Midterm Exam

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Question 1 :

$$y[n] - 6y[n-1] + 8y[n-2] = 2x[n-1]$$

→ Linearity & Delay Shift

$$y(e^{jw}) - 6y(e^{jw})e^{-jw} + 8y(e^{jw})e^{-2jw} = 2x(e^{jw})e^{-jw}$$

then →

$$y(e^{jw})(1 - 6e^{jw} + 8e^{-2jw}) = 2e^{-jw}x(e^{jw})$$

$$H(e^{jw}) = \frac{y(e^{jw})}{x(e^{jw})} = \frac{2e^{-jw}}{1 - 6e^{jw} + 8e^{-2jw}}$$

↳ $e^{-jw}(2)^n 6^n u[n] + 2(8)^n u[n-2]e^{-jw}$

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Question 2 :

$h_1[n] = \delta[n]$, $h_2[n] = \delta[n-1]$, $h_3[n] = a^n u[n]$

[a] Impulse responce

→ $h_1[n] * h_2[n]$, $h_3[n]$ "Cascade"

∴ $h_3[n] * (h_1[n] + h_2[n])$

$y[n] = X[n] * [h_2[n] * h_1[n] + h_3[n] * h_2[n]]$

then → $h_3[n] * h_1[n] = a^n u[n] . \delta[n]$

$= a^n u[n]$

→ $h_3[n] * h_2[n] = a^n u[n] . \delta[n-1]$ "Delay shift"

$= a^{n-1} u[n-1]$

→ $h_3[n] * h_1[n] + h_3[n] * h_2[n] = h[n]$

$= a^n u[n] + a^{n-1} u[n-1]$

$= a^n \frac{1}{a} [u[n] + u[n-1]) = \frac{a^n}{a} \delta[n]$

∴ $y[n] = x[n] * h[n] = \frac{a^n}{a} X[n]$

[b] Frequancy respance

- $h_1[n] \rightarrow H_1(e^{jw}) = 1$
- $h_2[n] \rightarrow H_2(e^{jw}) \cdot e^{-jw}$
- $h_3[n] \rightarrow H_3(e^{jw}) = e^{jw} / e^{jw} - a^n$

$\rightarrow Y(e^{jw}) = X(e^{jw}) \left((H_3(e^{jw}) H_1(e^{jw})) + (H_3(e^{jw}) \cdot H_2(e^{jw}) \right)$

Then $\rightarrow$

$$\frac{Y(e^{jw})}{X(e^{jw})} = H_3(e^{jw}) H_1(e^{jw}) + H_3(e^{jw}) H_2(e^{jw})$$

$$= \frac{e^{jw}}{e^{jw} - a^n} + \frac{1}{e^{jw} - a^n} \rightarrow \left(e^{jw} / e^{-jw} \right)$$

:

[a] The system is causal

The system is stable

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Question 3 :

$$H_{hp}(e^{j\omega}) = \begin{cases} 1 & |\omega| < 0.2\pi \\ 0 & 0.2\pi \leq |\omega| \leq \pi \end{cases}$$

$$\rightarrow h_2[n] = 2\, h_{hp}[n] \cos(0.5\pi n)$$

$$= 2\, h_{hp}[n] \left(\frac{e^{j0.5\pi} + e^{-j0.5\pi n}}{2} \right)$$

$$\because \quad h_2[n] = e^{j0.5\pi n} h_{hp}[n] + e^{-j0.5\pi n} h_{hp}[n]$$

$$\rightarrow H_2(e^{j\omega}) = H(e^{j\omega}) * (\delta(\omega - 0.5\pi) + \delta(\omega + 0.5\pi))$$

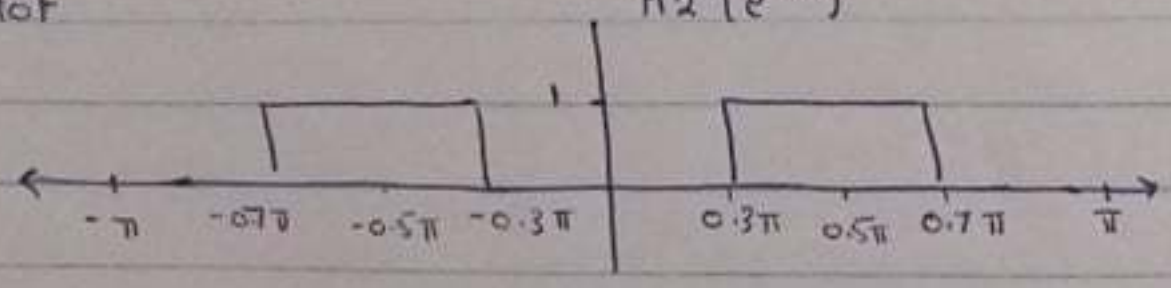
$$\& \quad H_2(e^{j\omega}) = H_1(e^{j\omega}) + H_1(e^{j\omega})$$

$$= H_1(e^{j(\omega - 0.5\pi)}) + H_1(e^{j(\omega - 0.5\pi)})$$

(a) The Respance equation

$$H_2(e^{j\omega}) = \begin{cases} 0 & |\omega| < 0.3\pi \ , \ 0.7\pi < |\omega| \leq \pi \\ 1 & 0.3\pi \leq |\omega| \leq 0.7\pi \end{cases}$$

(b) Plot



(c) Kind of new Filter is bandpass Filter