BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENCS431- Digital Signal Processing – Fall semester 2017

Midterm Exam – November 19th, 2017

Name: Key   ID:

Question	Student mark	ABET SO
1		a
2		c
3		k

Question 1: [10pts]

A causal LTI system has the system function

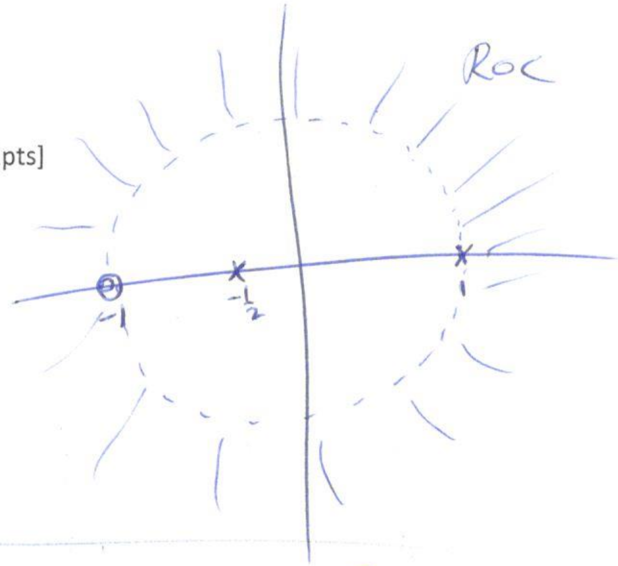
$$H(z)=\frac{1+2z^{-1}+z^{-2}}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$$

(a) Plot pole-zero diagram and indicate the region of convergence. [2pts]

Poles: $z=-\frac{1}{2}, 1$

Zeros: $z=-1,-1$

Causal $\Rightarrow$ ROC: $|z|>1$



(b) Find impulse response of the system, h(n). [3pts]

$M=2,\ N=2 \Rightarrow$ long division

$$H(z)=\frac{1+2z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}=-2+\frac{3+z^{-1}}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$$

$$\begin{array}{r} -2 \\ 1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}\ \big)\ 1+2z^{-1}+z^{-2} \\ -\ \ -2+z^{-1}+z^{-2} \\ \hline 3+z^{-1} \end{array}$$

using Partial Fraction Expansion:

$$H(z)=-2+\frac{A_1}{1+\frac{1}{2}z^{-1}}+\frac{A_2}{1-z^{-1}}$$

$$A_1=\frac{3+z^{-1}}{1-z^{-1}}\bigg|_{z=-\frac{1}{2}}=\frac{1}{3}$$

$$A_2=\frac{3+z^{-1}}{1+\frac{1}{2}z^{-1}}\bigg|_{z=1}=\frac{8}{3}$$

Causal $\Rightarrow$ Right-sided $\Rightarrow$ $\boxed{h(n)=-2\delta(n)+\frac{1}{3}(\frac{1}{2})^n u(n)+\frac{8}{3}u(n)}$

(c) Find the output of this system, y(n), for the input [3pts]

$$x(n) = e^{j(\frac{\pi}{2})n}$$

by using eigenfunction property of the input signal

$$y(n) = H(e^{j\frac{\pi}{2}})\,x(n)$$

Where,

$$H(e^{j\frac{\pi}{2}}) = -2 + \frac{\frac{1}{3}}{1+\frac{1}{2}e^{-j\frac{\pi}{2}}} + \frac{8/3}{1-e^{-j\pi/2}}$$

$$= -2 + \frac{\frac{1}{3}}{1-\frac{1}{2}j} + \frac{\frac{8}{3}}{1+j}$$

$$= \frac{-2j}{\frac{3}{2}+j\frac{1}{2}}$$

So, $y(n) = \frac{-2j}{\frac{3}{2}+j\frac{1}{2}}\,e^{j\frac{\pi}{2}n}$.

(d) Is this system stable? Justify? [2pts]

not stable because it has a pole at the unit circle.

Question 2: [10pts]

(a) For What values of ω is the signal $x(n) = \cos(\omega n)$ periodic with period of 4? [3pts]

$$N\omega_0 = 2\pi k$$

$$\omega = \frac{2\pi k}{N} = \frac{2\pi k}{4} = \frac{\pi}{2}k$$

value of ω

So, $\omega = \ldots, \frac{-3}{2}\pi, -\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

(b) Consider the following linear-phase FIR filter:

$$y(n) = ax(n) + bx(n-1) + cx(n-2)$$

Determine the coefficients a, b and c such that the filter rejects completely the frequency component at ω=π/2, and its frequency response is normalized so that $H(e^{j0}) = 1$. [3pts]

linear-phase ⇒ h(n) symmetric around mid-point

$x(n)=\delta(n) \Rightarrow y(n) = h(n) = a\delta(n) + b\delta(n-1) + c\delta(n-2) \Rightarrow a = c \quad \{a, \overset{\text{symmetric}}{b}, a\}$

$H(e^{jw}) = a + be^{-jw} + ae^{-j2w}$

$H(e^{j0}) = 1 \Rightarrow a + b + a = 1 \Rightarrow 2a + b = 1$

There is zero at $w = \frac{\pi}{2} \Rightarrow |H(e^{j\frac{\pi}{2}})| = 0 \Rightarrow |a + be^{-j\frac{\pi}{2}} + ae^{-j\pi}| = 0 \Rightarrow \cancel{a} + b(1) + \cancel{a}(-1) = 0$

$\Rightarrow \boxed{b = 0}$

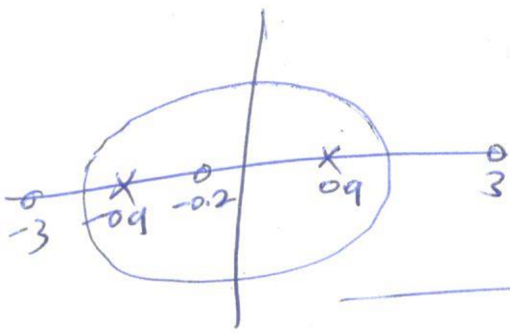
~~$|2a - jb| = 0 \Rightarrow 4a^2 + b^2 = 0 \Rightarrow b =$~~

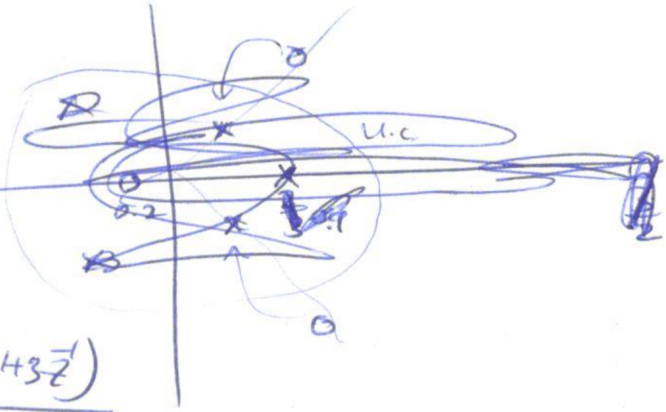
$\Rightarrow 2a + 0 = 1$

$2a = 1 \Rightarrow \boxed{a = \frac{1}{2}}$ and $\boxed{c = \frac{1}{2}}$

(c) Express the following system as a cascade of minimum phase and all-pass sub-systems. [4pts]

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})}, \text{ ROC } |z| > 0.9$$





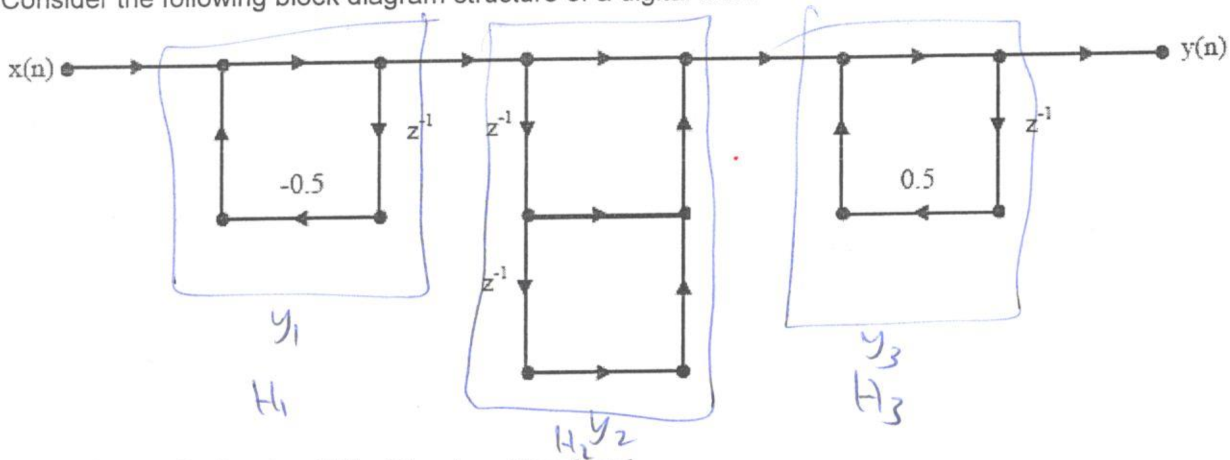
$$H(z) = \frac{(1+0.2z^{-1})(1-3z^{-1})(1+3z^{-1})}{(1+0.9z^{-1})(1-0.9z^{-1})}$$

$$H_{ap}(z) = \frac{(z^{-1}+\frac{1}{3})(z^{-1}-\frac{1}{3})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}$$

$$H_{min}(z) = \frac{-9(1+0.2z^{-1})(1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})}{(1+0.81z^{-2})}$$

Question 3 [10pts]:

Consider the following block diagram structure of a digital filter:



(a) Find the transfer function, H(z), of the above filter. [4pts]

$$H_1(z) = \frac{1}{1-0.5z^{-1}}$$

$$H_2(z) = 1 + z^{-1} + z^{-2}$$

$$H_3(z) = \frac{1}{1+0.5z^{-1}}$$

$$H(z) = H_1(z)H_2(z)H_3(z) = \frac{1+z^{-1}+z^{-2}}{(1-0.5z^{-1})(1+0.5z^{-1})} = \frac{1+z^{-1}+z^{-2}}{1-0.25z^{-2}}$$

(b) parallel form

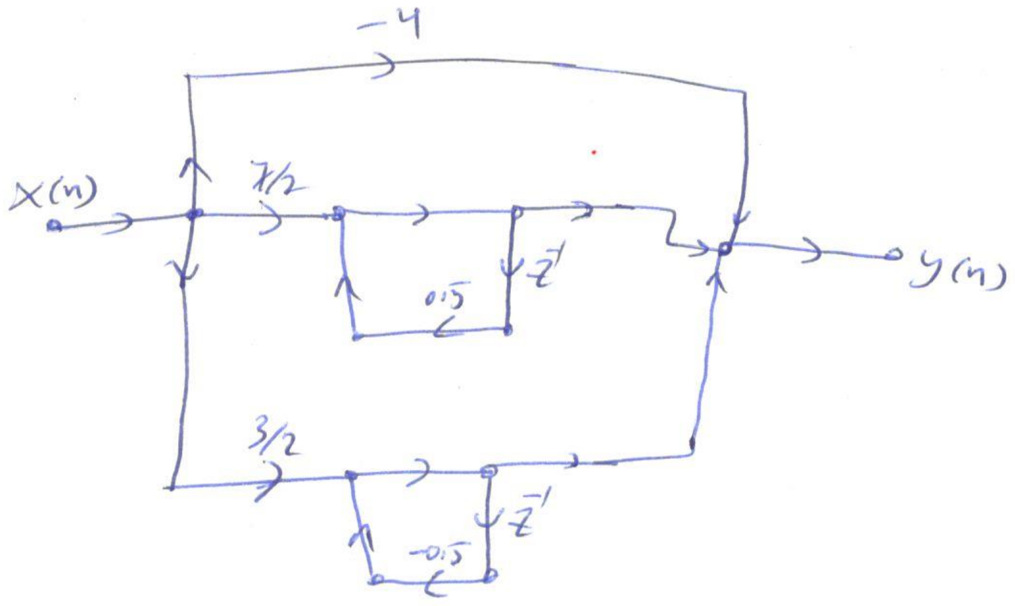
$$H(z) = -4 + \frac{5+z^{-1}}{1-0.25z^{-2}} = -4 + \frac{A_1}{(1-0.5z^{-1})} + \frac{A_2}{1+0.5z^{-1}}$$

$$-0.25z^{-2}+1 \;\overline{\big)\; z^{-2}+z^{-1}+1\;}\quad -4$$
$$z^{-2} - 4$$
$$z^{-1}+5$$

$A_1 = 7/2$
$A_2 = \frac{3}{2}$

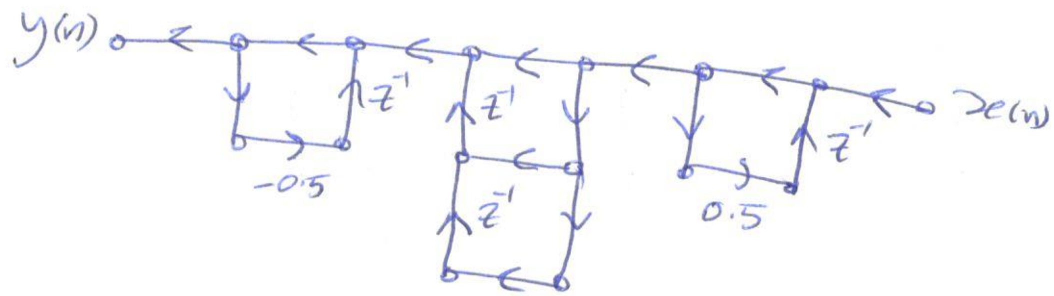
$$\text{So, } H(z) = -4 + \frac{7/2}{1-\frac{1}{2}z^{-1}} + \frac{3/2}{1+\frac{1}{2}z^{-1}}$$

(b) Draw the parallel-form structure of the above digital filter using 1st order filters. [3pts]



Parallel form of first order sub systems.

(c) Draw the transposed form of the above system. [3pts]



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