

Question#2:

3 cases for the convolution of $h_1[n]$ and $h_2[n]$:

1. $N > M$

$$h_1[n] * h_2[n] = \begin{cases} \sum_{k=0}^n a^k; 0 \leq n \leq M \\ \sum_{k=n-M}^n a^k; M \leq n \leq N \quad ; \text{and zero otherwise.} \\ \sum_{k=n-M}^N a^k; N \leq n \leq M + N \end{cases}$$

2. $N = M$

$$h_1[n] * h_2[n] = \begin{cases} \sum_{k=0}^n a^k; 0 \leq n \leq M \\ \sum_{k=n-M}^N a^k; N \leq n \leq M + N \\ 0; \text{elsewhere} \end{cases}$$

3. $N < M$

$$h_1[n] * h_2[n] = \begin{cases} \sum_{k=0}^n a^k; 0 \leq n \leq N \\ \sum_{k=0}^N a^k; N \leq n \leq M \quad ; \text{and zero otherwise.} \\ \sum_{k=n-M}^N a^k; M \leq n \leq M + N \end{cases}$$

Question#3:

N	0	1	2	3	4
H[n]	1	1/2	1/4	1/8	1/16
X[n]	A	B	C	D	E
H[0]X[n]	A	B	C	D	E
H[1]X[n]	-	A/2	B/2	C/2	D/2
H[2]X[n]	-	-	A/4	B/4	C/4
H[3]X[n]	-	-	-	A/8	B/8
H[4]X[n]	-	-	-	-	A/16
→ H[n]*X[n]	A	B+A/2	C+B/2+A/4	D+C/2+B/4+A/8	E+D/2+C/4+B/8+A/16
→ Y[n]	1	2	2.5	3	3
X[n]	1	1.5	1.75	1.625	1.5

→ By solving $H[n]*X[n] = Y[n]$ and solving for the coefficients.

Question#4:

a>

$$h[n] = h_1[n] * (h_2[n] + h_3[n] * h_4[n])$$

b>

$$\rightarrow h_3[n] * h_4[n] = -u[n] * \delta[n - 4] = -u[n - 4]$$

$$\rightarrow h_2[n] + h_3[n] * h_4[n] = u[n] - u[n - 4] = \{1, 1, 1, 1, 1\}; 0 \leq n \leq 4$$

$$\rightarrow h[n] = h_1[n] * (h_2[n] + h_3[n] * h_4[n])$$

n	0	1	2	3	4	5	6
$H_2[n]+H_3[n]*H_4[n]$	1	1	1	1	1	0	0
$H_1[n]$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	0	0
0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	-	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
2	-	-	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
H[n]	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{1}{2}$

$$\rightarrow h[n] = \{ \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}, \frac{5}{4}, \frac{3}{4}, \frac{1}{2} \}$$

c>

$$X[n] * h[n] = h[n + 2] + 3h[n - 1] - 4h[n - 3]$$

→ Shifted scaled h[n]'s added

n	-2	-1	0	1	2	3	4	5	6	7	8
$h[n+2]$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	0	0	0	0
$3h[n-1]$	0	0	0	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{15}{4}$	$\frac{15}{4}$	$\frac{15}{4}$	$\frac{9}{4}$	$\frac{3}{2}$	0
$-4h[n-2]$	0	0	0	0	$-\frac{4}{2}$	$-\frac{12}{4}$	$-\frac{20}{4}$	$-\frac{20}{4}$	$-\frac{20}{4}$	$-\frac{12}{4}$	$-\frac{4}{2}$
$x[n]*h[n]$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{11}{4}$	$\frac{6}{4}$	$\frac{6}{4}$	$-\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{11}{4}$	$-\frac{6}{4}$	$-\frac{4}{2}$

Question#5:

$$\Leftrightarrow \text{Even } n; |a| < 1$$

$$\Leftrightarrow \text{Odd } n; 0 \leq a < 1$$

Question#6:

n	0	1	2	3	4	5	6
X[n]	1	2	1	1	0	0	0
X[0]X[n]	1	2	1	1	0	0	0
X[1]X[n]	-	2	4	2	2	0	0
X[2]X[n]	-	-	1	2	1	1	0
X[3]X[n]	-	-	-	1	2	1	1
X[n]*X[n]	1	4	6	6	5	2	1