جامعة بيرزيت

BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENCS431- Digital Signal Processing – Fall semester 2016

Midterm Exam – Dec. 13, 2016

Name: Key **ID:** **Section:** 10:00 – 11:00, 12:00 – 1:00

Question	Student mark	ABET SO
1		a
2		c
3		k

Question 1: [10pts]

(a) The input signal

$x(n) = 0.6 - 0.5\cos(\frac{\pi n}{2} + \frac{\pi}{5})$ is applied to discrete-time system which has the following system impulse response.

$$h[n] = \left(\frac{1}{4}\right)^{n-1} u(n), \quad -\infty \le n \le \infty$$

Where u(n) is the unit step sequence. Determine the system output response y(n). [5pts]

$$H(e^{j\omega}) = F\{h(n)\} = \frac{4}{1-\frac{1}{4}e^{-j\omega}} \qquad e^{-j\pi/2} = -j$$

$$\Rightarrow \text{at } \omega = \pi/2 \Rightarrow H(e^{j\omega}) = \frac{4}{1-\frac{1}{4}e^{-j\pi/2}} = \frac{4}{1+\frac{1}{4}j} = 3.7647 - j0.9412$$

$$|H(e^{j\pi/2})| = 3.88 \quad \text{and} \quad \angle H(e^{j\pi/2}) = \tan^{-1}\frac{-0.9412}{3.7647} = -0.245 \approx -\pi/12.8$$

$$\Rightarrow \text{at } \omega = 0 \Rightarrow H(e^{j\omega}) = \frac{4}{1-\frac{1}{4}e^{j0}} = \frac{4}{1-\frac{1}{4}} = \frac{16}{3}$$

Therefore, $y(n) = (0.6)(\frac{16}{3}) - 0.5(3.88)\cos(\frac{\pi n}{2} + \frac{\pi}{5} - \frac{\pi}{12.8})$

$$\approx 3.2 - 1.94\cos(\frac{\pi n}{2} + \frac{\pi}{3.6}).$$

(b) The z-transform of a discrete sequence x(n) is denoted by X(z), derive the z-transform of sequence $nx(n)$. [5pts]

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\frac{dX(z)}{dz} = \sum_{-\infty}^{\infty} x(n)(-n) z^{-n-1}.$$

$$= -z^{-1} \sum (n x(n)) z^{-n} = -z^{-1} \cdot Z\{n x(n)\}$$

So, $Z\{n x(n)\} = -z \dfrac{dX(z)}{dz}$. #.

Question 2: [10pts]

a) A causal linear time-invariant system has the system function: [6pts]

$$H(z) = \frac{(1-1.5z^{-1} - z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1-j0.7z^{-1})(1+j0.7z^{-1})}$$

i) Write the difference equation that is satisfied by the input and output of the system?

$$H(z) = \frac{1 + 2.4 z^{-1} - 2.35 z^{-2} - 0.9 z^{-3}}{1 - z^{-1} + 0.49 z^{-2} - 0.49 z^{-3}} = \frac{Y(z)}{X(z)}$$

So, The difference eq. is:

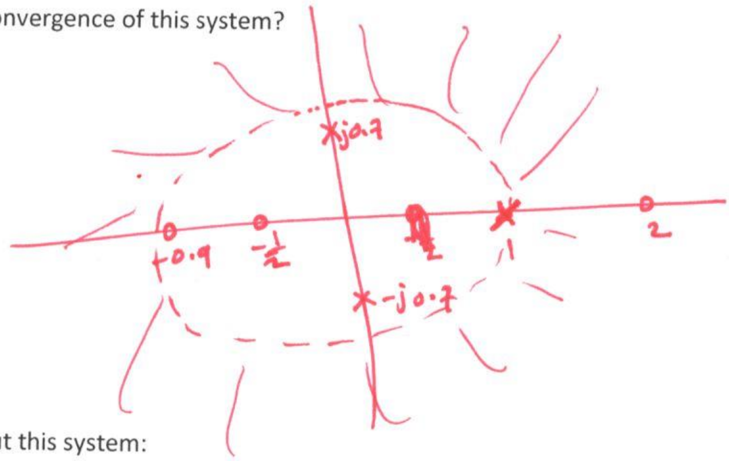
$$y(n) - y(n-1) + 0.49 y(n-2) - 0.49 y(n-3) = x(n) + 2.4 x(n-1) - 2.35 x(n-2) - 0.9 x(n-3).$$

ii) Plot zero-pole diagram and indicate region of convergence of this system?

Zeros: $-0.9, -\frac{1}{2}, 2$

poles: $1, j0.7, -j0.7$

ROC: $|z| > 1$
[Causal]



iii) State weather the following is true or false about this system:

⇨ The system is stable

False, pole at u.c.

⇨ The impulse response approaches a constant for large n.

False, h(n) is not absolutly summable.

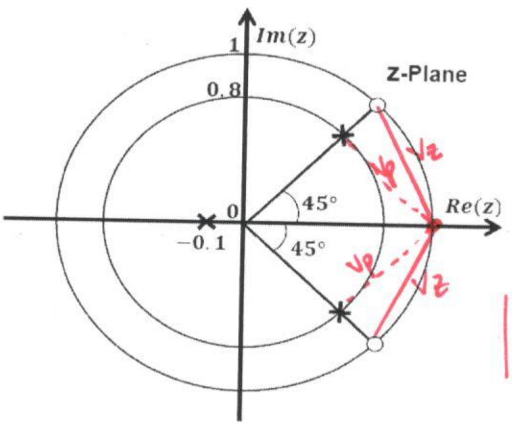
⇨ The magnitude of the frequency response has a peak at approximately $\omega = \pm\frac{\pi}{4}$

False, peak at $\omega = 0$.

⇨ The system has a stable and causal inverse.

False, zero outside unit circle.

b) Figure below shows the poles and zeros of the system function of a digital filter: [4pts]



$$|H(e^{j0})| = \frac{V_z^2}{1.1(V_p)^2}$$

Without explicitly finding $H(e^{j\omega})$, find magnitude and phase response of this system at ω=0.

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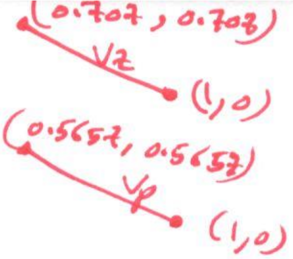
$V_z = \sqrt{(1-0.707)^2 + (-0.707)^2} = 0.7653$

$V_p = \sqrt{(1-0.5657)^2 + (-0.5657)^2} = 0.7132$

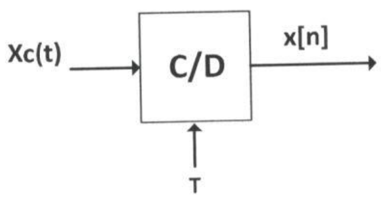


$$|H(e^{j0})| = \frac{|V_z|\cdot|V_z|}{|V_p||V_p|\cdot(1.1)} = \frac{(0.7653)^2}{1.1(0.7132)^2} = 1.3678$$

$$\angle H(e^{j0}) = 45^\circ + 45^\circ - 45^\circ - 45^\circ - 0^\circ = 0$$

Question 3 [10pts]:

a) Consider the ideal continuous-to-discrete convertor as shown below: [5pts]



Let the continuous-time signal, $x_c(t) = \cos(15\pi t) + \cos(40\pi t) + \cos(60\pi t)$, $\Rightarrow \Omega_{max} = 60\pi$ rad/sec, $f_{max} = 30$ Hz.

i) What is the range of sampling period, T that will avoid aliasing in the continuous-to-discrete converter? [1pt]

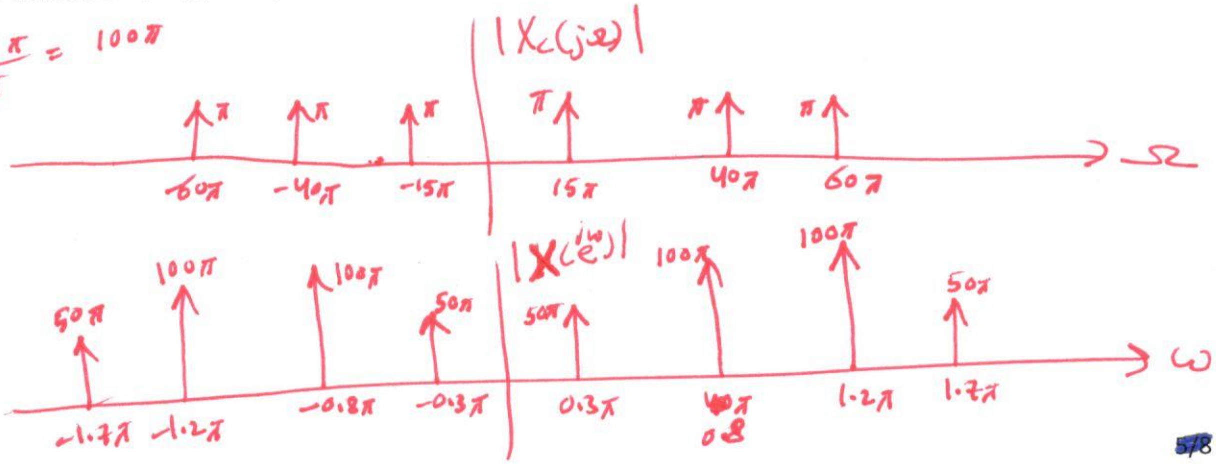
$\Omega_s \geq 2 * 60\pi$

$\frac{2\pi}{T} \geq 120\pi \Rightarrow T \leq \frac{1}{60}$ sec.

$T = \frac{1}{50} \Rightarrow$ There will be aliasing !

ii) Now assume sampling period, T is 20ms, plot spectrums of xc(t) and x[n]. i.e. $|X_c(j\Omega)|$ and $|\cancel{H}X(e^{j\omega})|$ [2pts]

$\Omega_s = \frac{2\pi}{T} = 100\pi$



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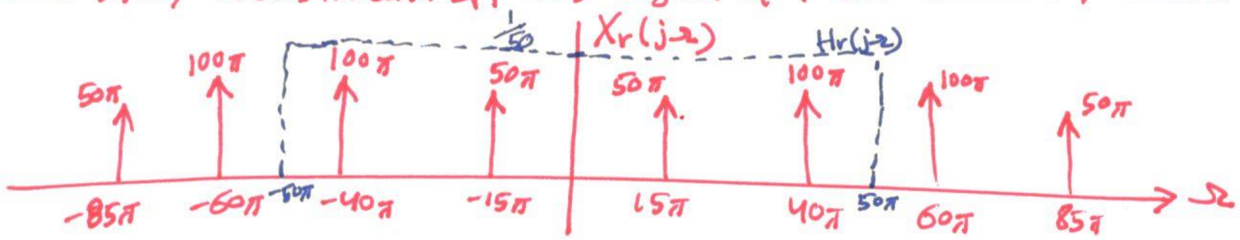
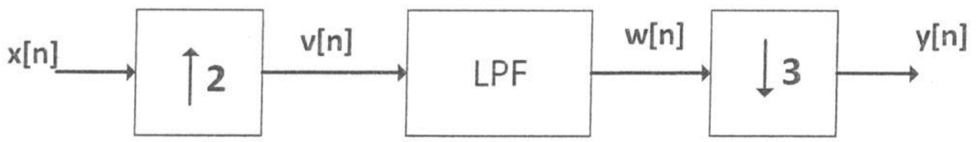
iii) If we use an ideal discrete-to-continuous converter (D/C), with gain T= 20ms to reconstruct a continuous-time signal from the resulted discrete-time signal, x[n], find the reconstructed signal $x_r(t)$? [1pt]

In Ideal D/C, reconstruction LPF has a gain of T and Cutoff freq. $\Omega_c = \frac{\Omega_s}{2} = 50\pi$



So, $x_r(t) = \cos(15\pi t) + 2\cos(40\pi t)$.

b) Consider the following system:



i) If x[n] = {1, 3, 5, 3, 7}, and impulse response of the Low-Pass filter is {0.5, 1, 0.5}, determine signals v[n], w[n] and y[n]. [3pts]
$\uparrow_{n=0}$ $\uparrow_{n=0}$

$v(n) = \{1, 0, 3, 0, 5, 0, 3, 0, 7, 0\}$

$w(n) = v(n) * h(n)$

$= \{1, 0, 3, 0, 5, 0, 3, 0, 7, 0\} * \{0.5, 1, 0.5\}$

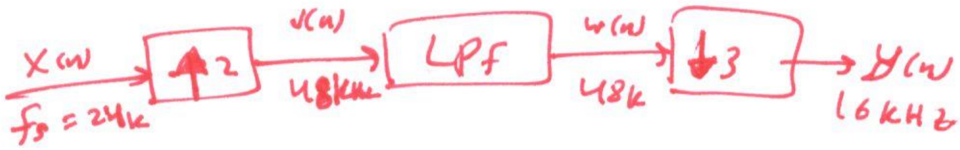
n:	0	1	2	3	4	5	6	7	8	9		
v(n) :	1	0	3	0	5	0	3	0	7	0		
h(n)	$\frac{1}{2}$	1	$\frac{1}{2}$									
	$\frac{1}{2}$	0	1.5	0	2.5	0	1.5	0	3.5	0		
	–	1	0	3	0	5	0	3	0	7	0	
	–	–	$\frac{1}{2}$	0	1.5	0	2.5	0	1.5	0	3.5	0
w(n)	$\frac{1}{2}$	1	2	3	4	5	4	3	5	7	3.5	0

$y(n) = \{0.5, 3, 4, 7\}$

OR $y(n) = \{1, 4, 3, 3.5\}$

ii) If x[n] is sampled at 24 KHz, find:

1) Sampling frequency of each signal. i.e. v[n], w[n] and y[n]. [1pt]



2) Gain and cut-off frequency of the Low-Pass filter. [1pt]

Gain = 3 to compensate attenuation of 3 due to decimation

$w_c = \frac{\pi}{3} \Rightarrow F_c = 8kHz$.

$\omega_c = \min\left\{\frac{\pi}{L}, \frac{\pi}{M}\right\} = \min\left\{\frac{\pi}{2}, \frac{\pi}{3}\right\} = \frac{\pi}{3}$

$\pi \rightarrow \frac{F_s}{2} = \frac{48k}{2} = 24kHz$.

$\frac{\pi}{3} \rightarrow \frac{24k}{3} = 8kHz$.

1 sec → 10^4 samples
10ms → ? ⇒ $\frac{0.01 * 10^4}{1}$ = 0.01 * fs
= 100 Samples.

10^4 Samples/Sec

MATLAB [5pts]

a) Write a MATLAB code for generating a discrete-time signal of length 10 ms and sampling frequency 10 KHz. The signal consists of the following three frequencies 1 KHz, 3 KHz and 5 KHz with amplitudes of 2, 4 and 10 respectively. Display this signal on a figure. [2pt]

```
Fs = 10000;
n=1:0.01*Fs;
x = 2*cos((2*pi*1000/fs).*n) + 4*cos((2*pi*3000/fs).*n) +
    10*cos((2*pi*5000/fs).*n);
stem(n,x);
```

b) Write MATLAB statement(s) to display zero-pole diagram of the following system: [1pt]

y(n) + 0.42y(n-2) = x(n) + 2x(n-1) + x(n-2)

```
zplane([1,2,1],[1,0,0.42]);
```

c) Write MATLAB statement(s) for finding and displaying the output signal if we apply the generated signal in part (a) to the discrete-time system in part (b): [2pts].

```
y= filter([1,2,1],[1,0,0.42],x);
```