BERZIET UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ENEE 3340

HW_2

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Q1:

a- Use a truth table (Enumeration) to prove modus tollens is sound for proposition logic:

P→Q, ¬Q
- - - - - - - - - - -
¬P

- **Answer:**

KB: P→Q, ¬Q

α: ¬P

P	Q	¬Q	P→Q	KB	α	KB and ¬ α
0	0	1	1	1	1	0
0	1	0	1	0	1	0
1	0	1	0	0	0	0
1	1	0	1	1	0	0

b- Prove the goal R for propositional logic using Resolution/Refutation.

1. P v Q,
2. P v R,
3. ¬P v R,
4. R v S,
5. R v ¬Q,
6. ¬S v ¬Q

- **Answer**:

Premises:

1. {P, Q}
2. {P, R}
3. {¬P, R}
4. {R, S}
5. {R, ¬ Q}
6. {¬S, ¬ Q}

Negated Goal:

7. {¬R}

Proves:

8. By 1,3 →{R}
9. By 8,7 →{ }

c- Show that the above knowledge base is the clausal form of:

a. (P →Q) →Q,

- **Answer:**

(P →Q) →Q
≡ (¬P U Q) → Q
≡ ¬ (¬P U Q) U Q
≡ (¬ ¬P ∩ ¬ Q) U Q
≡ (P ∩ ¬ Q) U Q
≡ (P U Q) ∩ (¬ Q U Q)
≡ (P U Q) ∩ 1
≡ (P U Q)

b. (P →P) →R,

- **Answer:**

(P →P) →R
≡ (¬P U P) → R
≡ ¬ (¬P U P) U R
≡ (P ∩ ¬ P) U R
≡ (P U R) ∩ (¬ P U R)

≡ R
≡ (P U Q) ∩ (¬ Q U Q)
≡ (P U Q) ∩ 1
≡ (P U Q)

c. (R →S) → ¬(S →Q)

- **Answer:**

(R →S) → ¬ (S →Q)
≡ (¬R U S) → ¬ (S →Q)
≡ (¬R U S) → ¬ (¬ S U Q)
≡ (¬R U S) → (¬ ¬ S ∩ ¬Q)
≡ (¬R U S) → (S ∩ ¬Q)
≡ ¬ (¬R U S) U (S ∩ ¬Q)
≡ (¬ ¬R ∩ ¬S) U (S ∩ ¬Q)
≡ (R ∩ ¬S) U (S ∩ ¬Q)
≡ (R U ¬Q)

Q2: Represent the following English sentences in first-order logic:

a. All swans are white.
b. There is a black swan.
c. All bowlers drink soda.
d. Some dogs have fleas.
e. There is somebody who loves everyone.
f. Everybody is loved by someone.
g. There is a barber in Ramallah who shaves all men in Ramallah who do not shave themselves.
h. Politicians can fool some of the people all of the time, and all of the people some of the time, but they can't fool

all of the people all of the time.

- **Answer**:

a. $\forall x\ , Sawn(x) \rightarrow White\ (x)$
b. $\exists x\ , Sawn(x) \rightarrow Black\ (x)$
c. $\forall x\ , Bowlers(x) \rightarrow Drink\ (x, soda)$
d. $\exists x\ , Dog(x) \cap has\ (x, fleas)$
e. $\exists x\ \forall y\ , Loves(x, y)$
f. $\forall y\ \exists x, LovedBy(y, x)$
g. $\exists x\ \forall y, Barber(x) \wedge Lives(x, Ramallah) \rightarrow Shaves(x, y) \wedge Lives(y, Ramallah) \wedge Shaves(y, y)$
h. $\forall x, Politician(x) \rightarrow$

$$\Big(\Big(\exists y, Person(y) \rightarrow \big(\forall z, Time(z) \rightarrow Fool(x, y, z)\big)\Big) \wedge \Big(\exists z, Time(z) \rightarrow \big(\forall y, Person(y) \rightarrow Fool(x, y, z)\big)\Big)$$

$$\wedge \big(\exists x\ \exists y, Time(z) \wedge Person(y) \rightarrow \neg Fool(x, y, z)\big)\Big)$$

Q3: Find the Most General Unifier (MGU), if one exists for the pairs:

1. f(g(x,y), c) and f(g(f(d,x),z),c)

- **Answer:** $S = \{\ \} \rightarrow S = \left\{\frac{f(d,x_2)}{x_1}\right\} \rightarrow S = \left\{\frac{f(d,x_2)}{x_1}, \frac{z}{y}\right\}$

2. h(c,d,g(x,y)) and h(z,d,g(g(a,y),z))

- **Answer:** $S = \{\ \} \rightarrow S = \left\{\frac{z}{c}\right\} \rightarrow S = \left\{\frac{z}{c}, \frac{g(a,y_2)}{x}\right\} \rightarrow S = \{\frac{z}{c}, \frac{g(a,y_2)}{x}, \frac{z}{y_1}\}$

3. P(f(a), g(X)) and P(Y,Y)

- **Answer:** $S = \{\ \} \rightarrow S = \left\{\frac{f(a)}{Y}\right\} \rightarrow S = \left\{\frac{f(a)}{Y}, \frac{g(X)}{Y}\right\} \rightarrow Notunifiable\ \times$

4. P(a,X,h(g(Z))) and P(Z,h(Y),h(Y))

- **Answer:** $S = \{\ \} \rightarrow S = \left\{\frac{Z}{a}\right\} \rightarrow S = \left\{\frac{Z}{a}, \frac{h(Y)}{X}\right\} \rightarrow S = \left\{\frac{Z}{a}, \frac{h(Y)}{X}, \frac{g(Z)}{Y}\right\}$

5. P(X,X) and P(Y,f(Y))

- **Answer:** $S = \{\,\} \rightarrow S = \left\{\frac{Y}{X}\right\} \rightarrow S = \left\{\frac{Y}{X}, \frac{f(Y)}{X}\right\} \rightarrow S = \left\{\frac{Y}{X}, \frac{f(Y)}{X}\right\} \equiv \left\{\frac{Y}{X}, \frac{f(Y)}{Y}\right\} \rightarrow Indefinite\ Recursion\ \times$

6. P(a, f(x, a)) and P(a, f(g(y), y))

- **Answer:** $S = \{\,\} \rightarrow S = \left\{\frac{g(y)}{x}\right\} \rightarrow S = \left\{\frac{g(y)}{x}, \frac{y}{a}\right\}$

Q4: Assume KB consists of the following rules:

R1: Soda(x) ^ Chips(y) → Cheaper(x, y)
R2: Chips(x) ^ Cereals(y) → Cheaper(x,y)
R3: Cheaper(x, y) ^ Cheaper(y, z) → Cheaper(x, z)

And the facts:
F1: Soda(Sprite)
F2: Chips(Ruffles)
F3: Cereals(Cheerios)
F4: Cereals(MiniWheats)

a. Assume that all facts F1-F4 are known at the beginning of the inference process. Illustrate the process of forward chaining by listing all newly inferred facts. Assume that both rules and facts are matched and tried in the order of their appearance.

- **Answer:**

$\boldsymbol{From\ F_1, F_2\ and\ R_1}$:

$$Soda(Sprite) \wedge Chips(Ruffels) \rightarrow Cheaper(Sprite, Ruffels) \Longrightarrow (1)$$

$\boldsymbol{From\ F_2, F_3\ and\ R_2}$:

$$Chips(Ruffels) \wedge Cereals(Cheerios) \rightarrow Cheaper(Ruffels, Cheerios) \Longrightarrow (2)$$

$$Chips(Ruffels) \wedge Cereals(MiniWheats) \rightarrow Cheaper(Ruffels, MiniWheats) \Longrightarrow (3)$$

$\boldsymbol{From\ 1, 2\ and\ R_3}$:

$$Cheaper(Sprite, Ruffels) \wedge Cheaper(Ruffels, Cheerios) \rightarrow Cheaper(Sprite, Cheerios)$$

$\boldsymbol{From\ A, C\ and\ R_3}$:

$$Cheaper(Sprite, Ruffels) \wedge Cheaper(Ruffels, MiniWheats) \rightarrow Cheaper(Sprite, MiniWheat)$$

b. Show how to prove Cheaper (Sprite, Cheerios) using backward chaining and the KB given in part

- **Answer**:

$$let\ Cheaper(Sprite, Cheerios) \Longrightarrow (1)$$

$\boldsymbol{From\ A\ and\ R_3}$:

$$Cheaper(Sprite, Ruffels) \Longrightarrow (2)$$
$$Cheaper(Ruffels, Cheerios) \Longrightarrow (3)$$

$\boldsymbol{From\ B\ and\ R_1}$:

$$Soda(Sprite)$$
$$Chips(Ruffels)$$

$\boldsymbol{From\ C\ and\ R_2}$:

$$Chips(Ruffels)$$
$$Cereals(Cheerios)$$

Note: we can show from above, in backward chaining we used less facts and effort to reach the proof unlike the forward chaining

Q5: Prove each of the **Goals:** grandmother(abe, carl) and grandmother(abe, mary) (each separately) byrefutation resolution from the following clause set. Is the clause set Horn?

- **Answer**:

1. $grandparent(X, Y) \wedge \neg parent(X, Z) \rightarrow \neg\, parent(Z, Y)$.
2. $parent(X, Y) \vee \neg father(X, Y)$
3. $parent(X, Y) \vee \neg mother(X, Y)$
4. $father(abe, bev)$.
5. $mother(bev, carl)$.
6. $mother(bev, mary)$.

	Resolution Proof	Refutation Proof
Goal: grandmother(abe,carl)	$We\ want\ to\ reach: grandparent(abe, carl)$ $5 + 3 = parent(bev, carl) \Longrightarrow 7$ $Let\ abe = X, carl = Y\ and\ bev = Z$ $1 + 7 = grandparent(abe, carl) \wedge \neg parent(abe, bev)$ $grandparent(abe, carl)$	$0. \neg grandparent(abe, carl)$ $Let\ abe = X, carl = Y\ and\ bev = Z$ $0 + 1 = \neg parent(bev, carl) \Longrightarrow 7$ $3 + 7 = \neg mother(bev, carl) \Longrightarrow 8$ $5 + 8 = [\quad]$

	Resolution Proof	Refutation Proof
Goal: grandmother(abe,mary)	$We\ want\ to\ reach: grandparent(abe, mary)$ $6 + 3 = parent(bev, carl) \Longrightarrow 7$ $Let\ abe = X, mary = Y\ and\ bev = Z$ $1 + 7 = grandparent(abe, mary) \wedge \neg parent(abe, bev)$ $grandparent(abe, mary)$	$0. \neg grandparent(abe, mary)$ $Let\ abe = X, mary = Y\ and\ bev = Z$ $0 + 1 = \neg parent(bev, mary) \Longrightarrow 7$ $3 + 7 = \neg mother(bev, mary) \Longrightarrow 8$ $6 + 8 = [\quad]$

The clause set is horn because we can write it in this form: $(P \wedge Q \wedge \ldots \wedge T) \rightarrow U$

$grandparent(X, Y) \wedge \neg parent(X, Z) \rightarrow \neg\, parent(Z, Y)$.
$parent(X, Y) \vee \neg father(X, Y) \equiv father(X, Y) \rightarrow parent(X, Y)$
$parent(X, Y) \vee \neg mother(X, Y) \equiv mother(X, Y) \rightarrow parent(X, Y)$
$father(abe, bev) \equiv TRUE \longrightarrow father(abe, bev)$
$mother(bev, carl) \equiv TRUE \longrightarrow mother(bev, carl)$
$mother(bev, mary) \equiv TRUE \longrightarrow mother(bev, mary)$