BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENCS431- Digital Signal Processing – Fall 2021

Midterm Exam   Time allowed 80 minutes

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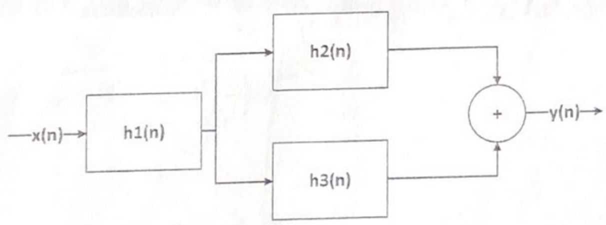
Question	Max mark	Student Mark	ABET SO
1	10	10	a
2	10	10	k
3	10		c
4	10		
SUM	40		

Tick your section: ☑ Dr. Abualsoud (SW 10:00), ☐ Dr. Alhareth(MW 11:25), ☐ Dr. Alhareth (TR 12:50)

Question 1: [10 marks]

Consider the system in the following figure:



If $h1(n) = \delta(n-2)$, $h2(n) = \left(\frac{3}{8}\right)^n u(n)$, and $h3(n) = 2\delta(n)$

(a) Find the expression for the impulse response, h(n) for the overall system.[6pts]

$$h(n) = [h_2(n) + h_3(n)] * h_1(n) = \left[\left(\frac{3}{8}\right)^n u(n) + 2\delta(n)\right] * \delta(n-2)$$

$$= \left(\frac{3}{8}\right)^n u(n) * \delta(n-2) + 2\delta(n) * \delta(n-2).$$

$$= \left(\frac{3}{8}\right)^{n-2} u(n-2) + 2\delta(n-2)$$

6

(b) Find the frequency response of the whole system, $H(e^{j\omega})$. [6pts]

$$H(e^{iw}) = \frac{e^{-i(2w)}}{1-\left(\frac{3}{8}\right)e^{-iw}} + 2e^{-i(2w)}$$

6

(c) Check if this system stable and/or causal? Justify. [4pts]

4

Causal, it depends only on the past samples. $h(n) = 0$ when $n<0$

stable, $\sum_{n=-\infty}^{\infty} h(n) < \infty \Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{3}{8}\right)^{n-2} u(n-2) + 2\delta(n-2) < \infty$, BIBO

$\frac{3}{8} < 1$ & and $u(n-2)$ removes the left side.

(d) Use the frequency response expression in part (b) to write the difference equation that characterize this system. [4pts]

$$H(e^{iw}) = \frac{e^{-i(2w)} + 2e^{-i(2w)} - \left(\frac{3}{8}\right)e^{-iw}\left(2e^{-i(2w)}\right)}{1-\left(\frac{3}{8}\right)e^{-iw}}$$

$$\frac{Y(e^{iw})}{X(e^{iw})} = \frac{3e^{-i(2w)} - \frac{3}{4}e^{-i(3w)}}{1-\left(\frac{3}{8}\right)e^{-iw}} \Rightarrow \left[1-\left(\frac{3}{8}\right)e^{-iw}\right]Y(e^{iw}) = \left[3e^{-i(2w)} - \frac{3}{4}e^{-i(3w)}\right]X(e^{iw})$$

$$\Rightarrow y(n) - \frac{3}{8}y(n-1) = 3x(n-2) - \frac{3}{4}x(n-3).$$

4

Question 2: [20 marks]

When the input to an LTI system is

$$x(n) = \left(\frac{1}{3}\right)^n u(n) + (2)^n u(-n-1)$$

The corresponding output is

$$y(n) = 5\left(\frac{1}{3}\right)^n u(n) - 5\left(\frac{2}{3}\right)^n u(n)$$

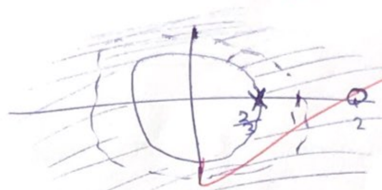
a) Find the system function H(z) of the system. Plot the pole-zero diagram of H(z) and indicate the region of convergence. [8pts]

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} = \frac{(1-2z^{-1})-(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

$$Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{2}{3}z^{-1}} = 5\frac{(-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})} = \frac{-\frac{5}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(1-2z^{-1})}{(1-\frac{2}{3}z^{-1})}$$

ROC : $|z| > \frac{2}{3}$



8

b) Find the impulse response h(n) of the system. [4pts]

$$H(z) = 3 - \frac{2}{1-\frac{2}{3}z^{-1}}$$

$$1-\tfrac{2}{3}z^{-1} \overline{\smash{\big)}\,1-2z^{-1}}\quad 3;\quad 3-2z^{-1};\quad -2$$

4

$$h(n) = 3\,\delta(n) - 2\left(\frac{2}{3}\right)^n u(n).$$

c) Write a difference equation that satisfied by the given input and output. [4pts]

$$\frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \Rightarrow \left[1-\frac{2}{3}z^{-1}\right]Y(z) = \left[1-2z^{-1}\right]X(z)$$

$$\Rightarrow y(n) - \frac{2}{3}y(n-1) = x(n) - 2x(n-1).$$

4

d) Is the system stable? Is it causal? [4pts]

Stable, ROC Contains the unit Circle.

Causal, ROC goes to infinity.

4

10 + 10

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Question 3: [10 marks]

a) If the frequency-response of a digital filter is given by the expression: [5pts]

$$H(\omega) = 0.5 + j\cos\left(\frac{2}{3}\omega\right) - \sin\left(\frac{8}{3}\omega\right)$$

$|Y(\omega)| = |H(\omega)||X(\omega)|$

$\angle Y(\omega) = \angle H(\omega) + \angle X(\omega)$

What would be its output when the input signal is {x(n)} with

$$x(n) = 3\cos\left(\frac{n\pi}{4} + \frac{\pi}{6}\right)$$

$w = \frac{\pi}{4}$

$H\left(\frac{\pi}{4}\right) = 0.5 + i\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{2\pi}{3}\right) = \left(0.5 - \frac{\sqrt{3}}{2}\right) + i\left(\frac{\sqrt{3}}{2}\right)$

$\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} - \frac{\sqrt{3}}{2} + \frac{3}{4} + \frac{3}{4}} = \sqrt{\frac{7}{4} - \frac{2\sqrt{3}}{4}} = \frac{\sqrt{7 - 2\sqrt{3}}}{2}$

$= 0.94$

$\angle H\left(\frac{\pi}{4}\right) = \angle \tan^{-1} \frac{\frac{\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}} = \angle \tan^{-1} \frac{\sqrt{3}}{1-\sqrt{3}} = \angle 67.08$

$y(n) = \left|H\left(\frac{\pi}{4}\right)\right| 3\cos\left(\frac{n\pi}{4} + \frac{\pi}{6}\right) \angle -67.08$

$= 2.82\cos\left(\frac{n\pi}{4}\right) \angle 30 - 67.08$

$= 2.82\cos\left(\frac{n\pi}{4} - 37.08\right).$

5

b) Determine whether the following discrete-time signal x(n) is periodic or non-periodic? If periodic, find the fundamental period. [5pts]

5

$$x(n) = 20e^{j0.48\pi n}$$

$w = 0.48\pi$

$wN = 2\pi K \Rightarrow N = \frac{2\pi K}{0.48\pi} = \frac{K}{0.24} = \frac{\cancel{100}K}{\cancel{24}} = \frac{25}{6}K$ $K = 6$

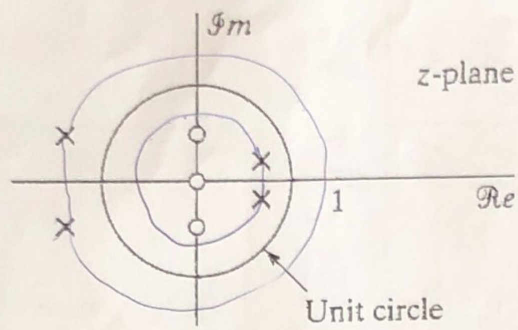
$\Rightarrow \boxed{N = 25}$

fundame. Period

Periodic with N = 25

Question 4: [10 marks]

a) The system unction of an LTI system has the pole-zero plot shown in the following figure. Specify whether each of the following statements is true, is false, or cannot be determined from the information given. [6pts]



i) The system is stable

Can't be determinated, we need the discrete function h(n).

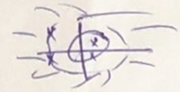
6

ii) The system is causal

Can't be determinated, we need h(n)

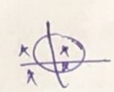
iii) If the system is causal, then it must be stable

False, the most outer range doesn't contain unit circle



iv) If the system is stable, then it must have a two-sided impulse response.

True, the only region that contains the unit circle is between the two poles.

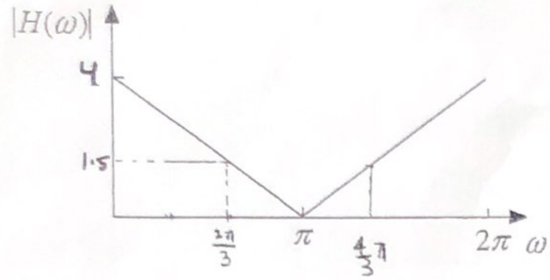


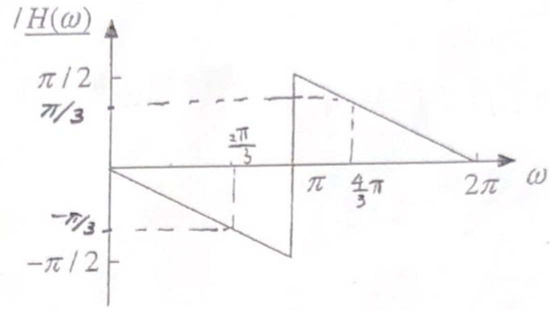
so its two-sided.

$N_1 = \frac{2\pi k}{\pi} \Rightarrow N_1 = 2$

$N_2 = \frac{2\pi k}{\frac{2\pi}{3}} = 3k \Rightarrow N_2 = 3$

b) The signal $x(n) = 2 + 4\cos(\pi n) + 2\cos(2\pi n/3)$ is input to a digital filter with frequency response magnitude and phase as shown: [4pts]

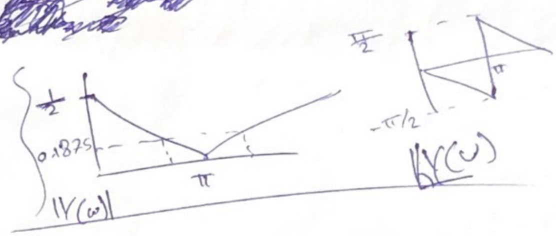




Find the output y(n) from this system.

$|H(\omega)| = |X(\omega)|\,|Y(\omega)|$, $\angle H(\omega) = \angle X(\omega) + \angle Y(\omega)$.

$|X(\omega)| = 8$, $\angle X(\omega) = 0$



4

$\cancel{y(n) = 2 \times 4 + 4 \times \cos(\pi n + \frac{\pi}{2}) + 2\cos(\frac{2\pi n}{3} + \frac{\pi}{3})}$

$y(n) = 2 \times 4 + 4 \times 0 \times \cos(\pi n + \frac{\pi}{2}) + 2 \times 1.5 \times \cos(\frac{2\pi n}{3} + \frac{\pi}{3})$

$= 8 + 16\cos(\pi n + \frac{\pi}{2}) + 3\cos(\frac{2\pi n}{3} + \frac{\pi}{3})$

$= 8 + 3\cos(\frac{2\pi n}{3} + \frac{\pi}{3})$