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a) A linear time-invariant system is described by the following difference equation:

$$y[n] = x[n] - \frac{1}{2}x[n-1]$$

(i) Determine $h[n]$, the impulse response of the system.

(ii) Is this a stable system?

(iii) Find the system response, $y[n]$, of the following input signal: $x[n] = \left(\frac{1}{2}\right)^n u[n]$

(i) $x(u) = \delta(u) \Rightarrow y(u) = \delta(u) - \frac{1}{2}\delta(u-1) = h(u)$

$h(u) = [1, -0.5]$

(ii) stable, BIBO.

when $\lim_{n \rightarrow \infty} x[n] < K < \infty$, $\lim_{n \rightarrow \infty} y[n] < K < \infty$
 It doesn't depend on past output (No feedback)

(iii) $y(u) = x(u) * h(u) = \left[\left(\frac{1}{2}\right)^u u(u)\right] [\delta(u) - \frac{1}{2}\delta(u-1)]$

$= \left(\frac{1}{2}\right)^u u(u) - \frac{1}{2}\left(\frac{1}{2}\right)^{u-1} u(u-1) = \left(\frac{1}{2}\right)^u u(u) - \left(\frac{1}{2}\right)^u u(u-1)$
 $= \left(\frac{1}{2}\right)^u [u(u) - u(u-1)] = \left(\frac{1}{2}\right)^u \delta(u) = \delta(u)$

b) Determine if the following discrete-time signal $x(n) = \sin\left(\frac{15}{36}\pi n + \frac{\pi}{4}\right)$ is periodic or non-periodic? If periodic, find its fundamental period? [4pts]

$\omega = \frac{15}{36}\pi$

$x(u) = \cos\left(\omega u - \frac{\pi}{4}\right)$, $\omega = \frac{15}{36}\pi$

$\omega_0 N = 2\pi K \Rightarrow N = \frac{2\pi K}{\frac{15}{36}\pi} = \frac{24}{15}K$

$K=5 \Rightarrow$ the period of $N = 24 \Rightarrow$ fundamental period = 24

\hookrightarrow Periodic.