.0152m	-494.7710u

21) a) $S = \frac{0.18\mu}{0.12\mu} = 1.5$

Fixed voltage scaling

$A' = \frac{A}{(1.5)^2} = \frac{0.7mm^2}{2.25} = 0.311mm^2$

$P' = \frac{0.4mW}{1MHz} \times 100MHz = 40mW$

$\left(\frac{P}{A}\right)' = 12.44\ mW/mm^2$

b) General scaling

$U = \frac{1.8}{1.5} = 1.2$

$P' = \frac{40mW}{(1.2)^2} = 27.78mW$

$\left(\frac{P}{A}\right)' = 89.29\ mW/mm^2$

c) $f' = Sf = 100MHz \cdot 1.5 = 150MHz$

Assuming dynamic power dominates

$P'_{150} = P'_{100} S = (27.78mW)(1.5)$
$= 41.67\ mW$

$\left(\frac{P_{150}}{A}\right)' = \left(\frac{P_{100}}{A}\right)' S = 89.29 \times 1.5$
$= 133.9\ mW/mm^2$

d) $\left(\frac{P_{150}}{A}\right)' = \left(\frac{P_{100}}{U^2}\right)\left(\frac{S^2}{A}\right) S = \frac{P_{100}}{A}$

$U^2 = S^3$

$U = S^{3/2} = (1.5)^{3/2} = 1.837$

$U = \frac{1.8}{V'} = 1.837$

$V' \approx 1.0V$

22) a) $S = 0.25/0.1 = 2.5$

speed scales inversely to t_p which scales as $1/S^2 \Rightarrow$ Speed scale with S^2 so $\underline{f = 625MHz}$.

Power scales $\propto S \Rightarrow \underline{P = 25W}$

b) in full scaling speed scales with S

$\Rightarrow \underline{f = 250MHz}$. Power scales as $1/S^2$ & thus $\underline{P = 1.6W}$

c) We want to keep power constant

$\frac{S}{U^3} = 1 \Rightarrow U = S^{1/3} = (2.5)^{1/3} = 1.36$

$\Rightarrow$ Voltage becomes $\underline{V = 1.842V}$

speed scales as $S^2/U = 4.6$

$\underline{f = 460MHz}$