BIRZEIT UNIVERSITY

Faculty of Information Technology

Computer Systems Engineering Department

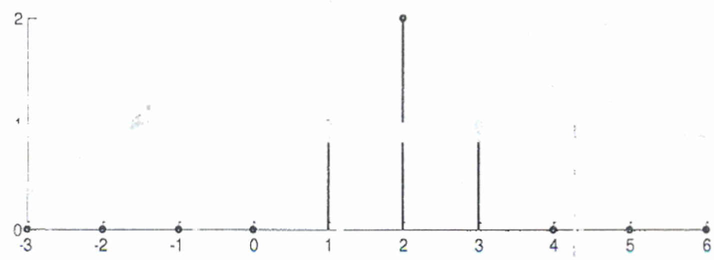
Digital Signal Processing - First Exam

Fall Semester 2013/2014

Name: key ID: section:

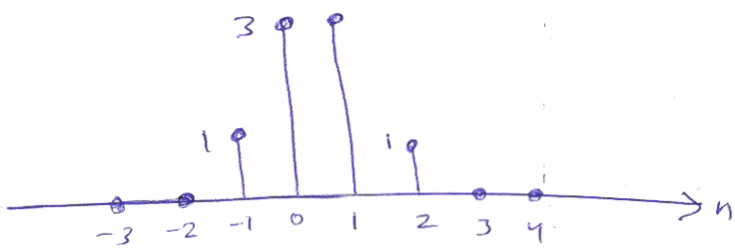
Question one: [34 marks]

(a) For the signal x(n) below: [*24 pts*]

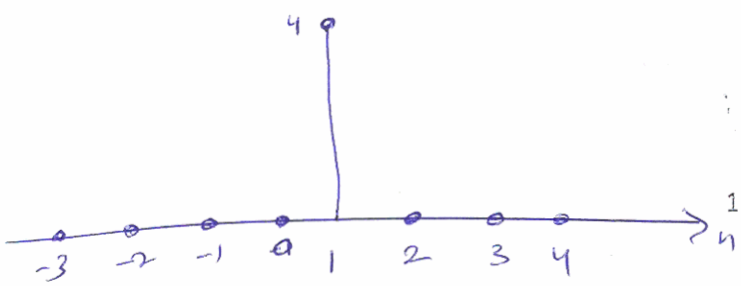


Plot the following signals:

(i) $y_1(n) = x(n+1) + x(-n+2)$ [6 pts]



(ii) $y_2(n) = x(2n)x(n+1)$ [6pts]



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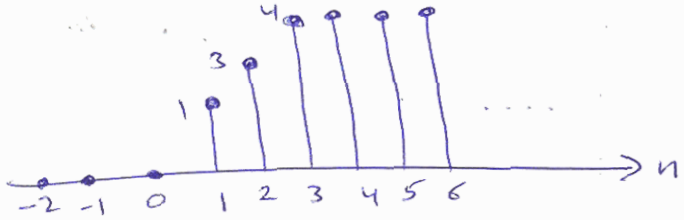
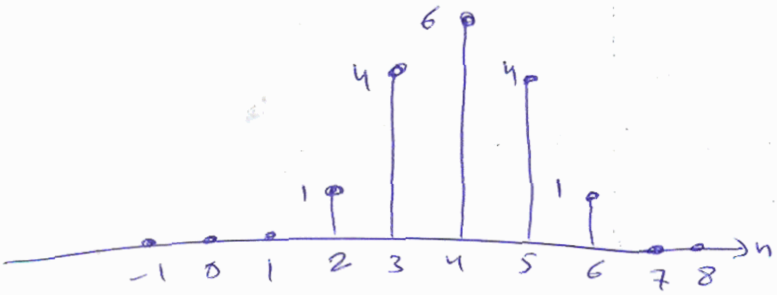
(iii) $y_3(n) = \sum_{k=-\infty}^{n} x(k)$ [6 pts]



(iv) $y_4(n) = x(n) * x(n)$ [6 pts]



(b) For What ***values*** of **ω** is the signal $x(n) = e^{(j\omega n)}$ periodic with period of 5? [10 pts]

$\omega N = 2\pi k$, k integer

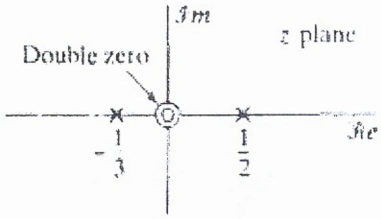
$\omega = \frac{2\pi}{N} k$

$= \frac{2\pi}{5} k = 0.4\pi k$, $k = 0, 1, 2, 3 \ldots$

$\omega = 0, 0.4\pi, 0.8\pi, 1.2\pi, 1.6\pi, 2\pi$

Question 2: [33 marks]

(a) The system function $H(z)$ of a causal LTI system has the pole-zero configuration shown in the following figure. It is also known that $H(z) = 6$ when $z=1$.



(i) Determine $H(z)$? [4pts]

$$H(z) = \frac{A}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}, \quad |z| > \frac{1}{2} \ (\text{causal } h(n)).$$

$$H(1) = 6 \Rightarrow \frac{A}{(\frac{1}{2})(\frac{4}{3})} = 6 \Rightarrow \boxed{A = 4} \ [1]$$

$$\text{So, } H(z) = \frac{4}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} \text{ or } \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})} [2], \ |z| > \frac{1}{2} \ [1]$$

(ii) Determine the output sequence, y(n), if the following input is applied to the system: [7pts]

$$x(n) = u(n) - \frac{1}{2}u(n-1). \Rightarrow X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}}, \ |z| > 1 \ [2]$$

$$Y(z) = H(z)\,X(z)$$

$$= \frac{4}{(1-\cancel{\frac{1}{2}z^{-1}})(1+\frac{1}{3}z^{-1})} \cdot \frac{(1-\cancel{\frac{1}{2}z^{-1}})}{(1-z^{-1})}, \ |z| > 1$$

$$= \frac{4}{(1-z^{-1})(1+\frac{1}{3}z^{-1})}, \ |z| > 1 \ [2]$$

$$= \frac{A_1}{1-z^{-1}} + \frac{A_2}{1+\frac{1}{3}z^{-1}}$$

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$$A_1 = Y(z)(1-z^{-1})\Big|_{z=1} = 3 \ [1]$$

$$A_2 = Y(z)(1+\tfrac{1}{3}z^{-1})\Big|_{z=-\frac{1}{3}} \Rightarrow A_2 = 1 \ [1] \Rightarrow y(n) = 3u(n) + (-\tfrac{1}{3})^n u(n) \ [1]$$

(b) An LTI-system has a system function [*11 pts*]

$$H(z) = \frac{6Z-2}{6Z-3}, |z| > \frac{1}{2}$$

Find impulse response of **stable inverse system**. Is this inverse system causal?

$H(z)$ Can be written as $H(z) = \dfrac{z - \frac{1}{3}}{z - \frac{1}{2}} = \dfrac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$

$H_i(z) = \dfrac{1}{H(z)} = \dfrac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{3}z^{-1}}$ [3], $H_i(z)$ has a pole at $z = \frac{1}{3}$

so There are two possibilities for ROC $\Rightarrow$ $|z| > \frac{1}{3}$ or $|z| < \frac{1}{3}$

Since $H_i(z)$ is stable (and it should intersect with Roc of $H(z)$ $|z| > \frac{1}{2}$)

therefore, Roc of $H_i(z)$ is $|z| > \frac{1}{3}$ [3], $H_i(z)$ is Causal because its Roc extends from outtermost pole to ∞. [2]

$h_i(n) = (\frac{1}{3})^n u(n) - \frac{1}{2}(\frac{1}{3})^{n-1} u(n-1)$ [3]

(c) Define the following systems and give an example for each one:

(i) Linear phase system [3pts]

phase Response is a linear function of frequency, therefor it has a Constant group delay. e.g. ideal delay system $y(n) = x(n - n_0)$, n_0 in

(ii) Minimum phase system [4pts]

Causal and Stable system and its Inverse is a Causal and Stable

or System which has all poles and zeros inside unit circle.

e.g. $H(z) = \dfrac{(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{4}z^{-1})}$.

(iii) All pass system [4pts]

System which has a magnitude response $|H(e^{jw})| = 1$ or Constant

or System which has poles and zeros in Conjugate reciprocal pair

e.g.

$$H(z) = \frac{z^{-1} - 2}{1 - 2z^{-1}}$$

Q3 (a)

$$H(e^{j\omega}) = a + b e^{-j2\omega}$$

at $\omega = 0 \Rightarrow H(e^{j\omega}) = \boxed{1 = a + b \quad \text{———} ①}$ [3]

at $\omega = \frac{\pi}{3} \Rightarrow \cancel{|H(e^{j\omega})|}$

[2] $20\log_{10}|H(e^{j\omega})| = -6 \Rightarrow |H(e^{j\omega})| = 10^{-0.3} = 0.5$

$$H(e^{j\omega}) = a + b e^{-j\frac{2\pi}{3}} = a + b\left[\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}\right]$$

[2] $= a - 0.5b - j\frac{\sqrt{3}}{2}b$

$$|H(e^{j\omega})| = \sqrt{(a-0.5b)^2 + \left(\frac{\sqrt{3}}{2}b\right)^2} = \frac{1}{2}$$ [1]

$$a^2 - ab + \frac{1}{4}b^2 + \frac{3}{4}b^2 = \frac{1}{4}$$

$$a^2 - ab + b^2 = \frac{1}{4}$$

but $a = \cancel{b}\ 1 - b$

$$(1-b)^2 - b(1-b) + b^2 = \frac{1}{4}$$

$$1 - 2b + b^2 - b + b^2 + b^2 = \frac{1}{4}$$

$$3b^2 - 3b + \frac{3}{4} = 0$$

$$b^2 - b + \frac{1}{4} = 0$$

$$(b - \tfrac{1}{2})(b - \tfrac{1}{2}) = 0 \Rightarrow \left.\begin{aligned} b &= \tfrac{1}{2} \\ a &= \tfrac{1}{2} \end{aligned}\right\}$$ [3]

Question 3: [33 marks]

(a) Consider the following FIR second order filter:

$$y[n] = ax[n] + bx[n-2], a \neq 0, b \neq 0$$

Determine coefficients **a** and **b** such that the filter attenuates the signal magnitude **-6dB** at $\omega = \frac{\pi}{3}$ and its frequency response is normalized so that $H(0) = 1$. [11pts]

$$H(e^{j\omega}) = a + b e^{-j2\omega}$$

at $\omega = 0 \Rightarrow H(e^{j\omega}) = 1 \Rightarrow \boxed{a + b = 1}$ — ①

at $\omega = \frac{\pi}{3} \Rightarrow 20 \log_{10} |H(e^{j\omega})| = -6 \Rightarrow |H(e^{j\omega})| = 10^{-0.3} \approx 0.5$

$H(e^{j\omega})$ at $\omega = \frac{\pi}{3}$ $= a + b e^{-j\frac{2\pi}{3}} = a + b\left[\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}\right]$

$= a - \frac{1}{2}b - j\frac{\sqrt{3}}{2}b$

$|H(e^{j\omega})| = \sqrt{(a - \frac{1}{2}b)^2 + (\frac{\sqrt{3}}{2}b)^2} = \frac{1}{2}$ — ②

$a^2 - ab + \frac{1}{4}b^2 + \frac{3}{4}b^2 = \frac{1}{4} \Rightarrow a^2 - ab + b^2 = \frac{1}{4}$. but $a = 1 - b$

$(1-b)^2 - b(1-b) + b^2 = \frac{1}{4}$

$3b^2 - 3b + \frac{3}{4} = 0 \Rightarrow b^2 - b + \frac{1}{4} = 0$

$(b - \frac{1}{2})(b - \frac{1}{2}) = 0 \Rightarrow b = \frac{1}{2} \Rightarrow a = \frac{1}{2}$

(b) Show that the z-transform of a sequence $nx(n) = -z\frac{dX(z)}{dz}$. [11pts]

[3] $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \Rightarrow \frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x(n) \frac{d(z^{-n})}{dz}$ [4]

$= \sum_{n=-\infty}^{\infty} x(n)(-n) z^{-n-1} = -z^{-1} \sum_{-\infty}^{\infty} n x(n) z^{-n}$ [2]

so, $-z\frac{dX(z)}{dz} = \sum_{-\infty}^{\infty} n x(n) z^{-n}$ [2]

$\therefore -z\frac{dX(z)}{dz} = Z\{n x(n)\}$ [1]

(c) Determine the impulse response of the following causal system: [11pts]

$$y(n) = 0.4y(n-1) - 0.08y(n-2) + x(n)$$

$z^2 - 0.4z - 0.08$

$$H(z) = \frac{1}{1 - 0.4z^{-1} - 0.08z^{-2}} \quad [3]$$

$$z = \frac{0.4 \mp \sqrt{(0.4)^2 - (4)(1)(6)}}{2}$$

$$= \frac{1}{(1 - 0.54z^{-1})(1 + 0.14z^{-1})} \quad [2]$$

$$z = \frac{0.4 \mp \sqrt{0.48}}{2}$$

$= 0.54$ and -0.14

Since Sys. is Causal $\Rightarrow$ ROC $|z| > 0.54$ and $h(n)$ is Right-sided seq.

$$H(z) = \frac{A_1}{1 - 0.54z^{-1}} + \frac{A_2}{(1 + 0.14z^{-1})} \quad [2]$$

$$A_1 = H(z)(1 - 0.54z^{-1})\Big|_{z=0.54} \simeq 0.788 \quad (1)$$

$$A_2 = H(z)(1 + 0.14z^{-1})\Big|_{z=-0.14} \simeq 0.211 \quad [1]$$

So, $h(n) = 0.788\,(0.54)^n u(n) + 0.211\,(-0.14)^n u(n)$ [2]

Question 4: [Bonus]

Given the following system function:

$$H(z)=\frac{2z^4-5z^3+13.48z^2-7.78z+9}{4z^4+7.2z^3+20z^2-0.8z+8}$$

Write a MATLAB statements to:

(a) Display the magnitude response and the phase response of the system.

[3] freqz([2 -5 13.48 -7.78 9],[4 7.2 20 -0.8 8]);

(b) Plot zero-pole diagram of the system.

[3] zplane([2 -5 13.48 -7.78 9],[4 7.2 20 -0.8 8]);

(c) Evaluate impulse response, h[n], of the system.

or any 50 no.

[3] [h,n] = impz([2 -5 13.48 -7.78 9],[4 7.2 20 -0.8 8],50);

or h = filter([2 -5 13.48 -7.78 9],[4 7.2 20 -0.8 8],[1 0 0 0 0 0]);

(d) Find the output sequence if the following input is applied to the system x[n] = { 1, 0, 3, 1, 2,1}, for n=0 to 5.

[3] y(n) = filter([2 -5 13.48 -7.78 9],[4 7.2 20 -0.8 8],
[1 0 3 1 2 1]);