جامعة بيرزيت

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Electrical and Computer Engineering

DSP (Fall 2015)

Quiz (2)

Name: Key ID:

Consider a first-order digital filter, which is described by the following difference equation:

$y(n) = x(n) - ax(n-1),\ a \neq 0$

[2] 1- Is this filter IIR or FIR filter? Justify? FIR because it doesn't depend on output.

2- Find the coefficient **a** such that this filter attenuates signal -6 db at frequency $\omega = \frac{\pi}{4}$?

To find freq. Response, $|H(e^{jw})|$, of this system, we take F.T. for two sides.

$$Y(e^{jw}) = X(e^{jw}) - a e^{-jw} X(e^{jw})$$

[2] $$\frac{Y(e^{jw})}{X(e^{jw})} = 1 - a e^{-jw} = H(e^{jw})$$

$$H(e^{jw}) = 1 - a\cos w + aj \sin w$$

[2] $$|H(e^{jw})| = \sqrt{(1 - a\cos w)^2 + (a \sin w)^2}$$

$$20 \log_{10} |H(e^{jw})|_{w=\frac{\pi}{4}} = -6$$

$$|H(e^{j\frac{\pi}{4}})| = 10^{-0.3} \cong 0.5$$

[2] $$\left(1 - a\cos\tfrac{\pi}{4}\right)^2 + \left(a \sin\tfrac{\pi}{4}\right)^2 = (0.5)^2$$

$$\left(1 - \frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{\sqrt{2}}\right)^2 = 0.25$$

$$1 - \frac{2a}{\sqrt{2}} + \frac{a^2}{2} + \frac{a^2}{2} = 0.25$$

$$1 - \sqrt{2}a + a^2 = 0.25$$

$$a^2 - \sqrt{2}a + 1 - 0.25 = 0$$

$$\boxed{a^2 - \sqrt{2}a + 0.75 = 0}$$

$$a = \frac{\sqrt{2} \mp \sqrt{2 - 4(0.75)}}{2}$$

$$= \frac{\sqrt{2} \mp j}{2}$$

$$= \frac{1}{\sqrt{2}} \mp \frac{1}{2}j \quad [2]$$

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Consider the following 2nd order digital filter:

$y(n) = ax(n) + bx(n-2), \ a \neq 0, b \neq 0$

[2] 1- Is this filter IIR or FIR? Justify? FIR because it doesn't depend on output

[8] 2- Determine coefficients; a and b such that filter attenuates signal magnitude by half at frequency $\omega = \frac{\pi}{3}$ and its frequency response is normalized so that H(0)=1.

$Y(e^{jw}) = a X(e^{jw}) + b e^{-j2w} X(e^{jw})$

[2] $\frac{Y(e^{jw})}{X(e^{jw})} = H(e^{jw}) = a + b e^{-j2w}$

at $w=0 \Rightarrow H(e^{jw}) = 1 \Rightarrow \boxed{a + b = 1}$ --- ① [2]

$|H(e^{jw})|_{\text{at } \omega = \frac{\pi}{3}} = \frac{1}{2} \Rightarrow |H(e^{j\frac{\pi}{3}})|^2 = \frac{1}{4} \Rightarrow (a + b\cos 2w)^2 + (b \sin 2w)^2 = \frac{1}{4}$

$(a + b\cos\frac{2\pi}{3})^2 + (b\sin\frac{2\pi}{3})^2 = \frac{1}{4}$

[2] $\boxed{(a - \frac{1}{2}b)^2 + \left(\frac{\sqrt{3}b}{2}\right)^2 = \frac{1}{4}}$ — ②

$a^2 - ab + \frac{1}{4}b^2 + \frac{3}{4}b^2 = \frac{1}{4}$

$\boxed{a^2 - ab + b^2 = \frac{1}{4}}$, but $b = 1 - a$

$a^2 - a(1-a) + (1-a)^2 = \frac{1}{4}$

$a^2 - a + a^2 + 1 - 2a + a^2 = \frac{1}{4}$

$3a^2 - 3a + \frac{3}{4} = 0$

$a^2 - a + \frac{1}{4} = 0$

$(a - \frac{1}{2})(a - \frac{1}{2}) = 0 \Rightarrow \boxed{a = \frac{1}{2}}$ [1]

So, $b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$

$\boxed{b = \frac{1}{2}}$ [1]