Problem #1: $y[n] - 6y[n-1] + 8y[n-2] = 2x[n-1]$

$$Y(e^{j\omega}) - 6Y(e^{j\omega})e^{-j\omega} + 8Y(e^{j\omega})e^{-j2\omega} = 2X(e^{j\omega})e^{-j\omega}$$

$$[1 - 6e^{-j\omega} + 8e^{-j2\omega}]Y(e^{j\omega}) = 2e^{-j\omega}X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{2e^{-j\omega}}{1 - 6e^{-j\omega} + 8e^{-j2\omega}}$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$= \frac{2e^{-j\omega}}{1-6e^{-j\omega}+8e^{-j2\omega}} \cdot \frac{1}{1-e^{-j\omega}}$$

$$= \frac{2e^{-j\omega}}{(1-2e^{-j\omega})(1-4e^{-j\omega})(1-e^{-j\omega})}$$

$$= \frac{A}{1-2e^{-j\omega}} + \frac{B}{1-4e^{-j\omega}} + \frac{C}{1-e^{-j\omega}}$$

$$= A(1-4e^{-j\omega})(1-e^{-j\omega}) + B(1-2e^{-j\omega})(1-e^{-j\omega}) + C(1-2e^{-j\omega})(1-$$

when $e^{-j\omega} = 1/2$

$$1 = A(1-2)(1-\tfrac{1}{2}) \Rightarrow 1 = A(-1)(\tfrac{1}{2}) \Rightarrow \boxed{A = -2}$$

when $e^{-j\omega} = \frac{1}{4}$

$$\frac{1}{2} = A(0) + B(1-\tfrac{1}{2})(1-\tfrac{1}{4}) + C(0)$$

$$\frac{1}{2} = B(\tfrac{1}{2})(\tfrac{3}{4}) \Rightarrow \boxed{B = 4/3}$$

when $e^{-j\omega} = 1$

$$2 = A(0) + B(0) + C(1-2)(1-4)$$

$$2 = C(-1)(-3)$$

$$\Rightarrow C = 2/3$$

$$y[n] = -2(2)^n u[n] + (\tfrac{4}{3})4^n u[n] + \tfrac{2}{3}u[n]$$

$h[n] = (h_1[n] + h_2[n]) * h_3[n]$

$= h_1[n] * h_3[n] + h_2[n] * h_3[n]$

$= \delta[n] * a^n u[n] + \delta[n-1] * a^n u[n]$

$= a^n u[n] + a^{n-1} u[n-1]$

$$H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} + \frac{e^{-j\omega}}{1-ae^{-j\omega}}$$

$$= \frac{1+e^{-j\omega}}{1-ae^{-j\omega}}$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1+e^{-j\omega}}{1-ae^{-j\omega}}$$

$$(1-ae^{-j\omega})Y(e^{j\omega}) = (1+e^{-j\omega})X(e^{j\omega})$$

$$y[n] - a y[n-1] = x[n] + x[n-1]$$

Since we study the impulse response $h[n] = 0$ for $n < 0$

$\Rightarrow$ the system is causal

since

$y[n] = x[n] + x[n-1] + a y[n-1]$

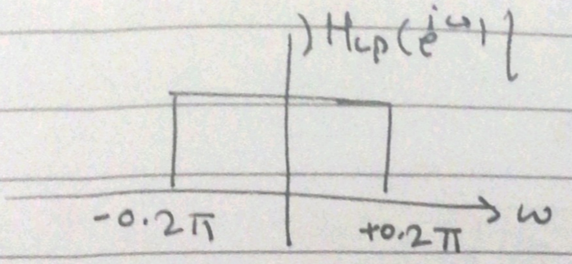
depends on the present and previous n-values at input.

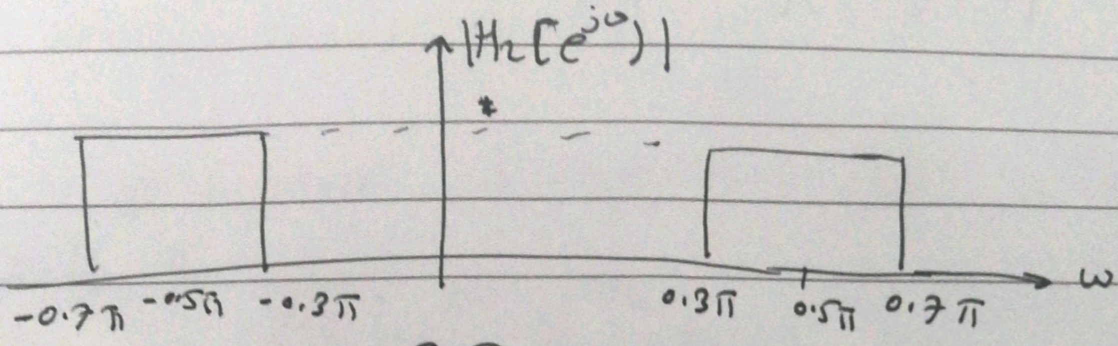
$\Rightarrow$ the system will be stable if $|a| < 1$

since we have to check $\sum_{n=0}^{\infty} |h[n]| < \infty$ Bound input - Bound output.

Q#3: $h_2[n] = 2h_{Lp}[n]\cos(0.5\pi n)$

$h_1[n]$





B.P.F

$$H_2(e^{j\omega}) = H_{Lp}(e^{j\omega}) * \left(\delta(\omega-0.5\pi) + \delta(\omega+0.5\pi)\right)$$

$$= H_{Lp}(e^{j\omega}) * \delta(\omega-0.5\pi) + H_{Lp}(e^{j\omega}) * \delta(\omega+0.5\pi)$$

$$= H_{Lp}(e^{j(\omega-0.5\pi)}) + H_{Lp}(e^{j(\omega+0.5\pi)})$$