Key

جامعة بيرزيت
BIRZEIT UNIVERSITY

Faculty of Information Technology

Computer Systems Engineering

Digital Signal Processing – ENCS431

Second Exam

Fall Semester 30/1/2014 Key Time allowed: 90 minutes

Name: ID: section: ☐ 9:30-11:00, ☐ 12:30 -2:00

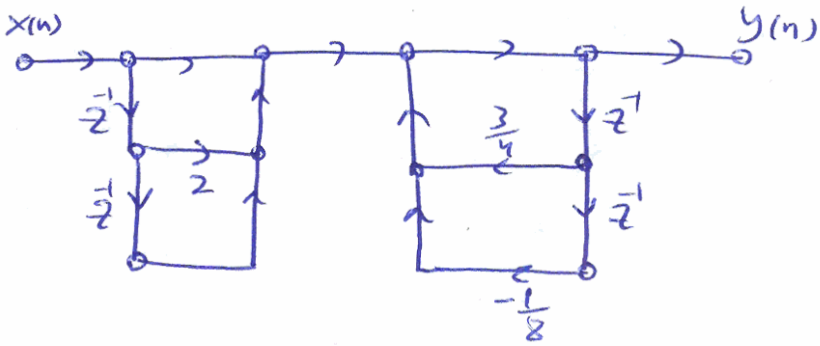
Question 1: [35 marks] Structures for DT systems

(a) Consider a causal LTI system whose system function is

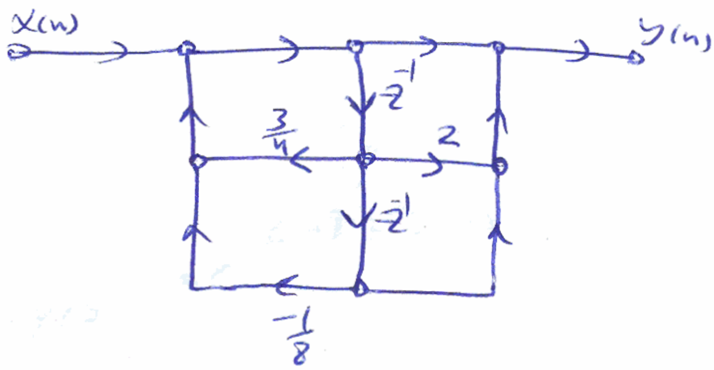
$$H(z)=\frac{1+2z^{-1}+z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

Draw the signal flow graph for implementations of the system in each of the following forms: [5pts each]

(i) Direct form I.



(ii) Direct form II (Canonical form)



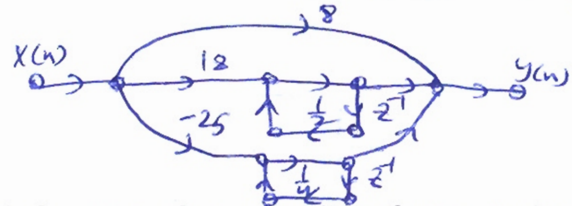
(iii) Parallel form using first order sections

by long division $H(z)$ can expressed as

$$H(z) = 8 + \frac{8z^{-1} - 7}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

by using partial fraction expansion

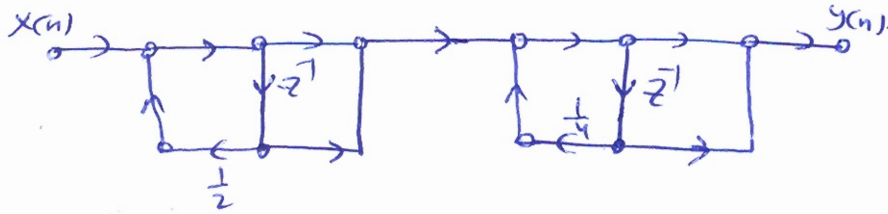
$$\frac{8z^{-1} - 7}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{A_1}{(1 - \frac{1}{2}z^{-1})} + \frac{A_2}{(1 - \frac{1}{4}z^{-1})} = \frac{18}{1 - \frac{1}{2}z^{-1}} - \frac{25}{1 - \frac{1}{4}z^{-1}}$$



(iv) Cascade form using first-order direct form II sections

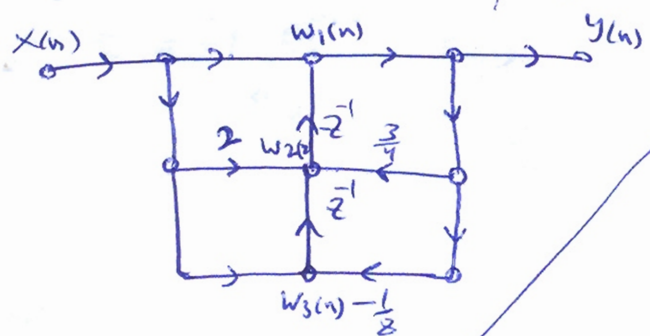
$$H(z) = \frac{(1 + z^{-1})^2}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}$$

$$= \underbrace{\frac{(1 + z^{-1})}{(1 - \frac{1}{2}z^{-1})}}_{H_1} \cdot \underbrace{\frac{(1 + z^{-1})}{(1 - \frac{1}{4}z^{-1})}}_{H_2}$$



(v) Transposed direct form II

from this graph we write a set of diff. eq.



$w_1 = x(n) + w_2(n-1)$ — ①

$w_2 = 2x(n) + w_3(n-1) + \frac{3}{4}y(n)$ — ②

$w_3 = x(n) - \frac{1}{8}y(n)$ — ③

$y(n) = w_1$ — ④

$$z^{-1}W_2 = Y(z) - X(z)$$

$$W_2 = 2X(z) + z^{-1}(X(z) - \tfrac{1}{8}Y(z)) + \tfrac{3}{4}Y(z)$$

$\cancel{zY(z) - zX(z) = X(z)} +$

(vi) Write the difference equation for the flow graph of **(v)** above and show that this system has the correct system function? [10 pts]

$$zY(z) - zX(z) = 2X(z) + z^{-1}X(z) - \tfrac{1}{8}z^{-1}Y(z) + \tfrac{3}{4}Y(z)$$

$$\left(z + \tfrac{1}{8}z^{-1} - \tfrac{3}{4}\right)Y(z) = \left(z + 2 + z^{-1}\right)X(z)$$

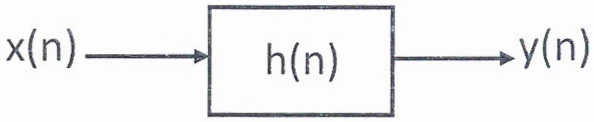
$$\left(1 - \tfrac{3}{4}z^{-1} + \tfrac{1}{8}z^{-2}\right)Y(z) = \left(1 + 2z^{-1} + z^{-2}\right)X(z)$$

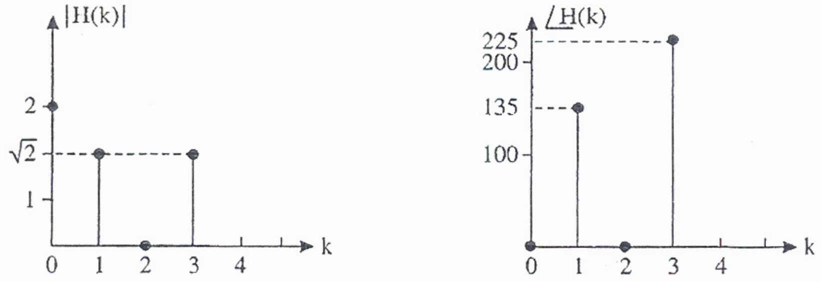
$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$ which is same of $H(z)$ giv

Question 2: [30 marks] DFT

(a) The figure below shows the block diagram of an LTI system, where x(n) is the input sequence, h(n) is the impulse response of the system and y(n) is the output sequence. The magnitude and phase (in degrees) of the Discrete Fourier Transform H(k) of h(n) is shown in figures (a) and (b), respectively.

Assume the input sequence **x(n) = { 1, 0, 2 , 3}.**





(a) (b)

(i) Determine the impulse response of this system, h(n)? [5pts]

2pts

$H(0) = 2e^{j0} = 2$

$H(1) = \sqrt{2}e^{j135} = \sqrt{2}(\cos 135 + j\sin 135)$
$= -1 + j$

$H(2) = 0$

$H(3) = \sqrt{2}e^{j225} = \sqrt{2}(\cos 225 + j\sin 225)$
$= -1 - j$

$h(n) = \frac{1}{4}\sum_{k=0}^{3} H(k) W_4^{-nk}$

$h(0) = \frac{1}{4}\sum_{k=0}^{3} H(k) e^{j\frac{2\pi}{4}k\cdot 0} = \frac{1}{4}(2 - 1 + j - 1 - j) = 0$

$h(1) = \frac{1}{4}\sum_{k=0}^{3} H(k) e^{j\frac{2\pi}{4}k}$

$= \frac{1}{4}[2e^{0} + (-1+j)e^{j\frac{\pi}{2}} + (-1-j)e^{j\frac{3\pi}{2}}]$

$= \frac{1}{4}[2 - j - 1 + j - 1] = 0$

$h(2) = 1$

$h(3) = 1$

$h(n) = \{0, 0, 1, 1\}$.

3pts

(ii) Determine the output sequence y(n) by computing the 4-point circular convolution? [5pts]

Using tabular method $y(n) = x(n) ④ h(n)$

n:	0	1	2	3
x(n)	1	0	2	3
h(n)	0	0	1	1
	0	0	0	0
	0	0	0	0
	2	3	1	0
	0	2	3	1

$\Rightarrow y(n) = \{2, 5, 4, 1\}, 0 \le n \le 3$

(ii) Determine the output sequence, y(n) using DFT? [10 pts]

$X(k) =$ DFT of $x(n)$

5 pts

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1+3j \\ 0 \\ -1-3j \end{bmatrix} \Rightarrow Y(k) = X(k) \cdot H(k)$$

$$= \begin{bmatrix} 6 \\ -1+3j \\ 0 \\ -1-3j \end{bmatrix} \begin{bmatrix} 2 \\ -1+j \\ 0 \\ -1-j \end{bmatrix} = \begin{bmatrix} 12 \\ -2-4j \\ 0 \\ -2+4j \end{bmatrix}$$

$y(n) =$ IDFT of $Y(k)$

5 pts

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 12 \\ -2-4j \\ 0 \\ -2+4j \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 4 \\ 1 \end{bmatrix}$$

(b) Let 10-length finite sequence ***x(n) = {1, 2, 3, a, 1, 3, 2, 1, 2, 3}*** for $0 \le n \le 9$. Assume x(n) is zero outside the interval $0 \le n \le 9$. Let $X(e^{j\omega})$ be the DTFT of x(n) and $X_1(k)$ be samples of $X(e^{j\omega})$ every $\frac{\pi}{3}$; i.e.,

$$X_1(k) = X(e^{j\omega})|_{\omega=(\pi/3)k}, \qquad 0 \le k \le 5$$

The 6-point sequence x1(n) that results from taking the 6-point inverse DFT of $X_1(k)$ is

x1(n)={3, 3, 5, 7, 1, 3} for $0 \le n \le 5$. Based on this sequence, x1, determine the value of the fourth sample of x(n), i.e. x(3) or a? is value of a unique? If so justify your answer. If not, find another choice of a consistent with the given information? [10 pts]

$X_1(k)$ is 6-point DFT of $x(n)$.

Since length of $x(n)$ is greater than 6 $\Rightarrow$ There will be time aliasing by 4 points. $\Rightarrow$ i.e. $x_1(n) = x((n))_6$

So

$x_1(0) = x(0) + x(6) = 3$

$x_1(1) = x(1) + x(7) = 3$

$x_1(2) = x(2) + x(8) = 5$

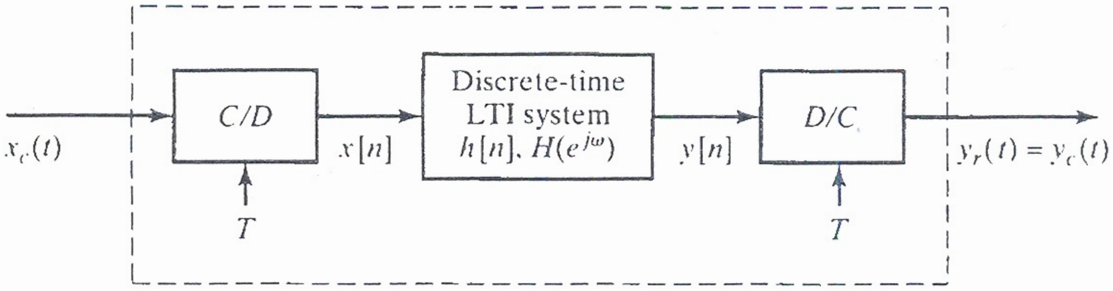
$x_1(3) = x(3) + x(9) = 7$

$\Rightarrow a + 3 = 7 \Rightarrow a = 4$ This value is unique.

It's clear from this relationship between $x(n)$ and $x_1(n)$ that a is unique.

Question 3:[35 marks] Sampling and reconstruction

Consider the system below:



(a) Let the continuous–time input signal $x_c(t) = \cos(25\pi t) + \cos(120\pi t) + \cos(50\pi t)$, what is the range of sampling frequency, in Hz, that will avoid aliasing in the C/D converter? [5 pts]

max. freq. of $x(t)$ is $\Omega_N = 120\pi$.

$\Omega_s \geq 2\Omega_N \Rightarrow \Omega_s \geq 240\pi$ } 2

$2\pi f_s \geq 240\pi \Rightarrow f_s \geq 120\,\text{Hz}$ } 3

(b) Now, assume the sampling frequency of C/D converter FS1=200Hz and the discrete system is an ideal low-pass filter with frequency response as follow:

$$H(e^{j\omega}) = \begin{cases} 0.005 & |\omega| \leq 0.5\pi \\ 0 & otherwise \end{cases}$$

2 pts { $\Omega_s = 2\pi f_s = 2\pi(200) = 400\pi > 240\pi$

2 pts { So, there is no aliasing

For $-\pi \leq \omega \leq \pi$, Find the discrete output $y(n)$,if the input signal $x_c(t)$ is the same as specified in part (c) above? Sketch spectrums of $x_c(t)$, x(n), y(n) and ~~y~~(t). Is there aliasing? [15 pts]

2 { $y(n) = 0.005\cos(\frac{\pi}{8}n) + 0.005\cos(\frac{\pi}{4}n)$

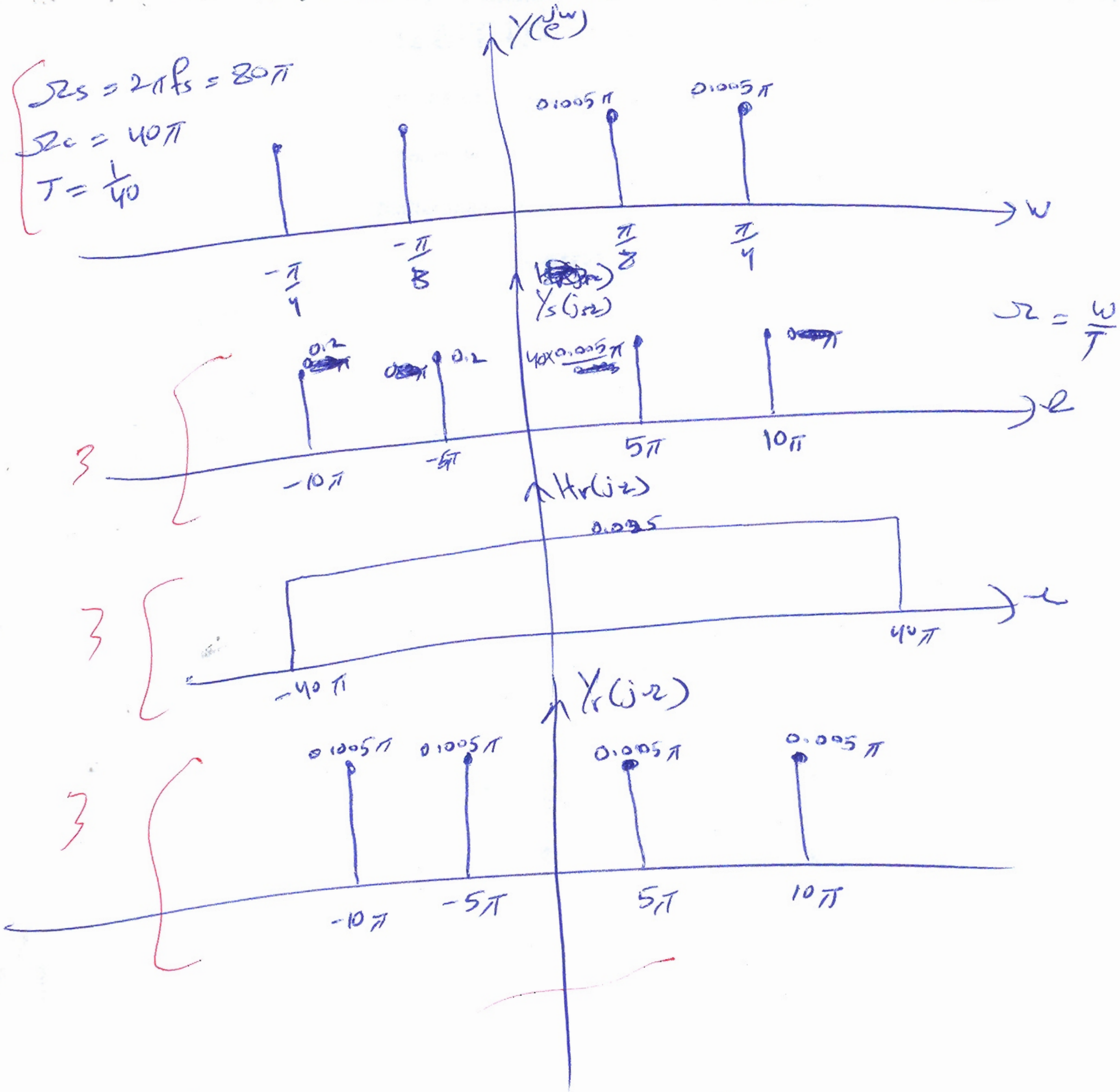
(c) If the sampling frequency of D/C converter FS2 = 40Hz, Find the reconstructed continuous-time signal if the analogue reconstruction filter in the D/C converter, $H_r(j\Omega)$, has a gain of (0.025) and cut-off frequency, Ω_c half of the sampling frequency. Sketch spectrums of all involved signals[15pts]

2 $\Omega_s = 2\pi f_s = 80\pi$

$\Omega_c = 40\pi$

$T = \frac{1}{40}$

$\Omega = \frac{w}{T}$



4 So, output $y_r(t) = 0.005[\cos(5\pi t) + \cos(10\pi t)]$