

DSP, Final Exam

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→ 1. The discrete signal $x[n] = e^{j(3\pi n/4)}$

a. Periodic with a period of 3

b. = = = = 4

c. = = = = 8

d. Non-periodic

$\frac{3\pi}{4}$ Period
 $WN = 2\pi K$

$$N = \frac{2\pi K}{W}$$

$$\Rightarrow N = \frac{4}{3\pi} \cdot 2\pi K = \frac{8}{3}K$$

to get $K = 3$

$$\Rightarrow N = 8$$

→ 2. If the Nyquist rate for $x_a(t)$ is Ω_s

What is the Nyquist rate for $x_a(2t)$

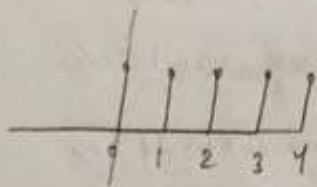
a. $2\Omega_s$

b. $\frac{\Omega_s}{2}$

c. Ω_s

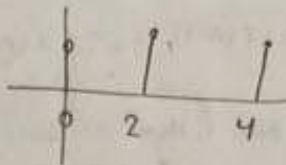
d. $\frac{\Omega_s}{4}$

Assume $x(t)$



$x(2t)$

$2t = \text{compression}$



→ 3. IIR Filter

a. use feed back

b. Are sometimes called recursive filters

c. Can oscillate (i.e. not stable) if not properly design

d. All of the above

①

→ 4. Find two different Continuous-time signals that will produce the
 $X[n] = \cos(0.15\pi n)$ when sampled with a sampling frequency of 8 kHz

- a. $\sin(1200\pi t)$ and $\cos(17200\pi t)$ ✗
- b. $\cos(1200\pi t)$ and $\sin(17200\pi t)$
- c. $\cos(1200\pi t)$ and $\cos(17200\pi t)$
- d. $\sin(1200\pi t)$ and $\sin(17200\pi t)$ ✗

$$\cos(0.15\pi n)$$

$$0.15\pi * f_s = 1200\pi \Rightarrow \text{Both a \& d wrong}$$

$$\sin\left(\frac{17200\pi}{8000}\right) = \sin(2.15\pi) = 0.0375$$

$$\cos\left(\frac{17200\pi}{8000}\right) = \cos(2.15\pi) = 0.999$$

$$\cos(0.15\pi) = 0.999$$

→ 5. A DSP system with the following difference equation:

$$Y[n] = 0.25X[n] + 0.25X[n-1] + 0.25X[n-2] + 0.25X[n-3] \text{ is}$$

- a. Low Pass filter
- b. High Pass filter
- c. Band Pass filter
- d. Band Reject filter

Small : Corresponding with input
 plus : = = output

$$H(z) = \frac{0.25 + 0.25z^{-1} + 0.25z^{-2} + 0.25z^{-3}}{1}$$

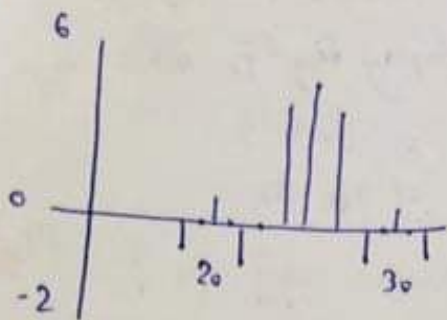
$$h[n] = \left\{ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\} \Rightarrow \text{Moving average}$$

\Rightarrow LPF

z^{-n} ← index

②

6. The following is an impulse response for



- a. An IIR high pass filter
- b. An FIR band pass filter
- c. An IIR low pass filter
- d. An FIR low pass filter

Symmetry \Rightarrow FIR

7. Coefficients symmetry is important in FIR filters because it provides

- a. A smaller transition bandwidth
- b. Less to avoid ripple
- c. = stopband =
- d. A linear phase response

8. Two digital filters can be operated in cascade. or the same effect can be achieved by

- a. Adding their coefficients
- b. Subtracting = =
- c. Convolution = =
- d. Averaging their = and then use Hamming window

9. If a linear phase filter has a phase response of 80 degrees at 400 Hz. what will its phase response freq. of 200 Hz (assuming that both freq. are in the passband of the filter)

- a. ~~35~~ 35°
- b. 40°
- c. 45°
- d. 80°

* Linear phase \Rightarrow $\omega = \omega_c \sin$

freq degree

400 \rightarrow 80

200 \rightarrow X

$$400X = 1600$$

$$X = 40$$

(3)

10. The continuous time signal $x(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling freq F_s to obtain the discrete time signal $X(n) = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$. The sampling freq F_s which is consistent with the information is
- a. 40 Hz b. 80 Hz **c. 100 Hz** d. 120 Hz

$$\frac{20\pi}{F_s} = \frac{\pi}{5} \Rightarrow F_s = 100 \text{ Hz}$$

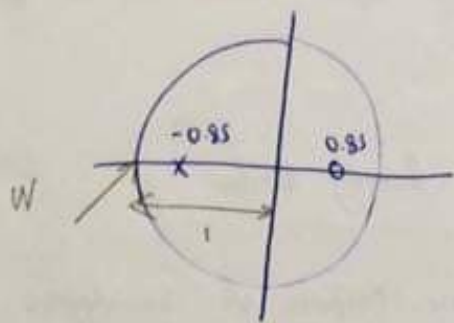
11. A DSP system with the following system function $H(z) = \frac{0.97(z-1)}{z-0.94}$ is a $\underline{\hspace{1cm}}$
- a. LPF b. ~~HPF~~ Band Pass Filter (Pass) c. HPF **d. Stop = (Notch)**



Zeros at $z=1$

Poles at $z=0.94$

12. The pole/zero diagram of DSP system is shown in the following figure. If this system implemented with $f_s = 10\text{KHz}$ the Mag. response of this system at $f_{\text{req}} = 5\text{KHz}$ is



a. 1 b. 0

c. 12.33 d. 0.81

~~12.33~~

$$\omega = \frac{2\pi f}{f_s} = \frac{2\pi \cdot 5\text{K}}{10\text{K}} = \pi$$

$$\pi \times \frac{180}{\pi} = 180^\circ$$

$$\text{Mag} = \frac{\text{Distance with zero}}{\text{Distance with pole}} = \frac{1.85}{0.15} = 12.33$$

4

* For question 16 & 17, $T = 0.2$

analogue filter is $H(s) = \frac{10}{s+10}$

$$h_a(t) = 10e^{-10t} u(t)$$

16. henzl designed using impulse invariance techniques is

- a. $2e^{-2n} u(n)$ sampling interval c. $20e^{-20n} u(n)$ e. $0.2e^{-0.2n} u(n)$
 b. $10e^{-10n} u(n)$ sampling interval d. $0.1e^{-0.1n} u(n)$

17. H(z) designed using bilinear transform is

- a. $\frac{1}{2}(1+z^{-1})$ c. $\frac{20z}{z-e^{-2}}$
 b. $\frac{2z}{z-e^{-2}}$ d. $\frac{1}{2}(1-z^{-1})$

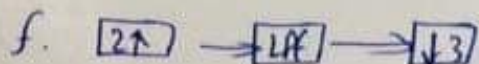
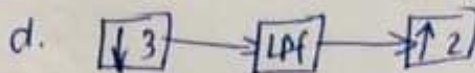
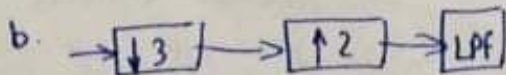
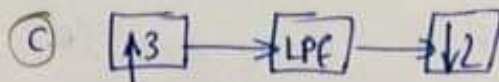
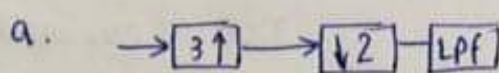
17 $s = \frac{z}{T} \left[\frac{1-z^{-1}}{1+z^{-1}} \right]$
 substitute in H(s)

$$\Rightarrow H(z) = \frac{10}{\frac{z}{0.2} \left[\frac{1-z^{-1}}{1+z^{-1}} \right] + 10}$$

$$\Rightarrow \frac{10}{\frac{z}{0.2} \left[\frac{1-z^{-1}}{1+z^{-1}} \right] + 10}$$

$$\Rightarrow \frac{1+z^{-1}}{1-z^{-1} + 1+z^{-1}} = \frac{1}{2}(1+z^{-1})$$

18. if input is a 300 Hz sinusoid, which Dsp system outputs only 450 Hz sinusoid $\frac{450}{300} = \frac{3}{2}$
 + LPF is an ideal low pass filter (not band pass) with a single cut off freq.



(6)

16 Impulse Invariance

Map Pole to Pole in Z-plane

$$\text{at } s = s_k \Rightarrow z = e^{s_k T}$$

$$H(s) = \frac{10}{s+10} = 10 \frac{1}{s+10} = 10e^{-10t} u(t)$$

$$h(n) = 10e^{-10nT} u(n) = 10e^{-2n} u(n)$$

$$H(z) = \frac{10}{1 - e^{-2} z^{-1}} = 10 \frac{z}{z - e^{-2}}$$

$x(n) = \cos(\frac{\pi}{2}n)$ is an input into LTI system with

→ 19. $H(e^{j\omega}) = 1 + \frac{1}{2}e^{j\omega} + e^{j2\omega}$. The output $y(n)$ is

a. $\sqrt{2} \cos(\frac{\pi}{2}n)$ b. $2.5 \cos(\frac{\pi}{2}n)$ c. $1.5 \cos(\frac{\pi}{2}n)$ d.

d. $\sin(\frac{\pi}{2}n)$ e. $-\frac{1}{2} \sin(\frac{\pi}{2}n)$

$$H(e^{j\omega}) = 1 + \frac{1}{2} [\cos \omega + j \sin \omega] + \cos 2\omega + j \sin 2\omega$$

$$|H(e^{j\omega})| = \sqrt{\left(1 + \frac{1}{2} \cos \omega + \cos 2\omega\right)^2 + \left(\frac{1}{2} \sin \omega + \sin 2\omega\right)^2}$$

$$= \sqrt{\left[1 + \frac{1}{2} \cos\left(\frac{\pi}{2}\right) + \cos\left(2 \cdot \frac{\pi}{2}\right)\right]^2 + \left[\frac{1}{2} \sin\left(\frac{\pi}{2}\right) + \sin\left(2 \cdot \frac{\pi}{2}\right)\right]^2}$$

$$= \sqrt{(1 + 0 - 1)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2}$$

$$\angle H(e^{j\omega}) = \tan^{-1} \frac{\sin \omega + \sin 2\omega}{1 + \frac{1}{2} \cos \omega + \cos 2\omega}$$

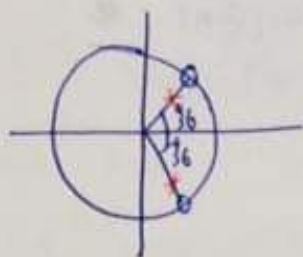
$$= \tan^{-1} \frac{1+0}{1+0-1} \Rightarrow \theta = \frac{\pi}{2}$$

$$y(n) = \frac{1}{2} \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}\right)$$

$$= \frac{1}{2} \times -\sin\left(\frac{\pi}{2}n\right)$$

(7)

→ 20. The pole zero diagram of a digital filter is shown in the figure below. This filter is



- a. Resonant filter
- b. Notch filter
- c. LPF
- d. HPF

Poles $\approx 0.937 \angle 36^\circ$

→ 21. The system having freq response

$$H(e^{j\omega}) = \frac{2e^{-j\omega}}{1 + 3e^{-j2\omega}}$$

Corresponding to input is

a. $y(n-1) - \frac{1}{2}x(n) + \frac{3}{2}x(n-2)$

b. $y(n) + 3y(n-2) - 2x(n-1)$

e. $y(n) + 3y(n-1) - 2x(n-2)$

Corresponding to output

c. $y(n) - x(n) + 3x(n-2)$

d. $y(n) + 3y(n-1) - 2x(n)$

→ 22. If a continuous time filter with an impulse response $h_c(t)$ has sampled freq = f_s , what happens to cutoff freq. ω_c of the discrete time filter as f_s increased.

- a. we increase
- b. we decrease
- c. we remain constant
- d. None

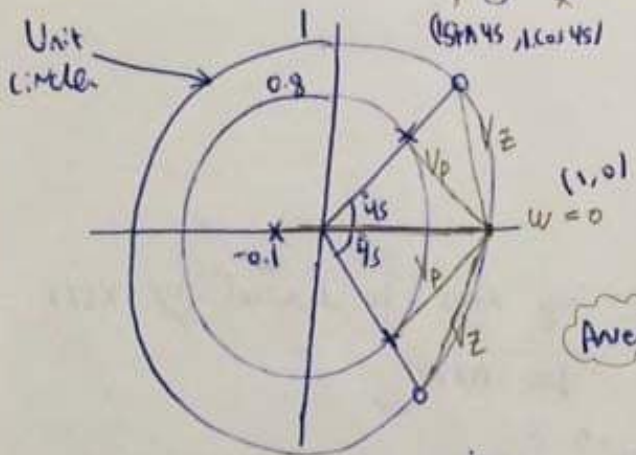
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R: Geometric Interpolation

Magnitude: $|H(e^{j\omega})|$ = is Product of lengths of Vectors from Zeros divide by Product of lengths of Vectors from Poles

Phase: $\angle H(e^{j\omega})$ = is sum of angles of Vectors from Zeros mins sum of angles of vector from Poles

Ex Consider the following figure



Find Mag & Phase at $\omega=0$ without finding $H(e^{j\omega})$

Answer $\angle H(e^{j0}) = 45^\circ + 45^\circ - 45^\circ - 45^\circ = 0^\circ$

$$|H(e^{j0})| = \frac{V_z^2}{1.1 \times V_p^2}$$

$$V_z = \sqrt{(1 - 1 \sin 45^\circ)^2 + (0 - \cos 45^\circ)^2} = 0.7653$$

$$V_p = \sqrt{(1 - 0.8 \sin 45^\circ)^2 + (0 - 0.8 \cos 45^\circ)^2} = 0.7132$$

$$\Rightarrow \text{Answer} = 1.3678$$

9

Ex The Z-transform of a discrete seq. $x(n)$ is denoted by $X(z)$
derive the Z transform of seq. $nx(n)$

Answer

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\frac{dX(z)}{dz} = \sum x(n) (-n) z^{-n-1}$$

$$= -z^{-1} \sum nx(n) z^{-n}$$

$$= -z^{-1} \underbrace{Z\{nx(n)\}}_{Z\text{-Transform}}$$

$$Z\{nx(n)\} = -z \frac{dX(z)}{dz}$$

(10)