

Name: \_\_\_\_\_

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When the input to a causal LTI system is

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}2^n u[-n-1],$$

The z-transform on the output is

$$Y(z) = \frac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})}$$

1- Find z-transform of  $x[n]$ ? [2pts]

$$X(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}}, \quad \frac{1}{2} < |z| < 2$$

2- What is the region of convergence ROC of  $Y(z)$ ? [2pts]

$$\text{ROC of } Y(z) = \text{ROC of } X(z) \cap \text{ROC of } H(z) = \frac{1}{2} < |z| < 2 \cap |z| > 1$$

3-is this system stable? Justify? [2pt]

$$= 1 < |z| < 2$$

ROC of  $H(z)$  is  $|z| > 1 \Rightarrow$  Unstable

4- Find impulse response of the system? [4pts]

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})} \Rightarrow \frac{-\frac{1}{3} + \frac{2}{3}z^{-1} + \frac{4}{3} - \frac{2}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$= \frac{(1+z^{-1})(1-\frac{1}{2}z^{-1})}{(1-z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{1-\frac{1}{2}z^{-1}+z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}$$

$$= \frac{1+\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}} = 1 + \frac{2+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})} = 1 + \frac{A_1}{1-z^{-1}} + \frac{A_2}{1+\frac{1}{2}z^{-1}}$$

~~$A_1 = \frac{3}{\frac{1}{2}}$~~   $A_1 = \frac{2}{3}, A_2 = -\frac{2}{3}$

~~$A_2 = 7$~~  So,  $h(n) = \delta(n) + \frac{2}{3}u(n) - \frac{2}{3}\left(\frac{1}{2}\right)^n u(n)$