جامعة بيرزيت

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Electrical and computer engineering

Digital Signal Processing – Fall2014

Midterm Exam

Name: Key ID: Section: 10-11, 12-1

Question 1: [10 marks]

a) For what values of ω is the signal $x(n) = A\cos(\omega n)$ periodic with period of 6? [4pts]

$\omega N = 2\pi k \Rightarrow \omega = \frac{2\pi k}{N}, \; N = 6 \Rightarrow$

$\omega = \frac{2\pi}{6}k = \frac{\pi}{3}k, \; k = 1, 2, 3, \ldots$

$\omega = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$

b) A causal discrete linear time invariant system has the following input, x(n) , and output, y(n) ,:

$$x(n) = (\frac{1}{2})^n u(n) - \frac{1}{4}(\frac{1}{2})^{n-1} u(n-1)$$

$$y(n) = (\frac{1}{3})^n u(n)$$

Where u(n) is the unit step sequence. Determine the system function H(z) and its Region of Convergence (ROC). Comment on the stability of this system. [6pts]

$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{\frac{1}{4}z^{-1}}{1 - \frac{1}{2}z^{-1}} = \frac{1 - \frac{1}{4}z^{-1}}{1 - \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$

$Y(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{3}z^{-1}} \times \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-1}} = \frac{1 - \frac{1}{2}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{4}z^{-1})}, \quad |z| > \frac{1}{3}$

Since ROC ($|z| > \frac{1}{3}$) Includes Unit Circle ($z = 1$) $\Rightarrow$

$\Rightarrow$ System is Stable.

Question 2: [10 marks]

a) Show that the z-transform of sequence $x(n-n_0)$ is $Z^{-n_0}X(Z)$? [2pts]

$$\mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = X(z)$$

$$\mathcal{Z}\{x(n-n_0)\} = \sum_{n=-\infty}^{\infty} x(n-n_0)\, z^{-n}, \text{ let } m=n-n_0 \Rightarrow \sum_{m=-\infty}^{\infty} x(m)\, z^{-(m+n_0)}$$

$$= z^{-n_0}\sum_{m=-\infty}^{\infty} x(m)\, z^{-m} = z^{-n_0}\,\mathcal{Z}\{x(n)\} = z^{-n_0}X(z) \; !!$$

b) Consider the following system function of an LTI system:

$$H(z) = \frac{(1-1.5z^{-1})(1-\frac{1+j}{2}z^{-1})(1-\frac{1-j}{2}z^{-1})}{(1-0.8e^{j\frac{2\pi}{3}}z^{-1})(1-0.8e^{-j\frac{2\pi}{3}}z^{-1})(1-\frac{2}{3}z^{-1})}, \text{ ROC } |z| > 0.8$$

i) Is this system stable? Why? [1pt]

Yes, stable because RoC $(|z|>0.8)$ Includes Unit Circle.

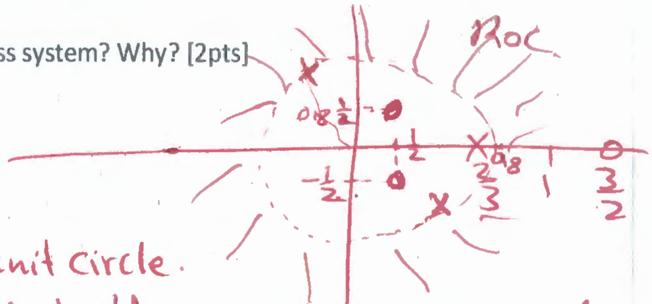
Is it an FIR or IIR system? Why? [1pt]

IIR because it has poles which are not cancelled by zeros.

ii) Plot the pole-zero diagram. Is this system a minimum-phase or all-pass system? Why? [2pts]

Poles: $\frac{2}{3}$, $0.8e^{\pm j\frac{2\pi}{3}}$

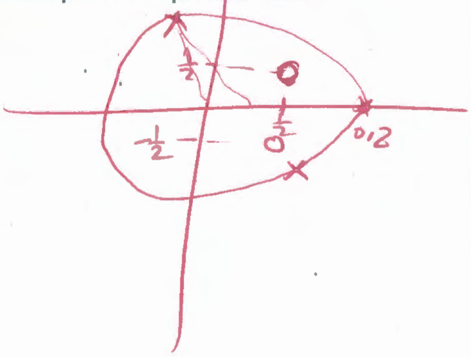
Zeros: 1.5, $\frac{1}{2}+\frac{1}{2}j$, $\frac{1}{2}-\frac{1}{2}j$



* Not min. phase because it has zero outside unit circle.
* Not All-pass because it has two zeros which don't have conjugate reciprocal poles.

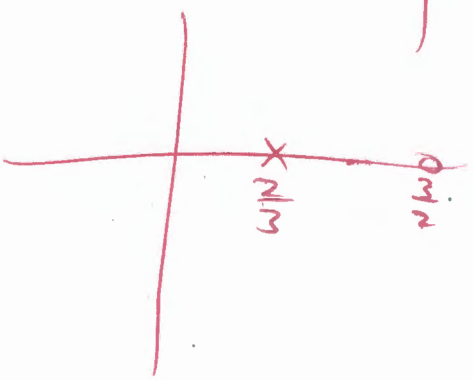
iii) Express H(z) as a combination of an all-pass filter and a minimum-phase filter. Draw the pole-zero plots for each of these filters. [4pts]

min. phase: $H_{min}(z) = \dfrac{-1.5\,(1-\frac{1+j}{2}z^{-1})(1-\frac{1-j}{2}z^{-1})}{(1-0.8e^{j\frac{2\pi}{3}})(1-0.8e^{-j\frac{2\pi}{3}})}$



All-Pass:

$$H_{ap}(z) = \frac{z^{-1}-\frac{2}{3}}{1-\frac{2}{3}z^{-1}}$$



Question 3: [10 marks]

a) Consider a first-order FIR digital filter, which is described by the following difference equation:

$$y(n) = x(n) - ax(n-1),\ a \neq 0$$

Find the coefficient (a) such that -3dB cut-off frequency is $\frac{\pi}{4}$? [4pts]

$H(e^{jw}) = 1 - a e^{-jw} = 1 - a(\cos w - j\sin w)$

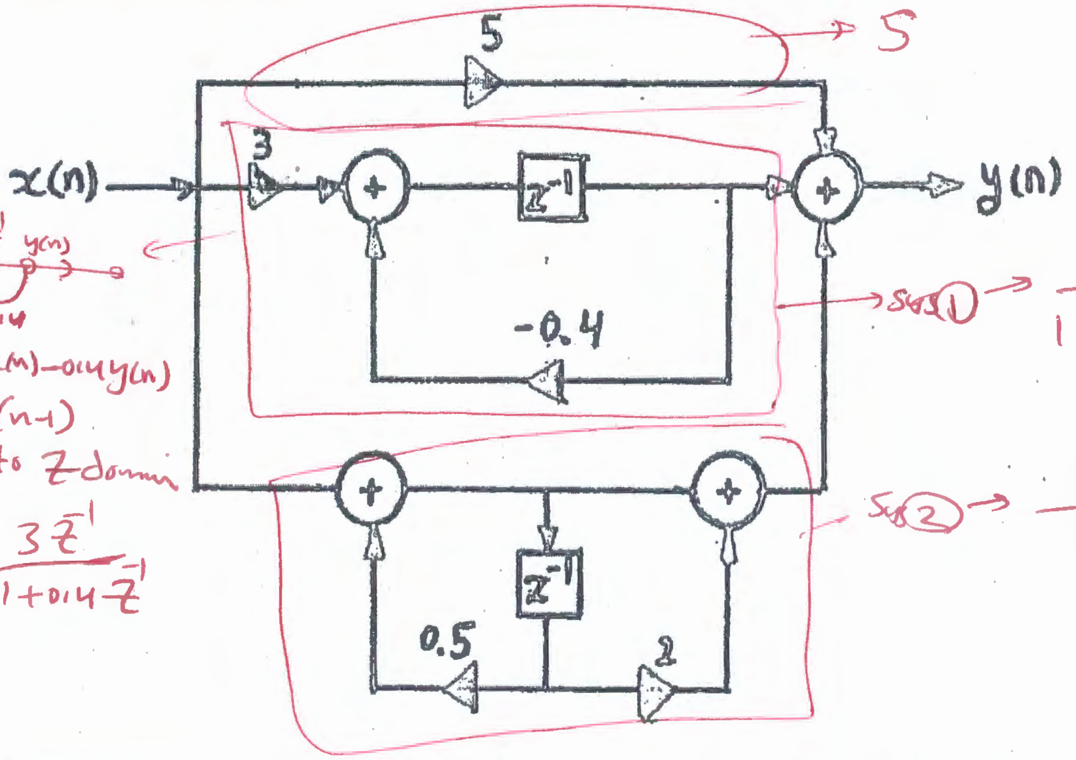
$= 1 - a\cos w - j a \sin w$

$|H(e^{jw})|^2 = (1 - a\cos w)^2 + a^2 \sin^2 w$

$10\log_{10}|H(e^{j\frac{\pi}{4}})|^2 = -3 \Rightarrow (1 - a\cos\frac{\pi}{4})^2 + a^2\sin^2\frac{\pi}{4} = 10^{-0.3} = 0.5$

$\Rightarrow a^2 - \sqrt{2}a + 0.5 = 0 \Rightarrow a = \frac{1}{\sqrt{2}} = 0.707$

b) Consider the following digital filter



$\rightarrow 5$

x(n) 3 w(n) z^{-1} y(n)
-0.4

$w(n) = 3x(n) - 0.4y(n)$

$y(n) = w(n-1)$

convert to Z domain

$\frac{Y(z)}{X(z)} = \frac{3z^{-1}}{1+0.4z^{-1}}$

Sys(1) $\rightarrow \frac{3z^{-1}}{1+0.4z^{-1}}$

Sys(2) $\rightarrow \frac{1+2z^{-1}}{1-0.5z^{-1}}$

i) Determine the impulse response, h(n), of this system and plot it for n=0,1,2 and 3? [3pts]

$H(z) = 5 + \frac{3z^{-1}}{1+0.4z^{-1}} + \frac{1+2z^{-1}}{1-0.5z^{-1}}$ (assume Causal).

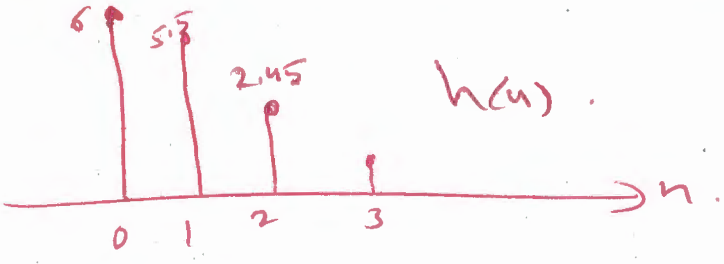
$\Rightarrow h(n) = 5\delta(n) + 3(-0.4)^{n-1}u(n-1) + (0.5)^n u(n) + 2(0.5)^{n-1}u(n-1)$

$h(0) = 6$

$h(1) = 5.5$

$h(2) = 2.45$

$h(3) =$



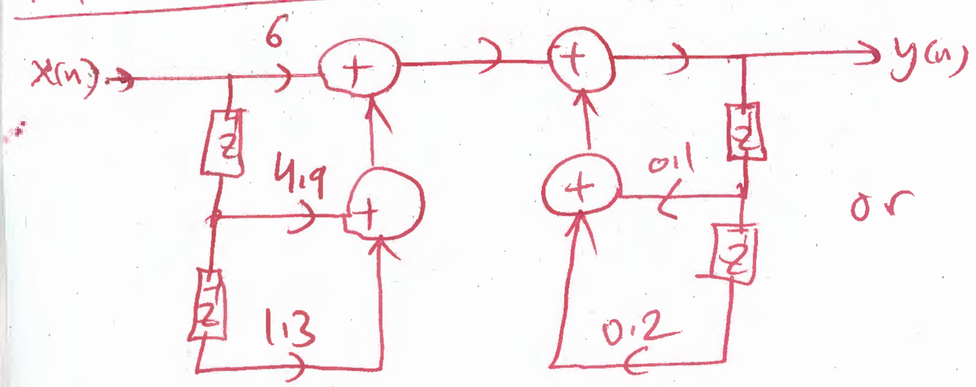
ii) Draw the realization block diagram of this system using

- Direct Form I [1pt]

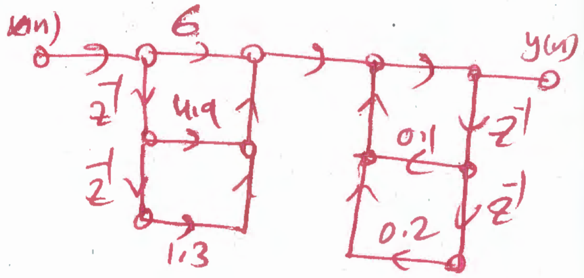
by Combining $H(z)$ into rational function:

$$\Rightarrow H(z) = \frac{6 + 4.9 z^{-1} + 1.3 z^{-2}}{1 - 0.1 z^{-1} - 0.2 z^{-2}}$$

* Direct form I.

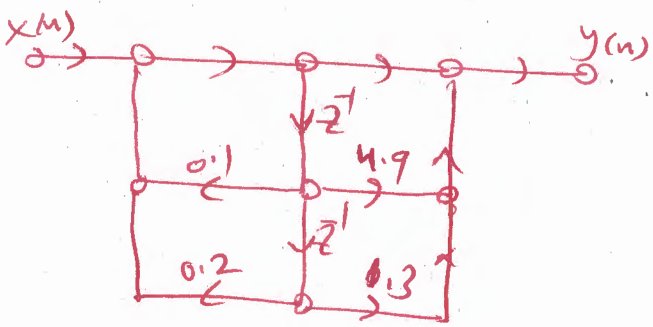


or

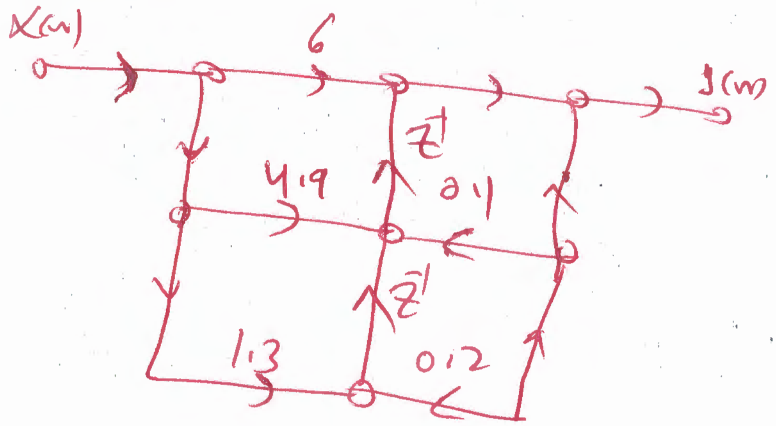


- Transposed Direct Form II [2pts]

* Direct form II :



Transposed Direct form II



Question 4: [*Bonus*]

Given the following system function:

$$H(z) = \frac{3 - 2z^{-1} + 1.5z^{-4}}{1 - z^{-1} - 4z^{-2} + z^{-4}}$$

b = [3, -2, 0, 0, 1.5]

a = [1, -1, -4, 0, 1]

Write **MATLAB** statements to:

(a) Display the magnitude response and the phase response of this system.

freqz(b, a);

(b) Plot zero-pole diagram of the system.

zplane(b, a);

(c) Evaluate impulse response, h[n], of the system.

impz(b, a,

(d) Find the output sequence if the following input is applied to the system x[n] = {2, 1, 0, 3, 2, 1}, for n=0 to 5.

y = filter(b, a, x);