جامعة بيرزيت
BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENCS431- Digital Signal Processing – Fall 2015

Midterm Exam – Nov. 10, 2015

Name: Solution **ID**: **Section**: 9:00 – 10:00, 12:00 – 1:00

Question	Student mark	ABET SO
1		a
2		k
3		c

Question 1: [10pts]

When the input to an LTI system is:

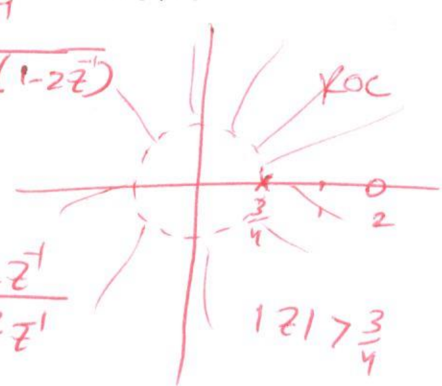
$$x(n) = (\frac{1}{2})^n u(n) + 2^n u(-n-1) \qquad and \qquad y(n) = 6(\frac{1}{2})^n u(n) - 6(\frac{3}{4})^n u(n)$$

a) Find the system function H(z) of the system. Plot the poles and zeros of H(z), and indicate the ROC. [4pts]

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}} \quad \frac{1}{2}<|z|<2 \Rightarrow X(z) = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}} = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}, \quad |z|>\frac{3}{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-3/2\,z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})} \cdot \frac{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}{-3/2\,z^{-1}} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$



$|z|>\frac{3}{4}$

b) Find the impulse response h(n) of the system. [3pts]

$$H(z) = \frac{1}{1-\frac{3}{4}z^{-1}} - \frac{2z^{-1}}{1-\frac{3}{4}z^{-1}}, \quad |z|>\frac{3}{4}$$

So $h(n) = (\frac{3}{4})^n u(n) - 2(\frac{3}{4})^{n-1} u(n-1)$

2^{12} 4096 $\frac{20V}{2^{12}-1}$

$-2^{n-1} \rightarrow +2^{n-1}$ $\frac{20}{4096}$

n-bit!

4.9mv

c) Write the difference equation that characterizes the system. [3pts]

$-8 \rightarrow +7$

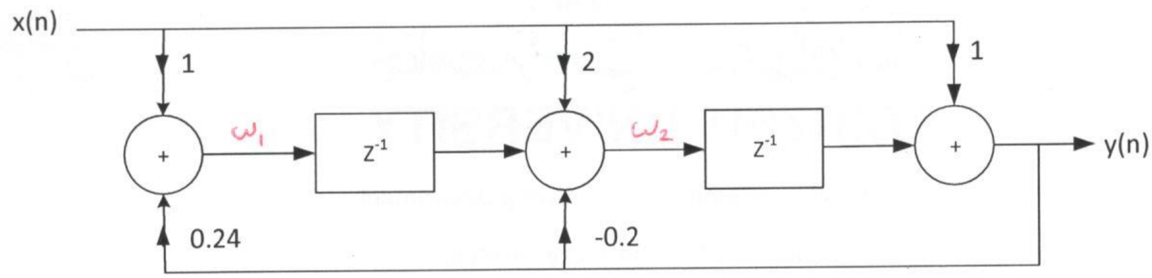
$$H(z) = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}} = \frac{Y(z)}{X(z)}$$

4.9 mv

$$y(n) - \frac{3}{4}y(n-1) = x(n) - 2x(n-1)$$

Question 2: [11pts]

Consider a causal digital filter shown in the following realization structure:



a) Is this a canonic structure? Explain? [1pt]

Yes, because it is 2nd-order system with only two delay units.

b) Determine system function, H(z), of the above structure? Plot pole-zero diagram and indicate if this system is stable or not? [4pts]

$w_1(n) = x(n) + 0.24y(n)$

$w_2(n) = 2x(n) - 0.2y(n) + w_1(n-1)$

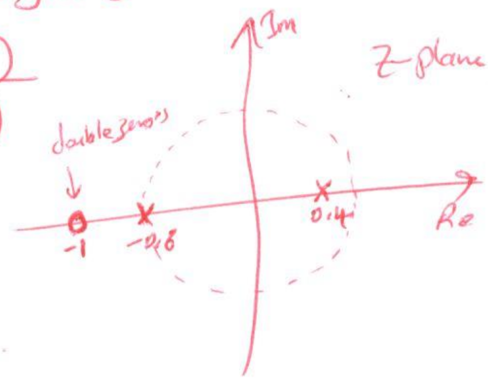
$y(n) = x(n) + w_2(n-1)$

$y(n) = x(n) + 2x(n-1) - 0.2y(n-1) + w_1(n-2)$

$y(n) = x(n) + 2x(n-1) - 0.2y(n-1) + x(n-2) + 0.24y(n-2)$

$y(n) = x(n) + 2x(n-1) + x(n-2) \cancel{+} - 0.2y(n-1) + 0.24y(n-2)$

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + 0.2z^{-1} - 0.24z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1+0.6z^{-1})(1-0.4z^{-1})}$$



ROC:

$|z| > 0.6 \Rightarrow$ Unit circle inside.

$\Rightarrow$ So, it is stable system.

c) Draw the following realization structure forms of the above filter: [6pts]

i) Parallel form using first order filters.

ii) Cascade form.

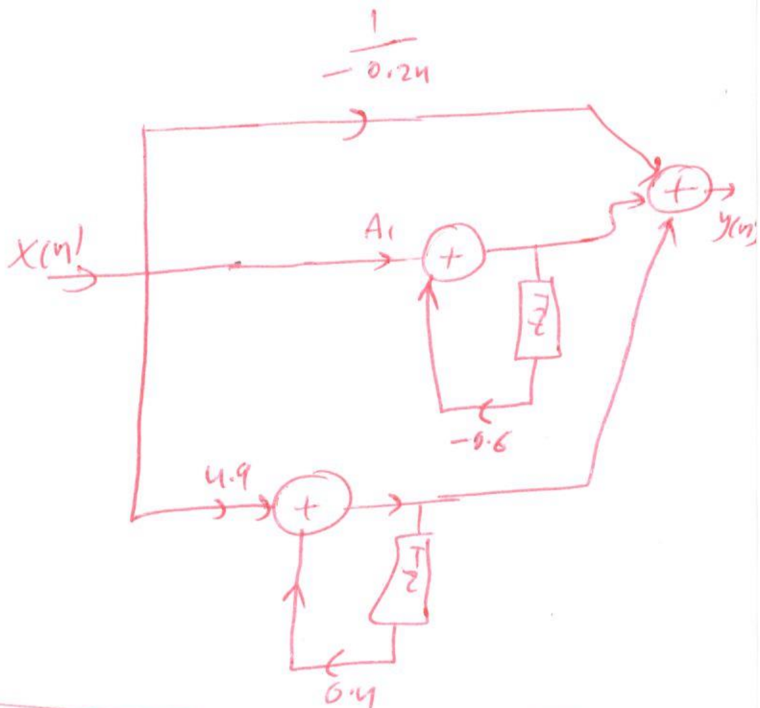
iii) Transposed direct form II.

i) $$H(z) = \frac{1+2z^{-1}+z^{-2}}{(1+0.6z^{-1})(1-0.4z^{-1})} = B_0 + \frac{A_1}{1+0.6z^{-1}} + \frac{A_2}{1-0.4z^{-1}}$$

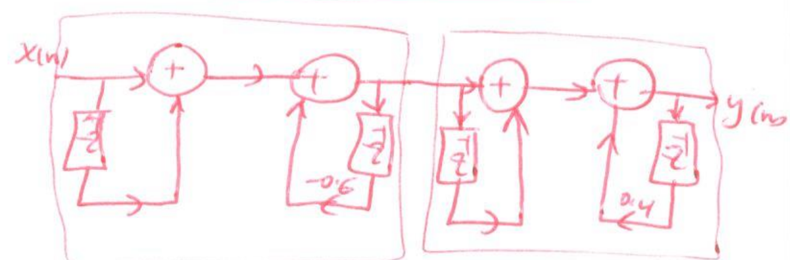
$$B_0 = \frac{-1}{0.24}$$

$$A_1 = H(z)(1+0.6z^{-1})\Big|_{z=-0.6} = \frac{64}{3}$$

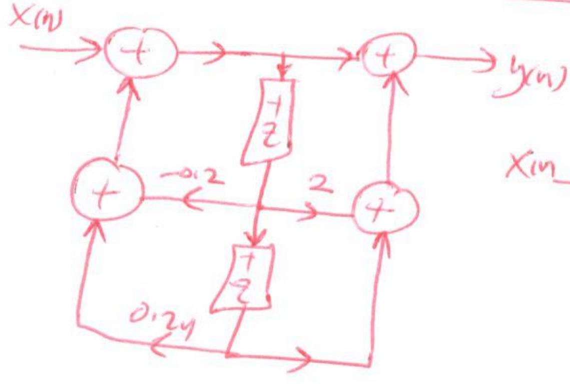
$$A_2 = H(z)(1-0.4z^{-1})\Big|_{z=0.4} = 4.9$$



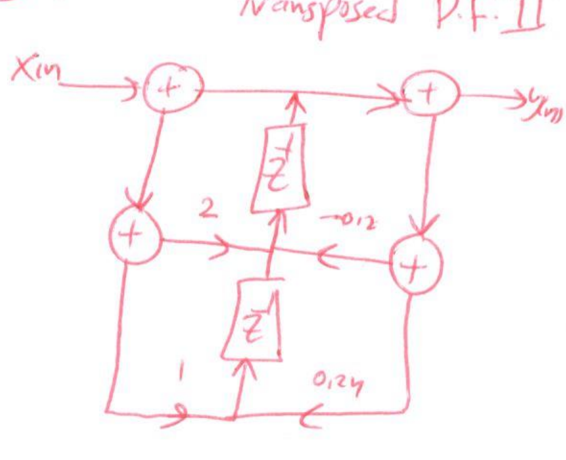
ii) $$H(z) = \left(\frac{1+z^{-1}}{1+0.6z^{-1}}\right)\cdot\left(\frac{1+z^{-1}}{1-0.4z^{-1}}\right)$$



iii) Direct form II



Transposed D.F. II



Question 3: [9pts]

a) Prove the convolution theorem of Z-transform, i.e. Z{x(n)*y(n)} = X(z)Y(z). [3pts]

Set $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n} = \sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x(k)\, h(n-k)\right) z^{-n} = \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(n-k)\, z^{-n}$$

Set $r = n-k$

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{r=-\infty}^{\infty} h(r)\, z^{-r-k} = \sum_{k=-\infty}^{\infty} x(k)\, z^{-k} \sum_{r=-\infty}^{\infty} h(r)\, z^{-r} = X(z) \cdot H(z).$$

#.

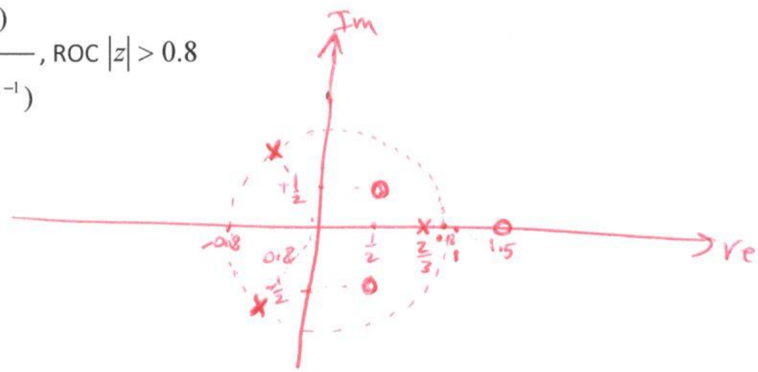
b) Consider the following transfer function of an LTI system.

$$H(z) = \frac{(1-1.5z^{-1})(1-\frac{1+j}{2}z^{-1})(1-\frac{1-j}{2}z^{-1})}{(1-0.8e^{j\frac{2\pi}{3}}z^{-1})(1-0.8e^{-j\frac{2\pi}{3}}z^{-1})(1-\frac{2}{3}z^{-1})}, \text{ ROC } |z| > 0.8$$

i) Plot the pole-zero diagram of this system. [2pts]

Zeros: $z = 1.5,\ \frac{1}{2}+\frac{1}{2}j,\ \frac{1}{2}-\frac{1}{2}j$

Poles: $0.8e^{j\frac{2\pi}{3}},\ 0.8e^{-j\frac{2\pi}{3}},\ \frac{2}{3}$



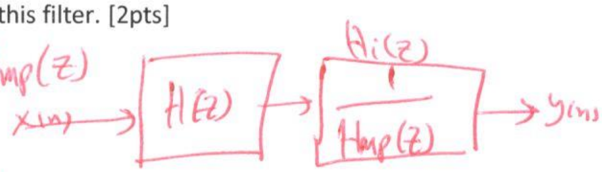
ii) Express H(z) as a cascade of an all-pass filter and a minimum-phase filter. [2pts]

$$H(z) = H_{AP}(z) \cdot H_{mp}(z)$$

$$H_{AP}(z) = \frac{z^{-1} - \frac{2}{3}}{1 - \frac{2}{3}z^{-1}}, \quad H_{mp}(z) = -1.5\frac{(1-\frac{1+j}{2}z^{-1})(1-\frac{1-j}{2}z^{-1})}{(1-0.8e^{j\frac{2\pi}{3}}z^{-1})(1-0.8e^{-j\frac{2\pi}{3}}z^{-1})}$$

iii) Suggest a method for eliminating magnitude distortion of this filter. [2pts]

Connect it in Cascade with inverse of $H_{mp}(z)$



$$H_i(z) = -\frac{2}{3}\frac{(1-0.8e^{j\frac{2\pi}{3}}z^{-1})(1-0.8e^{-j\frac{2\pi}{3}}z^{-1})}{(1-\frac{1+j}{2}z^{-1})(1-\frac{1-j}{2}z^{-1})}, \quad |z| > \frac{1}{\sqrt{2}}$$

ROC.

Bonus: MATLAB [5pts]

a) Write a MATLAB code for generating a discrete-time signal of length 1 second and sampling frequency 8KHz. The signal consists of the following two frequencies 200Hz and 3KHz with unity amplitude of each one. Display this signal on a figure. [1pt]

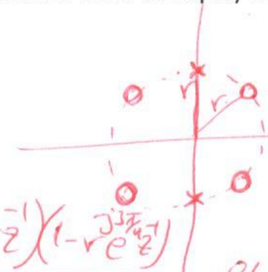
```
Fs = 8000;
t = 0 : 1/Fs : 1;
A = [1 1 1];
F = [1000 2000 3000];
x = A.*cos(2*pi*F.*t);
stem(t, x);
```

b) Suggest a filter for ~~removing 200Hz and keep 3KHz~~ passing only 2kHz and remove 1kHz and 3kHz? ~~Write MATLAB code to display the frequency response and phase response of your filter [2pts].~~

1000Hz ⇒ $\omega_1 = \frac{2\pi(1000)}{8000} = 0.25\pi$

2000 Hz ⇒ $\omega_2 = 0.5\pi$

3000 Hz ⇒ $\omega_3 = 0.75\pi$



→ we put two poles at ja and $-ja$.

→ we put two zeros at $re^{j\pi/4}$ and $re^{-j\pi/4}$

→ we put " " " $re^{j3\pi/4}$ and $re^{-j3\pi/4}$

$$H(z) = \frac{k(1 - re^{j\pi/4}z^{-1})(1 - re^{j\frac{3\pi}{4}}z^{-1})(1 - re^{j\frac{3\pi}{4}}z^{-1})(1 - re^{-j\frac{\pi}{4}}z^{-1})}{(1 - re^{j\frac{\pi}{2}}z^{-1})(1 - re^{-j\frac{\pi}{2}}z^{-1})}$$

set $k = 0.01$ and $r = 0.9$.

~~freqz(b,a)~~; Simplify it as $\frac{P(z^{-1})}{Q(z^{-1})}$

c) Write a MATLAB code for implementing your filter in part b) above and apply it on the generated signal in part a) above. [1pt]

coef. of P(z)

coef of Q(z)

```
y = filter(b, a, x);
stem(t, y);
```