

ANSWER BOOKLET

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Course: _____	Department: _____	Number: _____
Division: <u>2</u>	Instructor: _____	
Date: _____	_____	_____
Day	Month	Year

For Instructor's Use

Question	Grade
1	12
2	9
3	7 X
4	15 X
5	16 X
6	14
7	
8	
9	
10	
11	
12	
Total	77

+5



Q1) For $y(n] = 0.2 y(n-1) + x(n) - 0.3 x(n-1) + 0.02 x(n-2)$

$$\Rightarrow y(z) = 0.2 y(z) z^{-1} + x(z) - 0.3 x(z) z^{-1} + 0.02 x(z) z^{-2}$$

$$\Rightarrow y(z) - 0.2 y(z) z^{-1} = x(z) - 0.3 x(z) z^{-1} + 0.02 x(z) z^{-2}$$

$$\Rightarrow \frac{y(z)}{x(z)} = H_1(z) = \frac{1 - 0.3 z^{-1} + 0.02 z^{-2}}{1 - 0.2 z^{-1}}$$

For $y(n] = x(n] - 0.1 x(n-1)$

$$y(z) = x(z) - 0.1 x(z) z^{-1}$$

$$\Rightarrow H_2(z) = 1 - 0.1 z^{-1}$$

long
div.

$$\frac{1 - 0.2 z^{-1}}{1 - 0.3 z^{-1} + 0.02 z^{-2}} = \frac{1 - 0.1 z^{-1}}{1 - 0.2 z^{-1}}$$

$$\frac{1 - 0.3 z^{-1} + 0.02 z^{-2}}{1 - 0.2 z^{-1}} = \frac{1 - 0.1 z^{-1}}{1 - 0.2 z^{-1}}$$

$$0$$

$$\Rightarrow H_1(z) = \frac{1 - 0.3 z^{-1} + 0.02 z^{-2}}{1 - 0.2 z^{-1}} = \frac{1 - 0.1 z^{-1}}{1 - 0.2 z^{-1}} = H_2(z)$$

since $H(z)$ for the two systems is equal

\Rightarrow the systems equivalent

$$Q2) y(n] = x(n] + 0.3 y(n-2)$$

$$y(z) = x(z) + 0.3 y(z) z^{-2}$$

$$\Rightarrow \underline{H(z) = \frac{y(z)}{x(z)}}$$

9/10

$$\Rightarrow y(z) - 0.3 y(z) z^{-2} = x(z)$$

$$\Rightarrow \frac{y(z)}{x(z)} = \frac{1}{1 - 0.3 z^{-2}}$$

$$\underline{\underline{\frac{z}{z - 0.3 z^{-1}}}}$$

ROC: $0.3 < z^2 \Rightarrow z > \sqrt{0.3}$

$$\underline{\underline{\frac{z^2}{z^2 - 0.3}}}$$

$$= \frac{z^2}{(z - 0.55)(z + 0.55)}$$

$$z_{1,2} = \frac{\pm \sqrt{4 \times 0.3}}{2}$$

$$= \frac{\pm 1.09}{2} \approx \pm 0.55$$

$$= \frac{A}{z - 0.55} + \frac{B}{z + 0.55}$$

$$\Rightarrow A = \left. \frac{z^2}{z + 0.55} \right|_{z=0.55} = 0.275$$

$$B = \left. \frac{z^2}{z - 0.55} \right|_{z=-0.55} = -0.275$$

$$\Rightarrow H(z) = \frac{0.275}{z - 0.55} - \frac{0.275}{z + 0.55}$$

$$\Rightarrow H(z) = \frac{0.275z^{-1}}{1 - 0.55z^{-1}} - \frac{0.275z^{-1}}{1 + 0.55z^{-1}}$$

~~$$\Rightarrow h(n) = 0.275(-0.55)^n u[n]$$~~

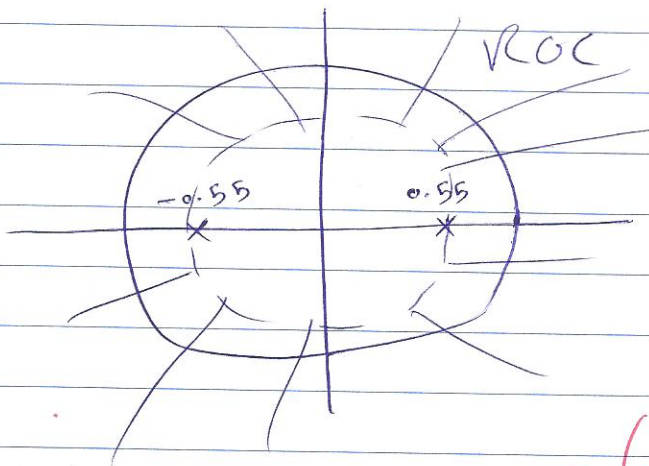
$$-0.55 < z$$

~~$$h(n) = 0.275(-0$$~~

~~$$h(n) = 0.275(0.55)^n u[n] - 0.275(-0.55)^n u[n]$$~~

$$\text{ROC} \left\{ \frac{0.275z^{-1}}{1 - 0.55z^{-1}} \right\} \Rightarrow |0.55z^{-1}| < 1 \Rightarrow \boxed{0.55 < z}$$

$$\text{ROC} \left\{ \frac{-0.275z^{-1}}{1 + 0.55z^{-1}} \right\} \Rightarrow |0.55z^{-1}| < 1 \Rightarrow \boxed{z < 0.55}$$



\Rightarrow Stable System

for only $|z| > \sqrt{-3}$

Q3) ~~$y(n) = x(n) + (-1)x(n)z^{-1} + y(n)z^{-1}(0.5)$~~

$y(z) = \cancel{x(z)} - x(z)z^{-1} + \cancel{y(z)z^{-1}(0.5)}$ -4
 $\frac{z}{1-0.5z^{-1}}$

$y(n) = x(n) - x(n-1) + 0.5y(n-1)$

~~$y(4) = x(4) - x(3) + 0.5y(3)$~~

~~$\Rightarrow y(4) = x(4) = 1$~~

~~$y(5) = x(5) - x(4) + 0.5y(4)$~~

~~$= 1 - 1 + 0.5 \cdot 1$~~

~~$= 0.5$~~

~~$y(6) = x(6) - x(5) + 0.5y(5)$~~

~~$= 1 - 1 + 0.5 \cdot 0.5 = 0.25$~~

~~$y(z) = \dots$~~

$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}} = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$

$y(z) = \dots$

~~$h(n) = \dots$~~

$= \frac{y(n)}{x(n)} = \frac{y(n)}{1}; n \geq 4$

$\Rightarrow y(n) = \begin{cases} (0.5)^n n[n] - (0.5)^{n-1} n[n-1]; & n \geq 4 \\ 0 & \text{otherwise} \end{cases}$

Q4) $x(n) = \begin{cases} n+1 & ; 0 \leq n \leq 3 \\ 4 & ; n \geq 4 \\ 0 & ; \dots \end{cases}$

15 + 20 = 35

~~$x(z) = \begin{cases} z+1 & ; 0 \leq z \leq 3 \\ 4 & ; z \geq 4 \\ 0 & ; \dots \end{cases}$~~

~~$\frac{z+1}{1} = \frac{1+z^{-1}}{z^{-1}} ; |z| > 0 \Rightarrow \text{ROC}$~~

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$0 \leq n \leq 3 \Rightarrow \sum_0^3 (n+1) z^{-n} = z^0 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

$0 \leq n \leq 3 \Rightarrow 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

$n \geq 4 \Rightarrow \sum_4^{\infty} 4z^{-n} = 4 \sum_4^{\infty} z^{-n}$

$= \frac{4}{1-z^{-1}} - (z^0 + z^{-1} + z^{-2} + z^{-3})$

$= \frac{4}{1-z^{-1}} - (1 + z^{-1} + z^{-2} + z^{-3})$

$= \frac{4}{1-z^{-1}} - \frac{1 + z^{-1} + z^{-2} + z^{-3} (1-z^{-1})}{(1-z^{-1})}$

$n \geq 4 \Rightarrow \frac{4 - [(1 + z^{-1} + z^{-2} + z^{-3})(1-z^{-1})]}{1-z^{-1}}$

ROC: $|z| > 1$

$$a) H(z) = 1 + 2z^{-1} + 3z^{-2} + 3.5z^{-3} + 3.5z^{-4}$$

$$\frac{16}{18} + 3.5z^{-5} + 3z^{-6} + 2z^{-7} + z^{-8}$$

$$H(e^{j\omega}) = 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 3.5e^{-3j\omega} + 3.5e^{-4j\omega} + 3.5e^{-5j\omega} \\ + 3e^{-6j\omega} + 2e^{-7j\omega} + e^{-8j\omega}$$

~~$$e^{-j\omega} (e^{+j\omega} + 2e^{0j\omega} + e^{-j\omega} + 3.5e^{-2j\omega} + 3.5e^{-3j\omega})$$~~

~~$$e^{2j\omega} (e^{-2j\omega} + 2e^{-3j\omega} + 3e^{-4j\omega})$$~~

~~$$e^{2j\omega} (e^{-2j\omega} + 2e^{j\omega} + 1) + 3.5e^{-j\omega} + 3.5e^{-2j\omega}$$~~

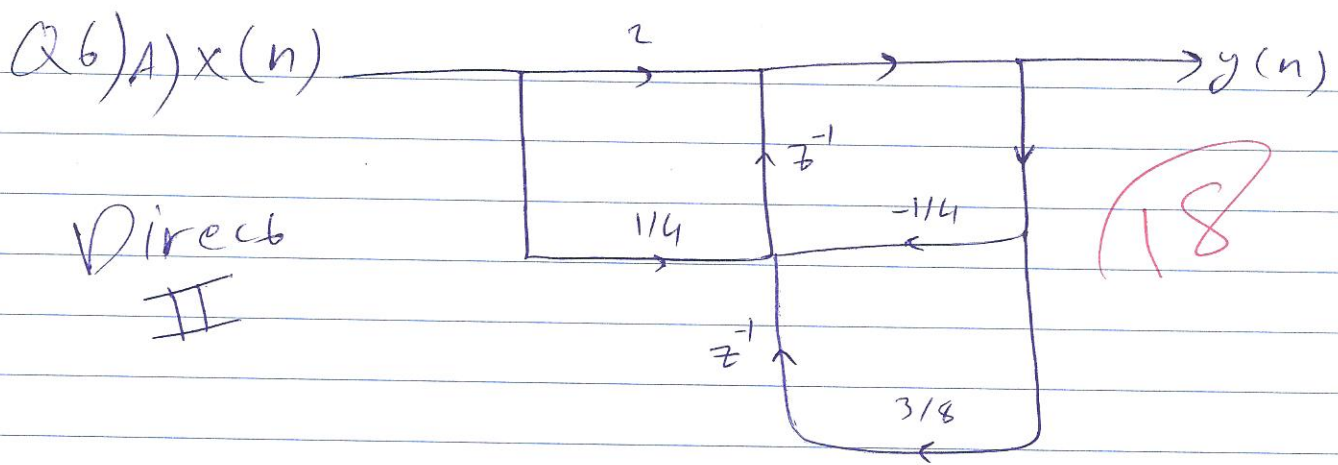
~~$$e^{-3j\omega} (e^{-3j\omega} + 2e^{-2j\omega} + 3e^{-j\omega} + 3.5)$$~~

$$\Rightarrow e^{-4j\omega} (e^{-4j\omega} + 2e^{-3j\omega} + 3e^{-2j\omega} + 3.5e^{-j\omega} + 3.5e + 3.5e^{j\omega} \\ + 3e^{2j\omega} + 2e^{3j\omega} + e^{4j\omega})$$

$$\Rightarrow e^{-4j\omega} (3.5 + 2\cos 4\omega + 4\cos 3\omega + 6\cos 2\omega + 7\cos \omega)$$

$$\Rightarrow \phi = -4\omega \Rightarrow \text{grad} = \frac{d(-4\omega)}{d\omega}$$

$$\Rightarrow +4$$



\Rightarrow this is transpose of original system

\Rightarrow Poles are zeros & zeros are ~~poles~~ poles

Poles \Rightarrow ~~1/4, -3/8~~ 1/4, -3/8

Zeros \Rightarrow 1/4

$$\Rightarrow H(z) = \frac{2 + 1/4 z^{-1}}{1 + 1/4 z^{-1} - 3/8 z^{-2}}$$

B)

$$\frac{y(z)}{x(z)} = \frac{2 + 1/4 z^{-1}}{1 + 1/4 z^{-1} - 3/8 z^{-2}}$$

$$\Rightarrow y(z) + \frac{1}{4} y(z) z^{-1} - \frac{3}{8} y(z) z^{-2} = 2x(z) + \frac{1}{4} x(z) z^{-1}$$

$$\Rightarrow y(n) + \frac{1}{4} y(n-1) - \frac{3}{8} y(n-2) = 2x(n) + \frac{1}{4} x(n-1)$$