Key

Faculty of Information Technology

Computer Systems Engineering

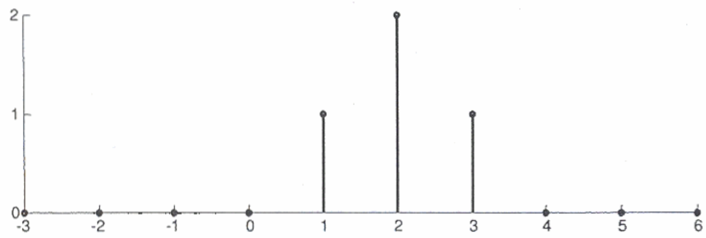
First Exam *[Time Allowed: 2 hours]*

ENCS431 (DSP) **Fall Semester 2012/2013**

Name: **ID:**

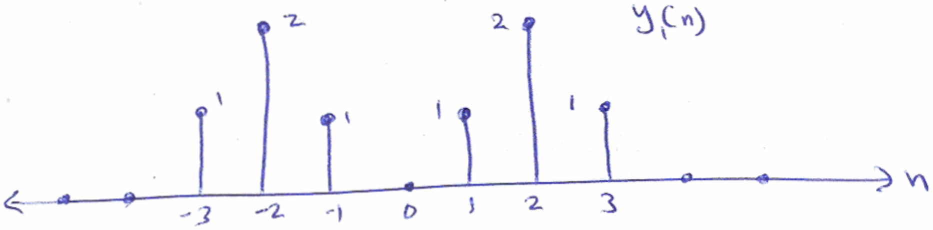
Question one: [25 marks]

(a) For the signal x(n) below: [*16 pts*]

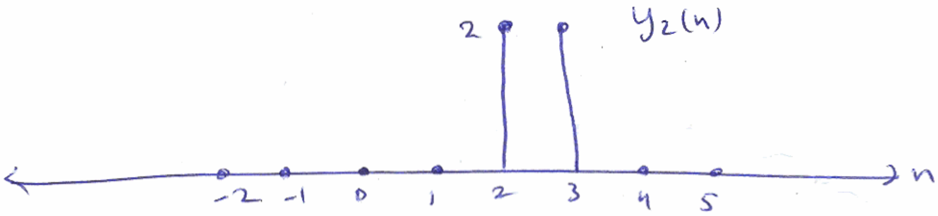


Plot the following signals:

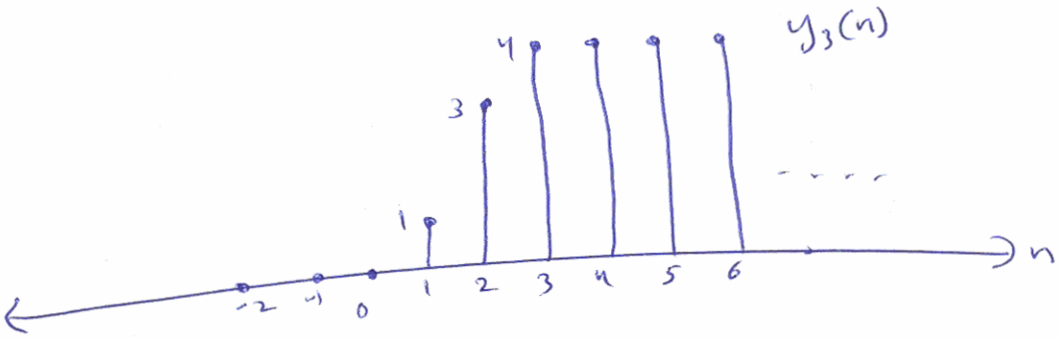
i) $y_1(n) = x(n) + x(-n)$ [4 pts]



ii) $y_2(n) = x(n)x(n-1)$ [4 pts]

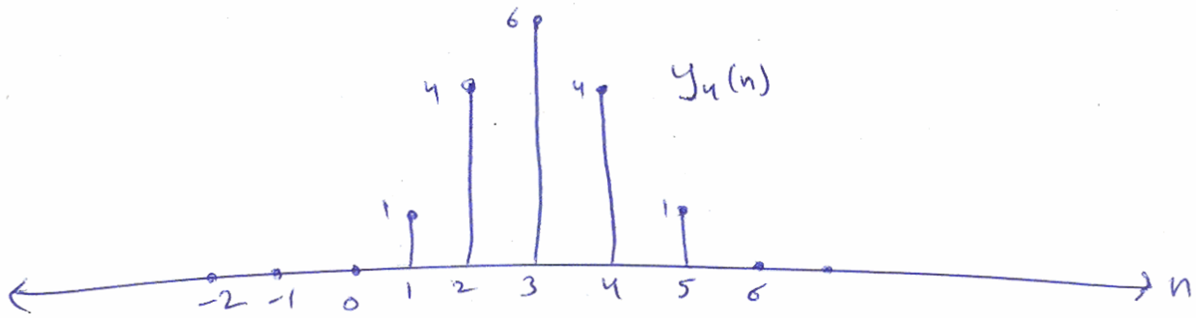


iii) $y_3(n) = \sum_{k=-\infty}^{n} x(k)$ [4 pts]



iv) $y_4(n) = x(n) * x(n)$ [4 pts]

n	1	2	3	4	5
X(n)	1	2	1		
X(n)	1	2	1		
	1	2	1		
	–	2	4	2	
	–	–	1	2	1
X(n) * X(n)	1	4	6	4	1



(b) For What values of **ω** is the signal $x(n) = e^{(j\omega n)}$ periodic with period of 8? [9 pts]

$X(n) = e^{j\omega n}$ is periodic iff $X(n) = X(n + kN)$ for any Integer k. $\Rightarrow N\omega = 2\pi k \Rightarrow \omega = \frac{2\pi}{N} k$

for $N = 8$

$\omega = \frac{2\pi}{8} k = \frac{\pi}{4} k$

So, Value of ω is $\{\ldots, -\frac{3\pi}{4}, -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \ldots\}$

<u>Question Two</u>: [25 marks]

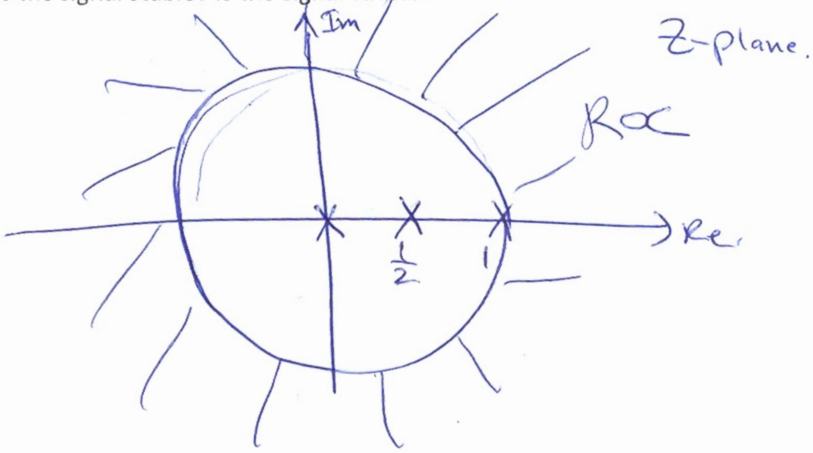
<u>(a)</u> A signal has the z-transform [*16 pts*]

$$X(z) = \frac{1}{z(z-1)(2z-1)}$$

with region of convergence $|z| > 1$, Draw a pole-zero plot of the signal in the z-plane, and use the method of partial fractions to recover the signal x(n) . Is the signal stable? Is the signal causal?

* Poles at $z = 0$
  $z = 1$
  $z = \frac{1}{2}$

* No zeros



Therefor, the signal is Causal (Roc is outside outermost pole) but it is unstable (Roc doesn't contain Unit Circle).

H(z) Can be written as;

$$H(z) = \frac{\frac{1}{2} z^{-3}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{1}{2} z^{-3} \left[\frac{A_1}{1-z^{-1}} + \frac{A_2}{1-\frac{1}{2}z^{-1}} \right]$$

$$A_1 = \left. \frac{1}{1-\frac{1}{2}z^{-1}} \right|_{z=1} = 2$$

$$A_2 = \left. \frac{1}{1-z^{-1}} \right|_{z=\frac{1}{2}} = -1$$

The terms in the square brackets inverts to $x_1(n) = 2u(n) - (\frac{1}{2})^n u(n)$

So, x(n) is $x_1(n)$ delayed by 3 samples ⇒

$$x(n) = \frac{1}{2}\left(2u(n-3) - (\tfrac{1}{2})^{n-3} u(n-3)\right)$$

$$\boxed{x(n) = u(n-3) - (\tfrac{1}{2})^{n-2} u(n-3)}$$

(b) An LTI-system has a system function [*9 pts*]

$$H(z) = \frac{6Z-2}{6Z-3}, |z| > \frac{1}{2}$$

Find impulse response of stable inverse system. Is this inverse system causal?

The System Function Can be written as

$$H(Z) = \frac{Z - 1/3}{Z - \frac{1}{2}} = \frac{1 - \frac{1}{3}Z^{-1}}{1 - \frac{1}{2}Z^{-1}}$$

So, Inverse Hi(Z) is

$$Hi(Z) = \frac{1 - \frac{1}{2}Z^{-1}}{1 - \frac{1}{3}Z^{-1}}$$

This has a pole at $Z = \frac{1}{3}$, so the two possible ROCs are

$|Z| < 1/3$ and $|Z| > 1/3$

Since the inverse is stable, its ROC must contain unit circle and overlap with the ROC of H(Z). ⇒ ROC is $|Z| > \frac{1}{3}$

So, Hi(Z) is Causal because ROC is outside of outermost pole.

The inverse in this case is right-sided sequence

$$hi(n) = \left(\frac{1}{3}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{3}\right)^{n-1} u(n-1).$$

Question Three: [25 marks]

A causal LTI system has a system function

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$

(a) Write the difference equation that is satisfied by input and output of this system. [*9 pts*]

$$H(z) = \frac{1+0.9z^{-1}-1.5z^{-1}-1.35z^{-2}-z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}}$$

$$= \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}} = \frac{Y(z)}{X(z)}$$

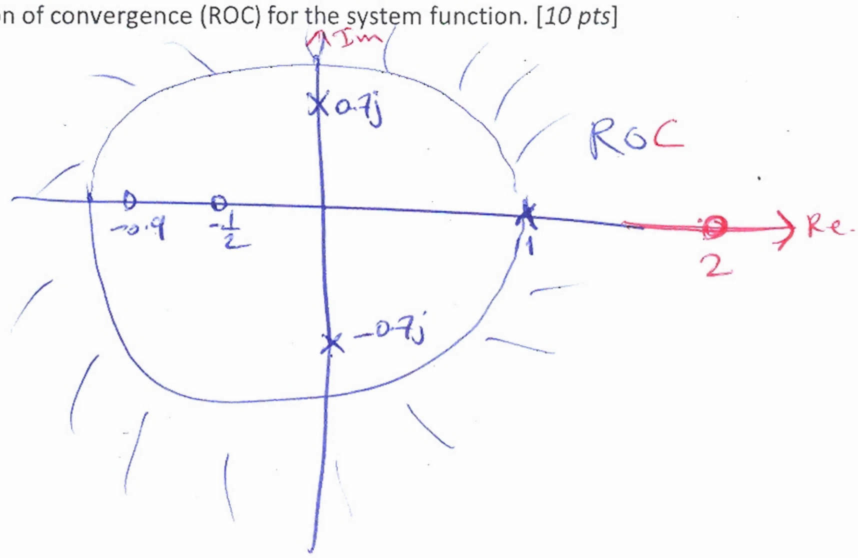
$$y(n)-y(n-1)+0.49y(n-2)-0.49y(n-3) = x(n)-0.6x(n-1)-2.35x(n-2)-0.9x(n-3)$$

(b) Plot the pole-zero diagram and indicate the region of convergence (ROC) for the system function. [*10 pts*]

Zeros at $z=-0.9$, $z=2$, $z=-\frac{1}{2}$

Poles at $z=1$, $z=-0.7j$, $z=0.7j$

Since Sys. is Causal ⇒ ROC: $|z|>1$



(c) State whether the following are true or false about the system: [*6 pts; 2 each*]

(i) The system is stable

False, because RoC doesn't include unit circle.

(ii) The impulse response approaches a constant for large n.

False, $n\to\infty \Rightarrow h(n)\to\infty$ (unstable).

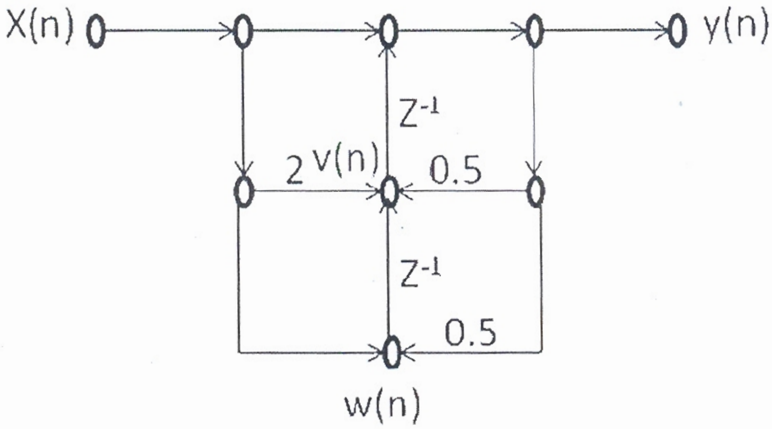
(iii) The system has a stable and causal inverse.

False. For a system to have stable and causal inverse, all of its zero's and poles must be inside unit circle.

Pole at $z=1$ is not inside unit circle.

Question Four: [25 marks]

(a) Consider the signal flow graph shown below: [*15 pts*]



(i) Using the node variables indicated, write the set of difference equations represented by this network [6 pts]

$$v(n) = 2x(n) + w(n-1) + \tfrac{1}{2}y(n)$$

$$w(n) = x(n) + \tfrac{1}{2}y(n)$$

$$y(n) = x(n) + v(n-1)$$

$$V(z) = 2X(z) + z^{-1}W(z) + \tfrac{1}{2}Y(z)$$

$$W(z) = X(z) + \tfrac{1}{2}Y(z)$$

$$Y(z) = X(z) + z^{-1}V(z)$$

$$V(z) = 2X(z) + z^{-1}\{X(z) + \tfrac{1}{2}Y(z)\} + \tfrac{1}{2}Y(z)$$

$$V(z) = (2 + z^{-1})X(z) + (\tfrac{1}{2} + \tfrac{1}{2}z^{-1})Y(z)$$

$$Y(z) = X(z) + z^{-1}[(2+z^{-1})X(z) + \tfrac{1}{2}(1+z^{-1})Y(z)]$$

$$= X(z) + (2z^{-1} + z^{-2})X(z) + \tfrac{1}{2}(z^{-1} + z^{-2})Y(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - \tfrac{1}{2}z^{-1} - \tfrac{1}{2}z^{-2}}$$

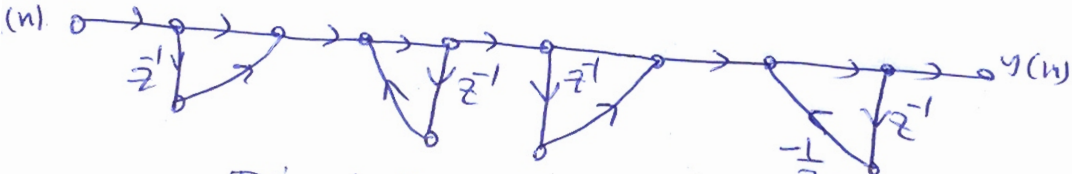
(ii) Draw the flow graph of an equivalent system that is cascade of two first-order systems. [6 pts]

from part (i)

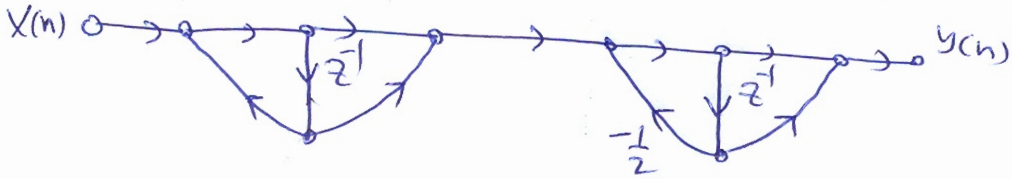
$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1-z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H_1(z) = \frac{1+z^{-1}}{1-z^{-1}} \text{ and } H_2(z) = \frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}$$



Direct form I

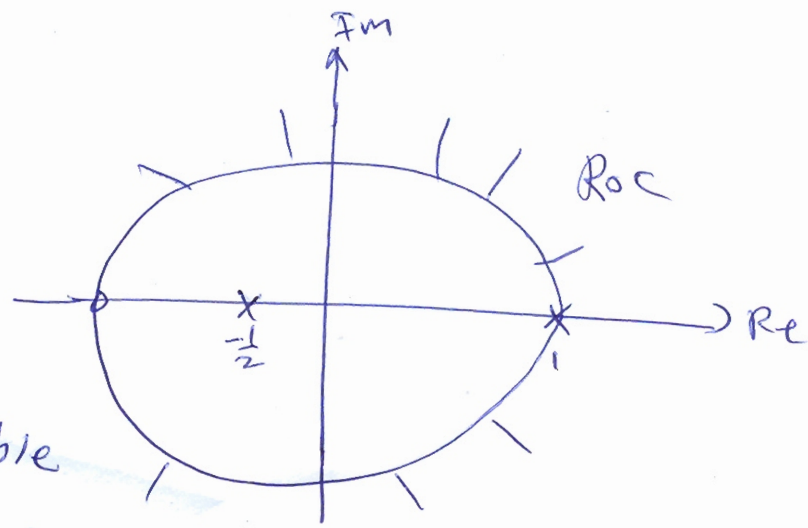


Direct form II

(iii) Is this system stable? Explain [3 pts]

Zero's at $z=-1$

Poles at $z=1$, $z=-\frac{1}{2}$



So, This system is unstable because unit circle ($z=1$) is not included in any of possible RoC's.

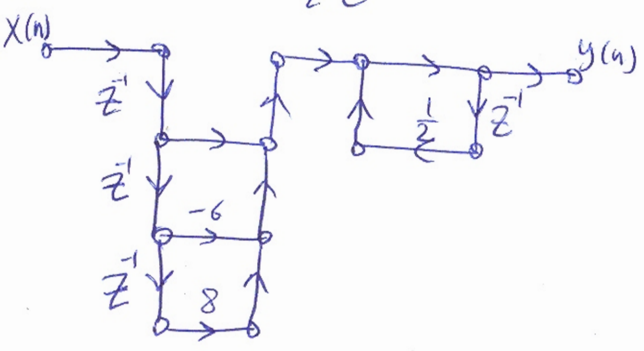
(b).Consider a causal LTI system with system function [10 pts]

$$H(z) = \frac{(1-2z^{-1})(1-4z^{-1})}{z(1-\frac{1}{2}z^{-1})}$$

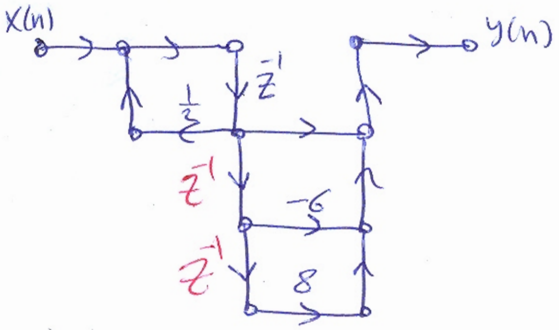
(i) Draw direct form II flow graph for the system [5 pts]

$$H(z) = \frac{1 - 4z^{-1} - 2z^{-1} + 8z^{-2}}{z - \frac{1}{2}} = \frac{1 - 6z^{-1} + 8z^{-2}}{z - \frac{1}{2}} = \frac{z^{-1}(1 - 6z^{-1} + 8z^{-2})}{1 - \frac{1}{2}z^{-1}}$$

$$= \frac{z^{-1} - 6z^{-2} + 8z^{-3}}{1 - \frac{1}{2}z^{-1}}$$

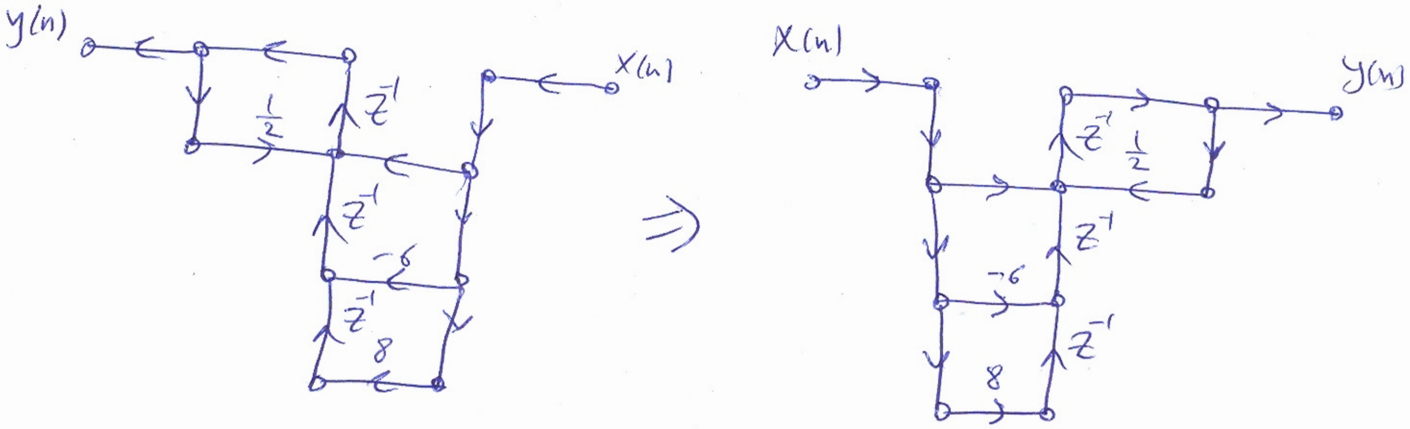


Direct form I



Direct form II

(ii) Draw the transposed form of the flow graph in part (i) [5 pts]



Transposed

Good Luck!