Quiz #1

Key solution.

Q#1: Check the Periodicity of the following signal

$$x[n] = 2\cos(0.5\pi n) + 4\sin(0.7\pi n) - \cos(0.2\pi n)$$

Ans:

$$x[n] = 2\cos(\underbrace{0.5\pi}_{\omega_1} n) + 4\sin(\underbrace{0.7\pi}_{\omega_2} n) - \cos(\underbrace{0.2\pi}_{\omega_3} n)$$

For ω_1 - $2\pi k_1 = 0.5\pi N_1$

$$\frac{N_1}{k_1} = \frac{2}{0.5} = \frac{20}{5} = \frac{4}{1}$$

$N_1 = 4$ and $k_1 = 1$

For ω_2 - $2\pi k_2 = 0.7\pi N_2$

$$\frac{N_2}{k_2} = \frac{20}{7} \Rightarrow N_2 = 20 \text{ and } k_2 = 7$$

For ω_3 - $2\pi k_3 = 0.2\pi N_3$

$$\frac{N_3}{k_3} = \frac{2}{0.2} = \frac{10}{1}$$

$N_3 = 10$ and $k_3 = 1$

$\Rightarrow N = \text{LCM}(4, 20, 10) = 20$

2: For each of the following systems, determine whether the system (1) stable, (2) causal, (3) linear, (4) time invariant, and (5) Mem[ory]
Justify your answer.

ⓐ $T\{x[n]\} = \sum_{k=n-2}^{n+2} [x[k]+2]$

Ans:

- Stability

$$|T\{x[n]\}| \le \left|\sum_{k=n-2}^{n+2} x[k] + 2\right|$$

$$\le \sum_{k=n-2}^{n+2} |x[k]| + \left|\sum_{k=n-2}^{n+2} 2\right|$$

$$\le 5M + 10 < \infty$$ stable.

- Causality

Since the output depends on the future values of the input $\Rightarrow$ non-causal.

- linearity

$$T\{\alpha_1 x_1[n]\} = \sum_{k=n-2}^{n+2} [\alpha_1 x_1[k] + 2\alpha_1]$$

$$T\{\alpha_2 x_2[n]\} = \sum_{k=n-2}^{n+2} [\alpha_2 x_2[k] + 2\alpha_2]$$

$$T\{\underbrace{\alpha_1 x_1[n] + \alpha_2 x_2[n]}\} = \sum_{k=n-2}^{n+2} (\underbrace{\alpha_1 x_1[k] + \alpha_2 x_2[k]}) + \sum_{k=n-2}^{n+2} (2\alpha_1 + 2\alpha_2)$$

$$T\{\alpha_3 x_3[n]\} = \sum_{k=n-2}^{n+2} (\alpha_3 x_3[k]) + \sum_{k=n-2}^{n+2} 2\alpha_3$$

Since $\alpha_3 \neq \alpha_1 + \alpha_2 \Rightarrow$ non-linear.

- Time-invariant or time-variant.

- Delay of the input sequence.

$$T\{x[n-n_0]\} = \sum_{k=n-2}^{n+2} [x[k-n_0]+2]$$

$$= \sum_{k=n-2}^{n+2} x[k-n_0] + \sum_{k=n-2}^{n+2} 2$$

$$= \sum_{k=n-2}^{n+2} x[k-n_0] + 10$$

let $u = k - n_0$

when $k = n-2 \Rightarrow u = n-2-n_0$

when $k = n+2 \Rightarrow u = n+2-n_0$

$$\Rightarrow \quad T\{x[n-n_0]\} = \sum_{u=n-n_0-2}^{n-n_0+2} x[u] + 10$$

$$\equiv$$

$$y[n-n_0] \equiv \sum_{k=n-n_0-2}^{n-n_0+2} x[k] + 10 \qquad [\text{time-shift}]$$

$\Rightarrow$ time-invariant.

Memory & Memoryless:

Since the system depends on future & past n^{th} values of input

$\Rightarrow$ memory.